NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Non-isomorphic graphs

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Look at this two graphs,
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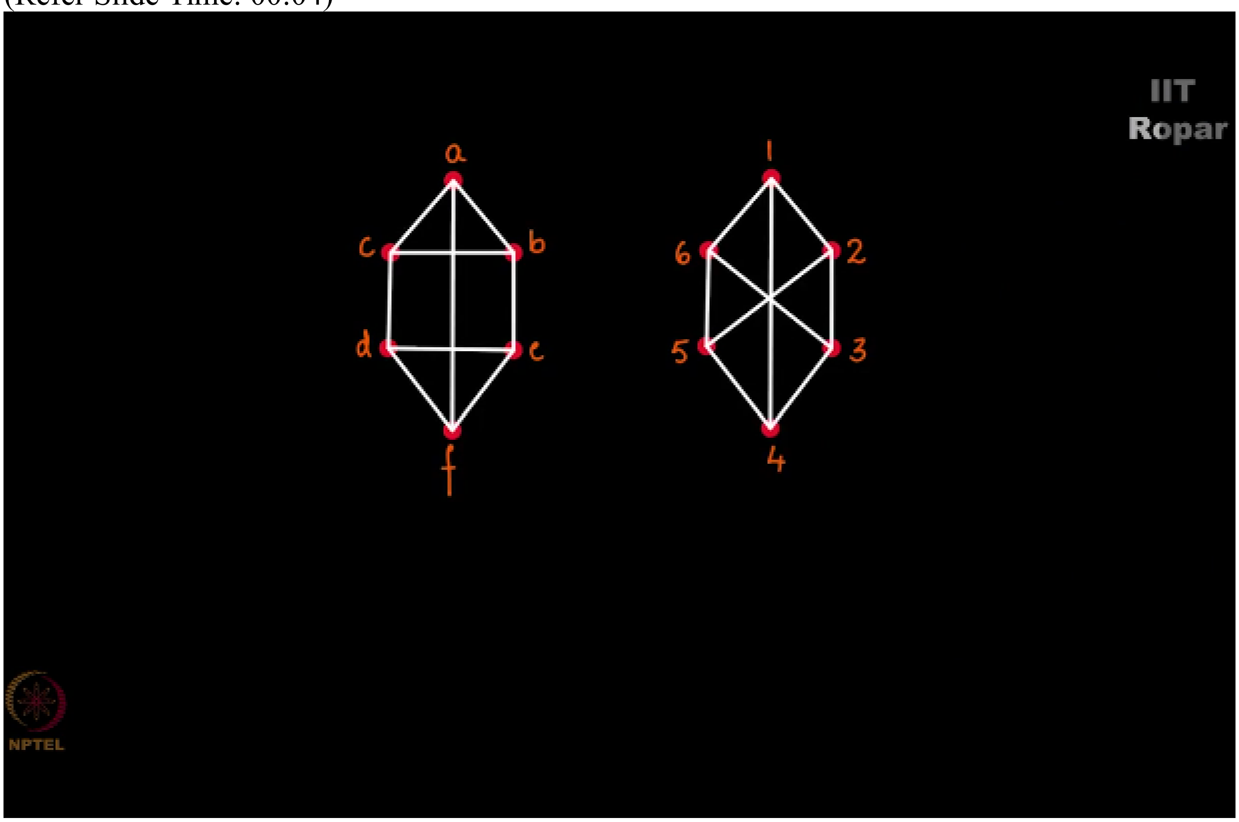

let me take a look at the number of vertices here, you have 6 vertices here, 1, 2, 3, 4, 5, 6, you have 6 vertices on this side as well,
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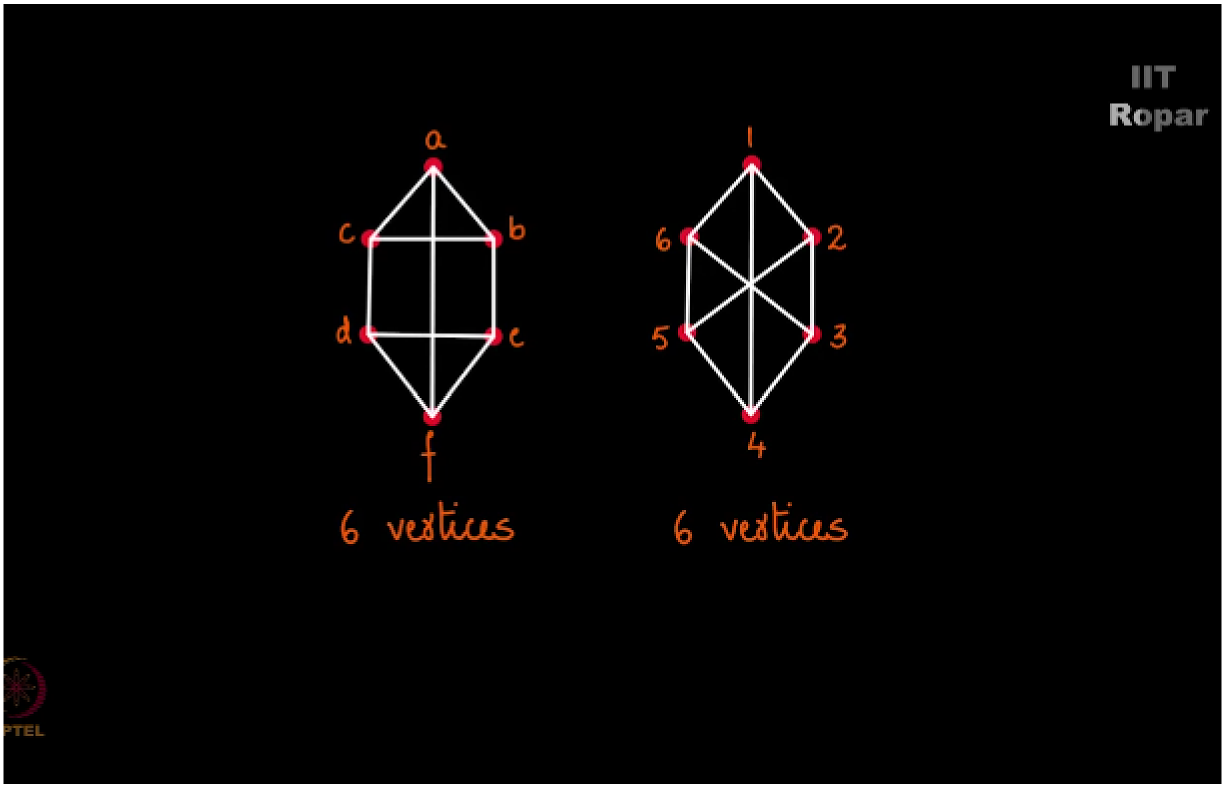

and looks like they are very similar, but let me just check the degree here, the degree is 3, 3, 3, 3, 3, 3 for all the nodes in G, and 3, 3, 3, 3, 3, 3 for all the nodes in H,

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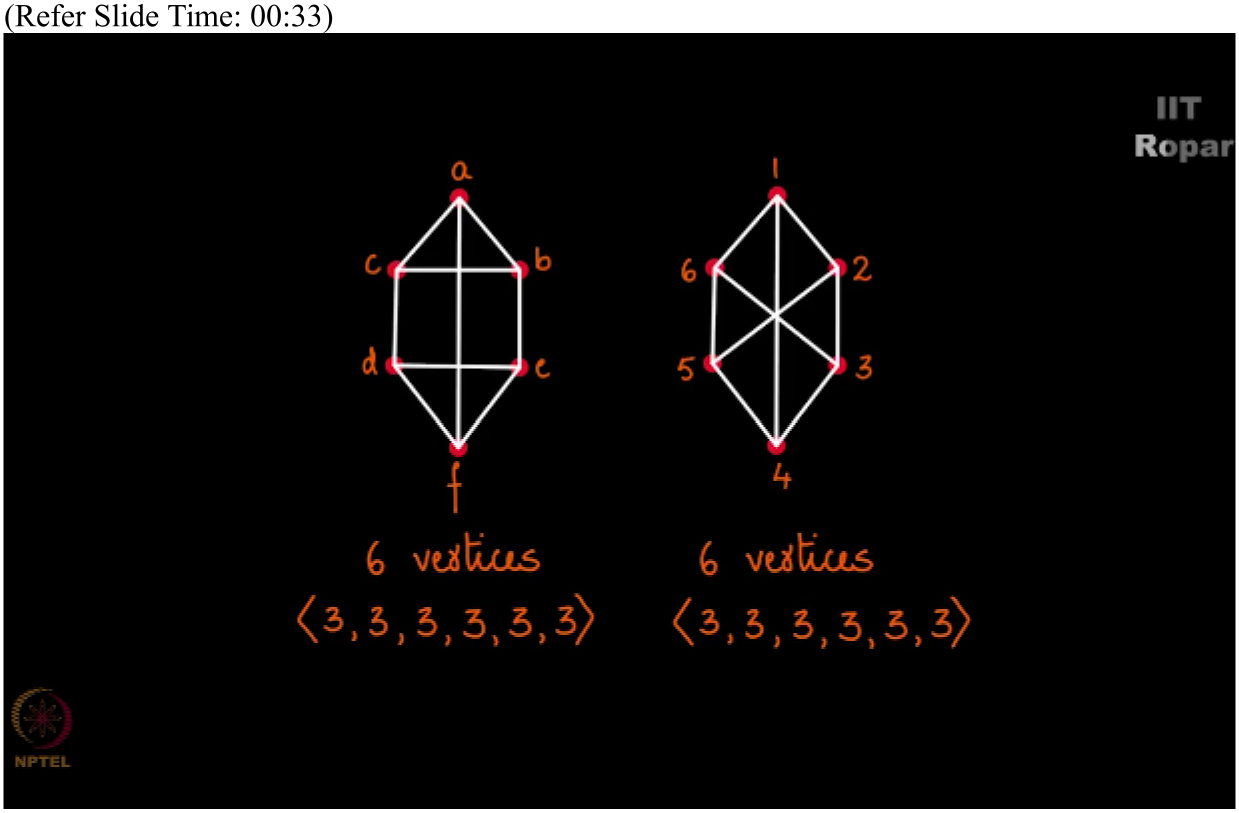

so the total number of vertices here is 6, the total number of edges here happens to be if I count 1, 2, 3, 4, 5, 6, 7, 8, 9, both have the same number of vertices, both have the same number of edges and both of them do have the same number of same type of degree sequence as well 3, 3, 3, 3, 3, 3,

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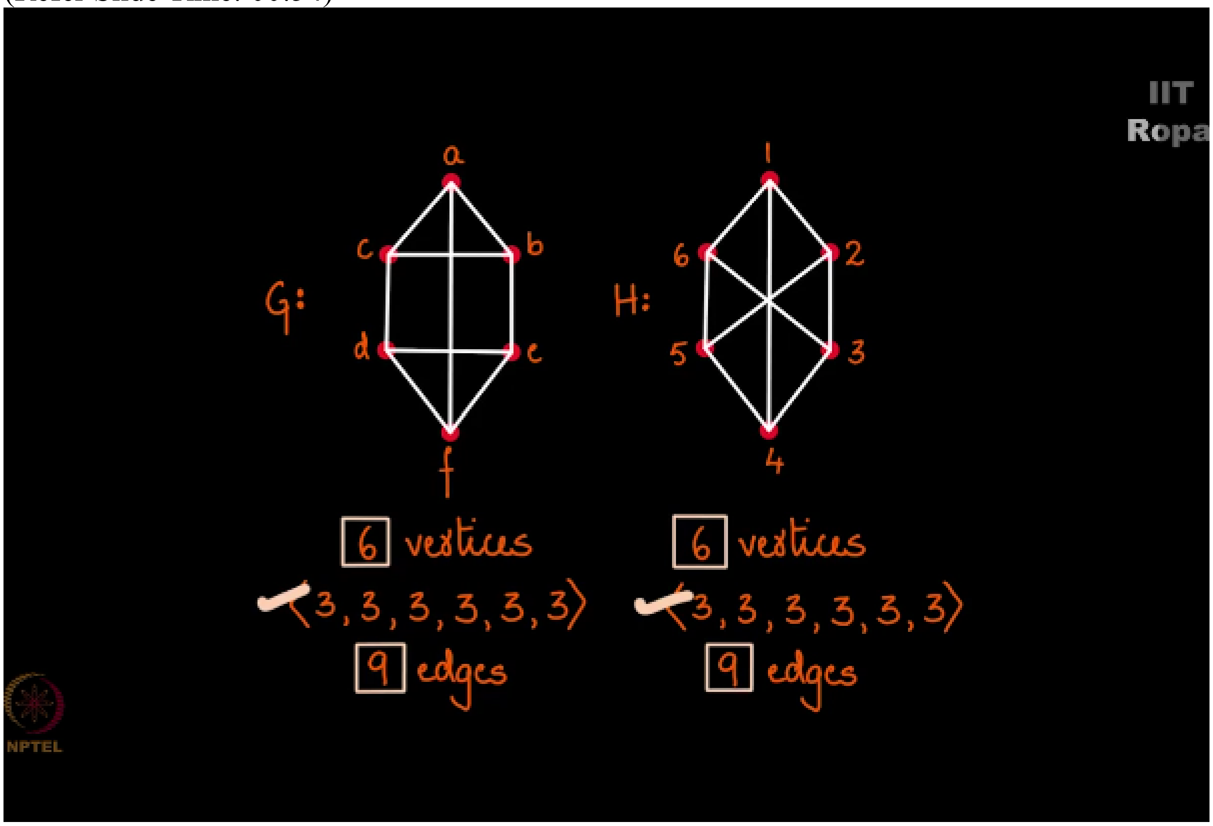

but my question would be are these two graphs isomorphic?

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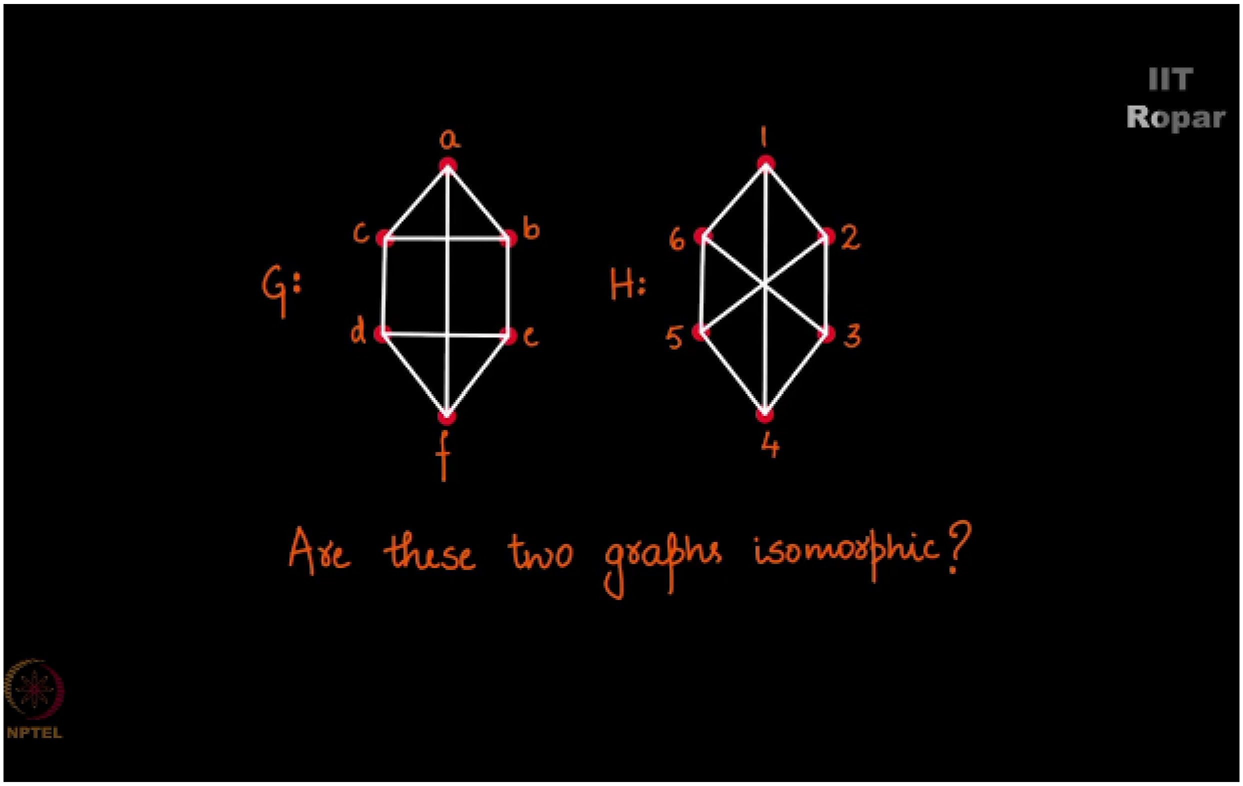

Think about it, you may want to pause the video here and then think.

Okay, the answer is no, they're not isomorphic, why? Do you see a triangle here A, B, C in the graph G that triangle is missing in H,

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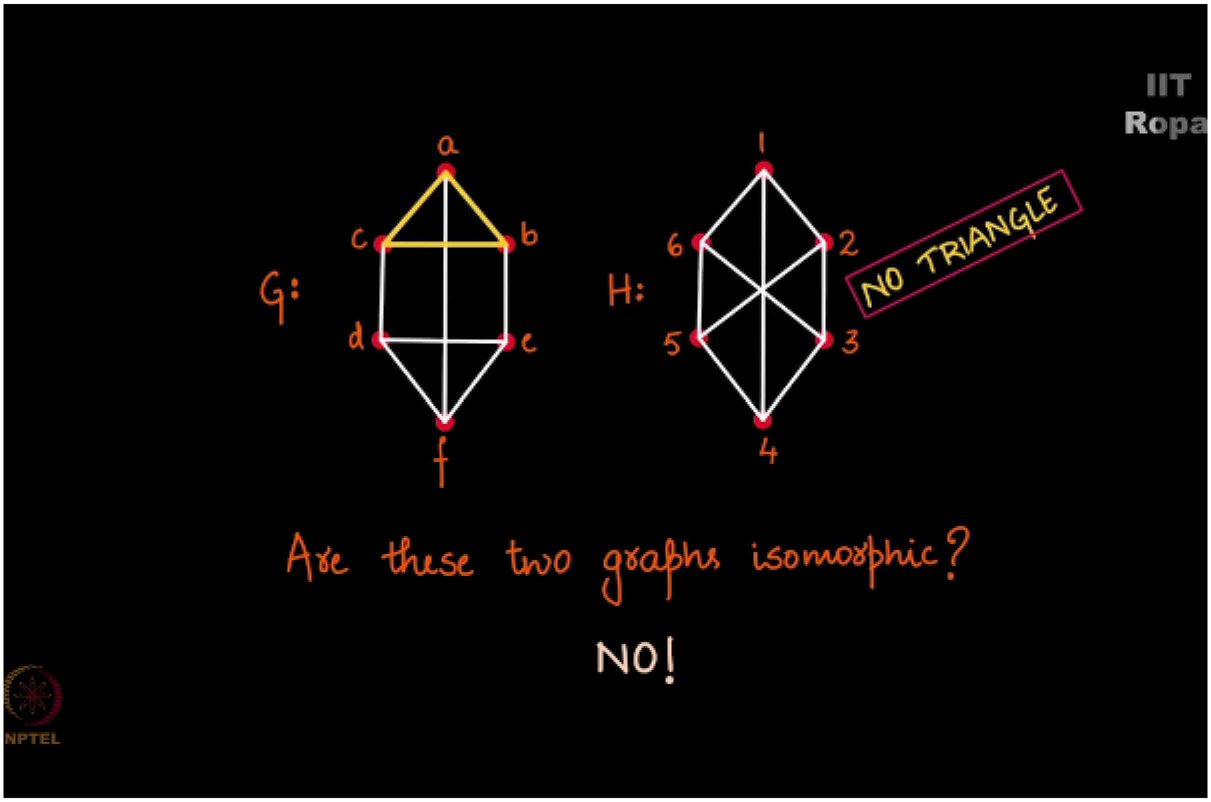

of course the labeling here is 1, 2, 3, 4, 5, 6, you will not be able to see A, B, C here, the point is there is a triangle here, but there isn't a triangle this side, correct, and hence the graphs are not isomorphic, why? They appear the same, but there are some structural differences in them,

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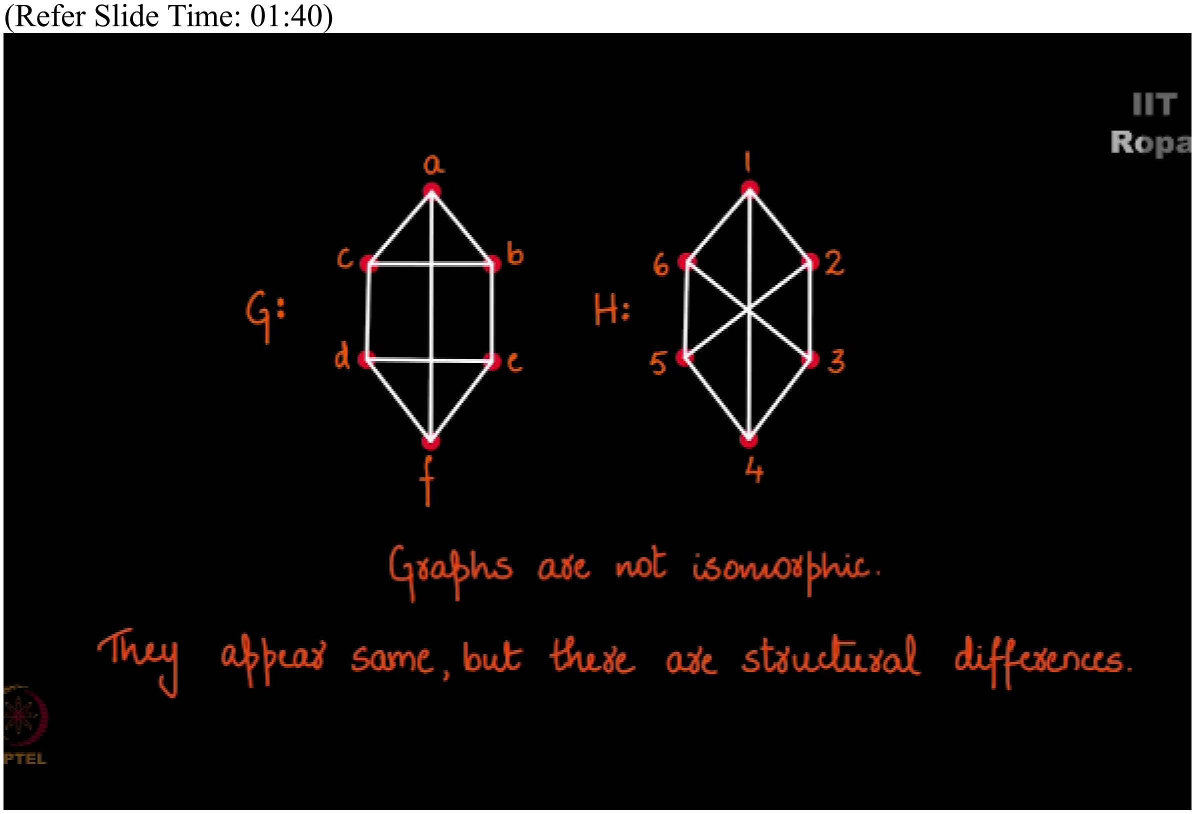

there is no way I can modify G and get H, by modifying I mean I will not disturb the adjacencies, but I'll just push and pull around the graph and make it appear like H, come what may this will not happen because a triangle is present here, but a triangle is not present here.

So the definition of isomorphism is seems to be a very wage one you see, if somehow you can modify a graph and make it appear like something else, then this two graphs are isomorphic is not a very mathematical way of statement.
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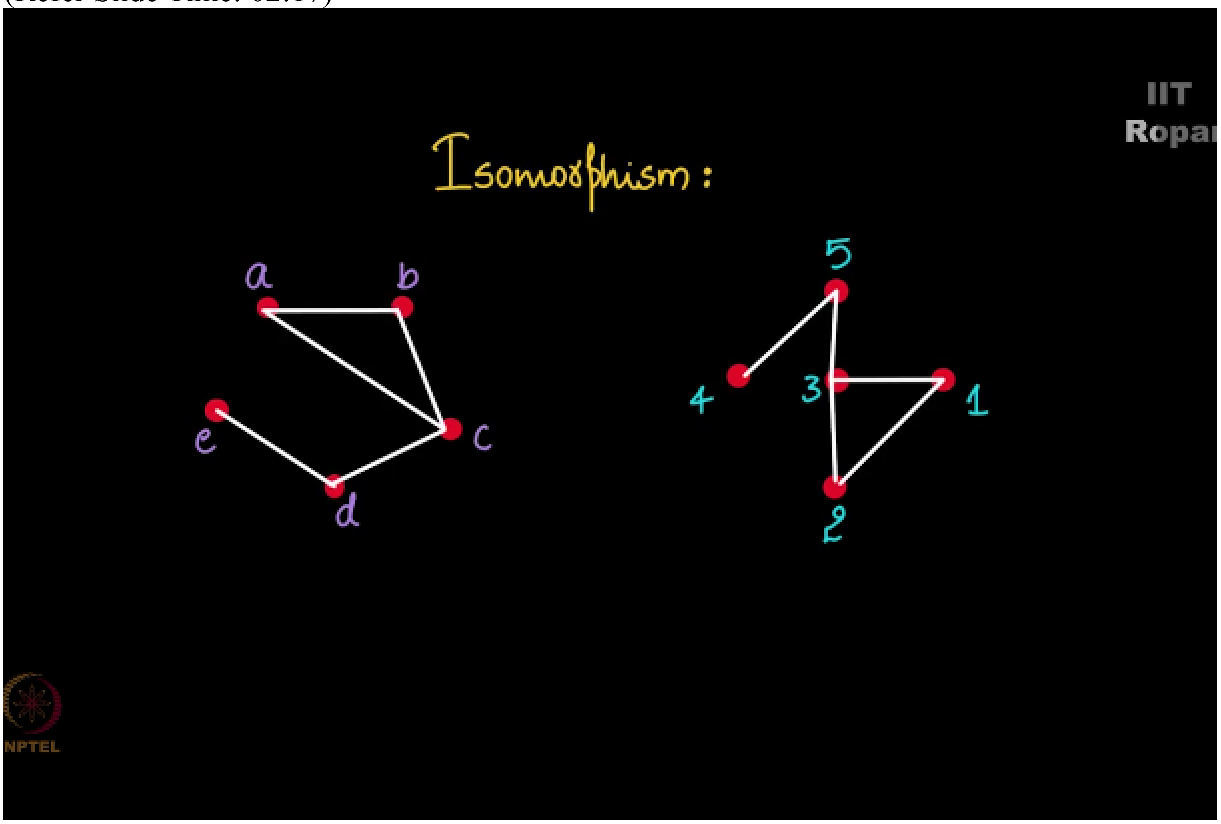

Let us see what is the mathematical method? The second graph is exactly the same as the first graph, but the labels are changed, think about it, the labels are changed, that's it otherwise the graph is the same,
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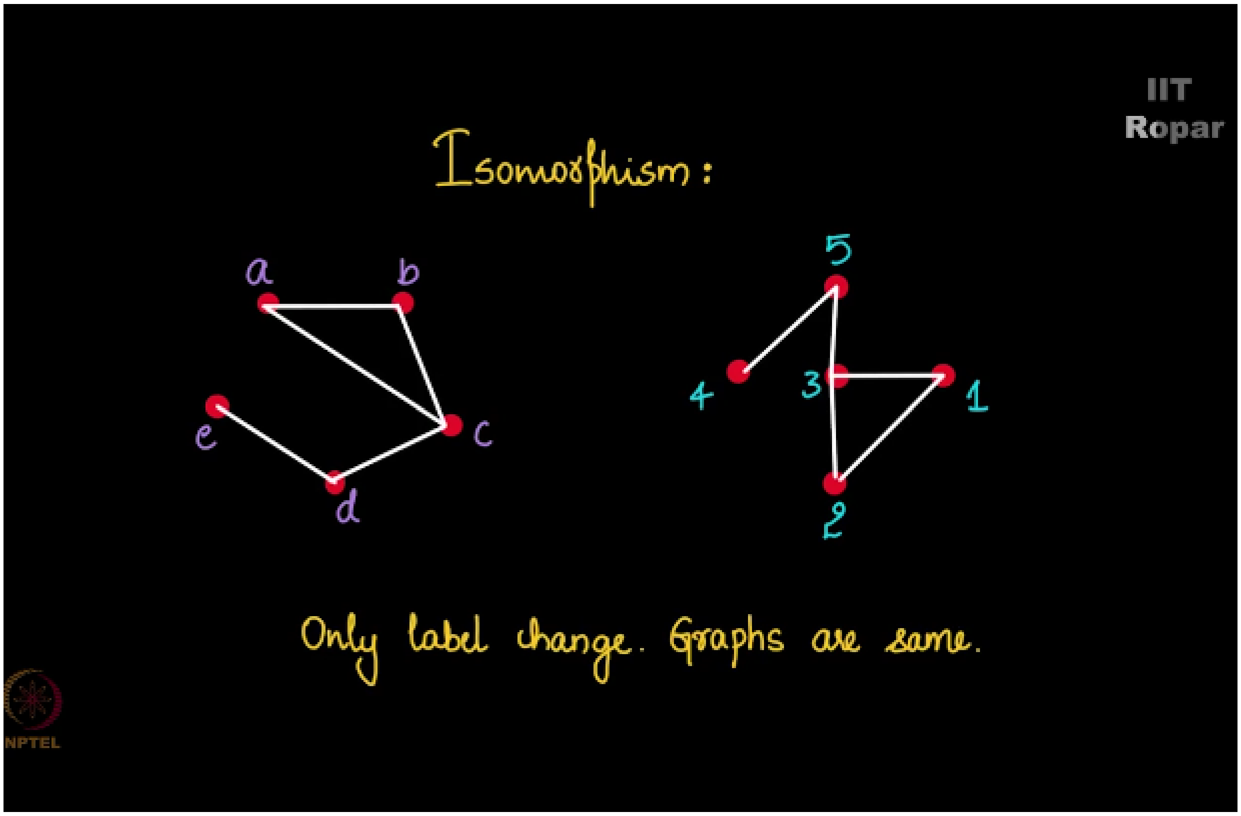

so if there is Rama and Deepak here on this side, that Rama and Deepak will become Peter and Mike on that side, if they're friends here these two will also be friends,

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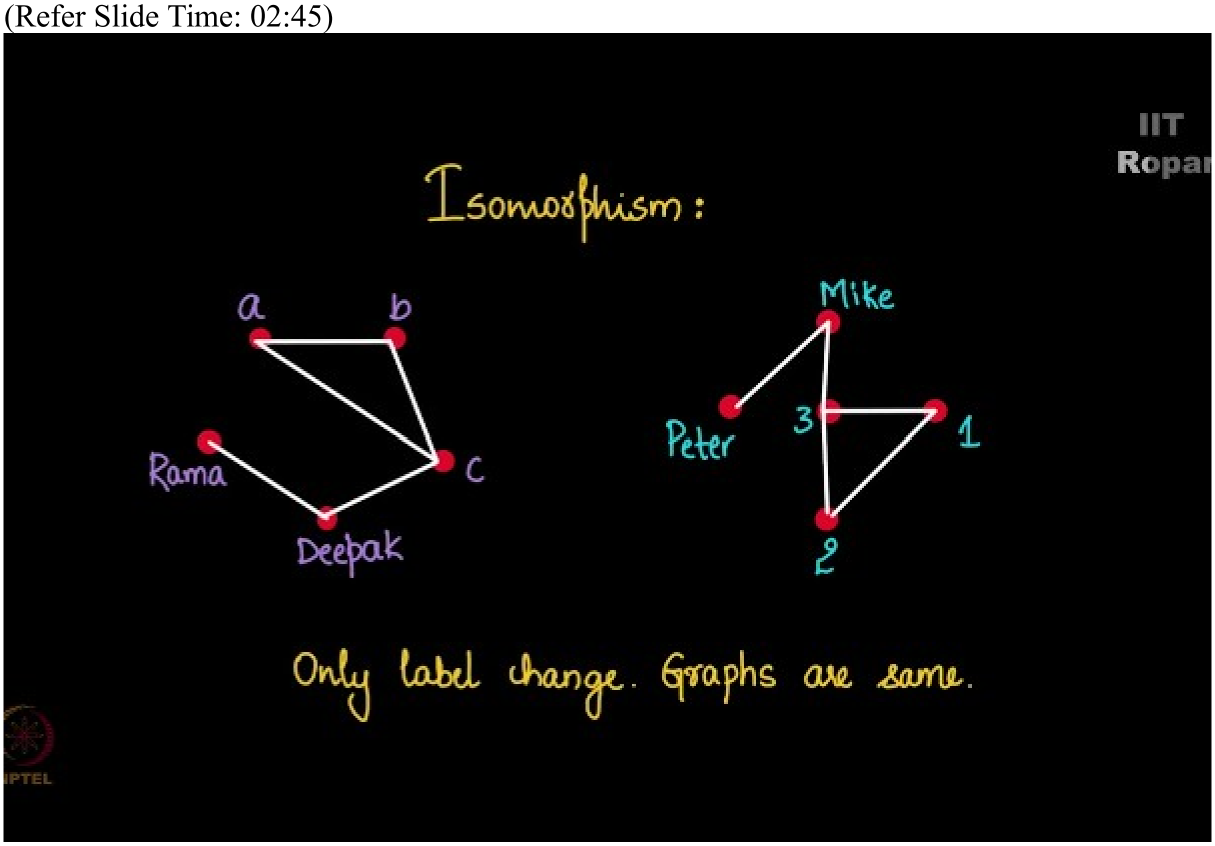

you morph the nodes to make it become someone else, something else but then in the morph world if they were friends this side they will be friends this side as well, that's what we mean by isomorphism, right.

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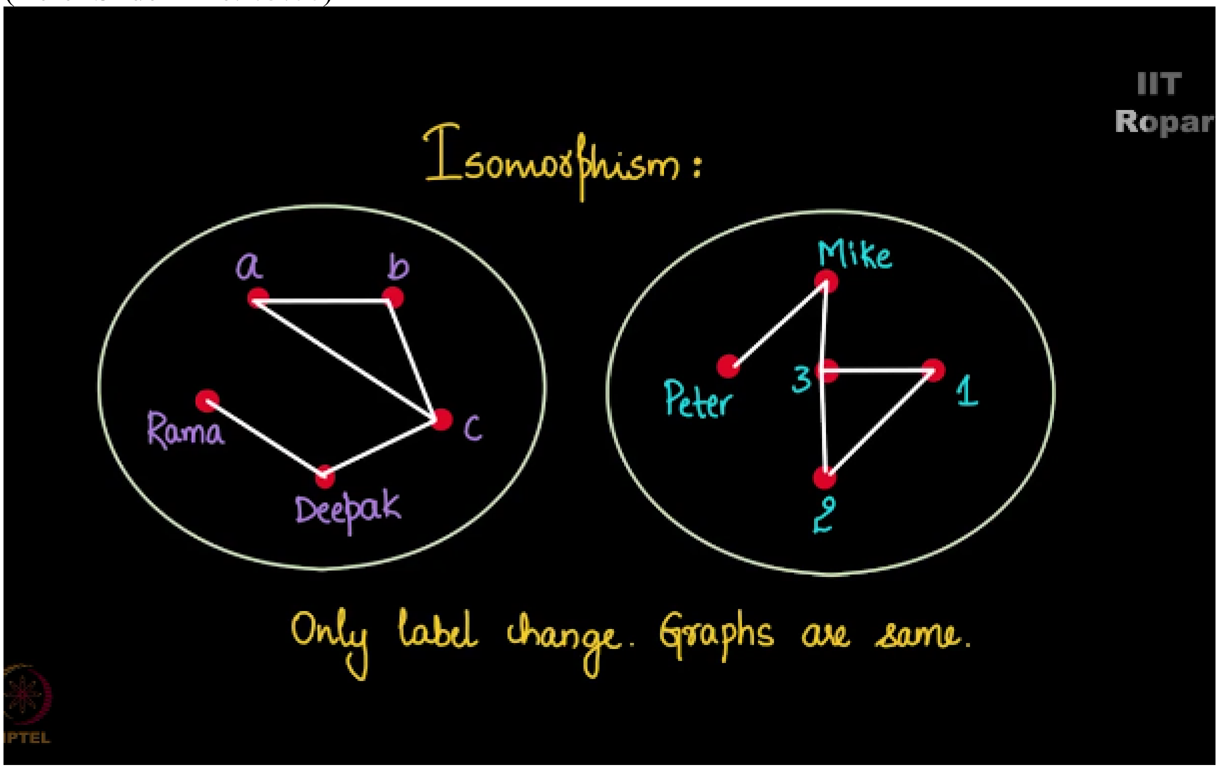

So mathematically speaking, given a graph G and another graph let's say G dash, the vertices of G is let's say V, and edges of G is denoted by E, vertices of G dash is denoted by V dash, and edges of G dash is denoted by E dash.

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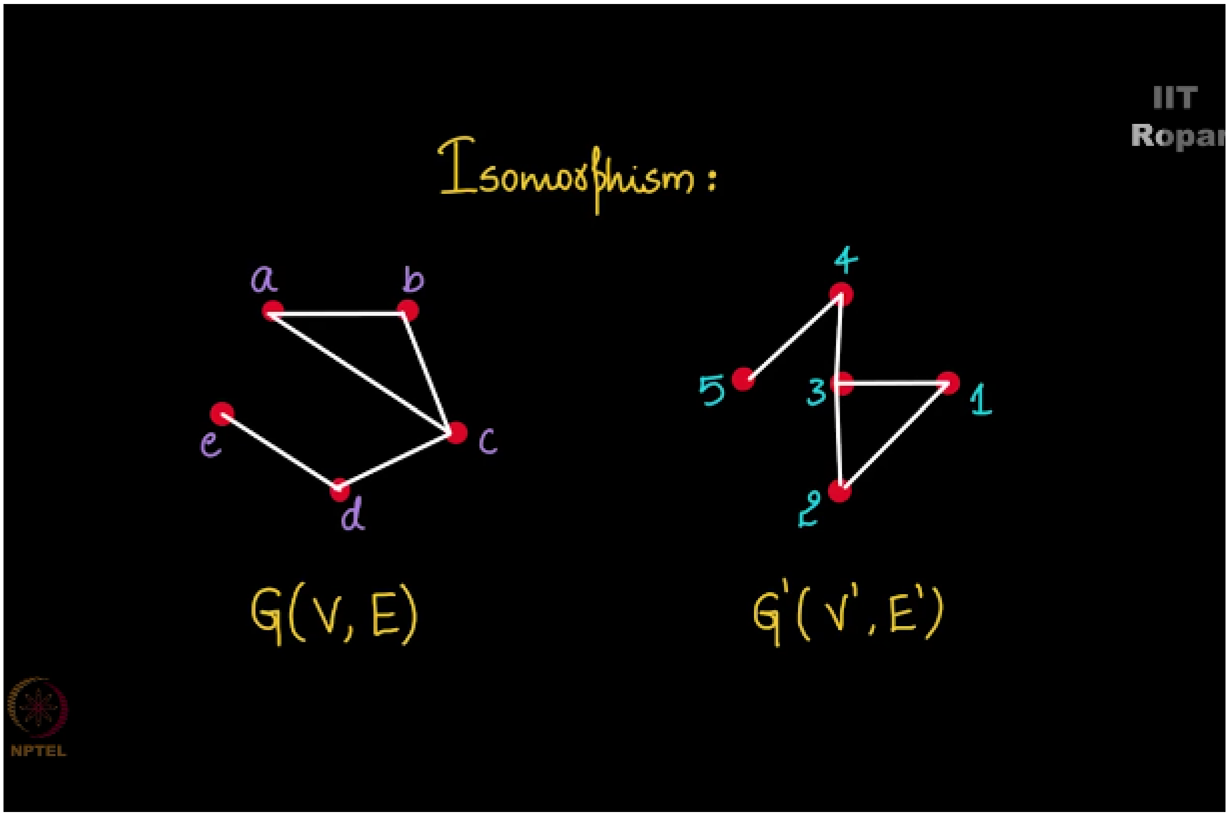

So V and V dash obviously are the same cardinality, isomorphism is basically a bijection of 1, 1,
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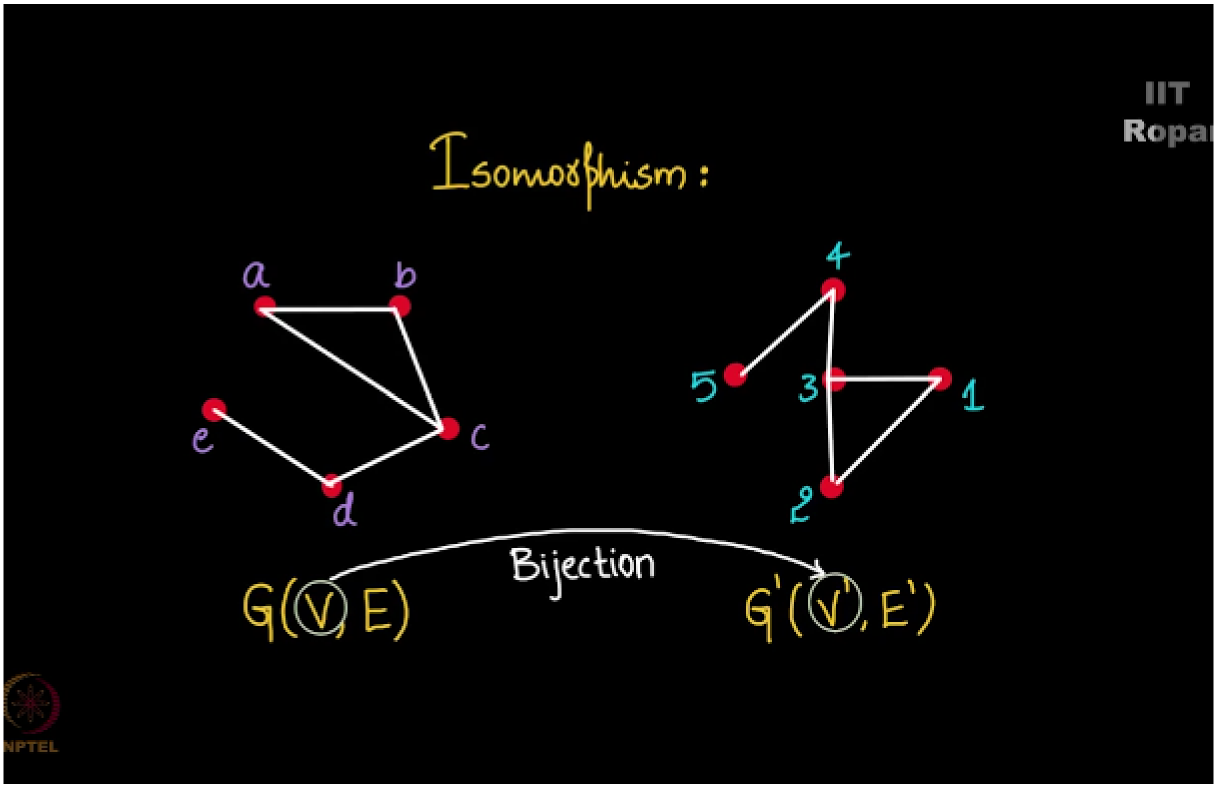

and on to function from V to V dash, why do I say that I want, all the nodes here to be mapped to all the nodes here and it should just be renaming of the nodes, so it should obviously be a bijection, correct, think about it, this part I'm not explaining much, it's actually commonsensical, think about it.

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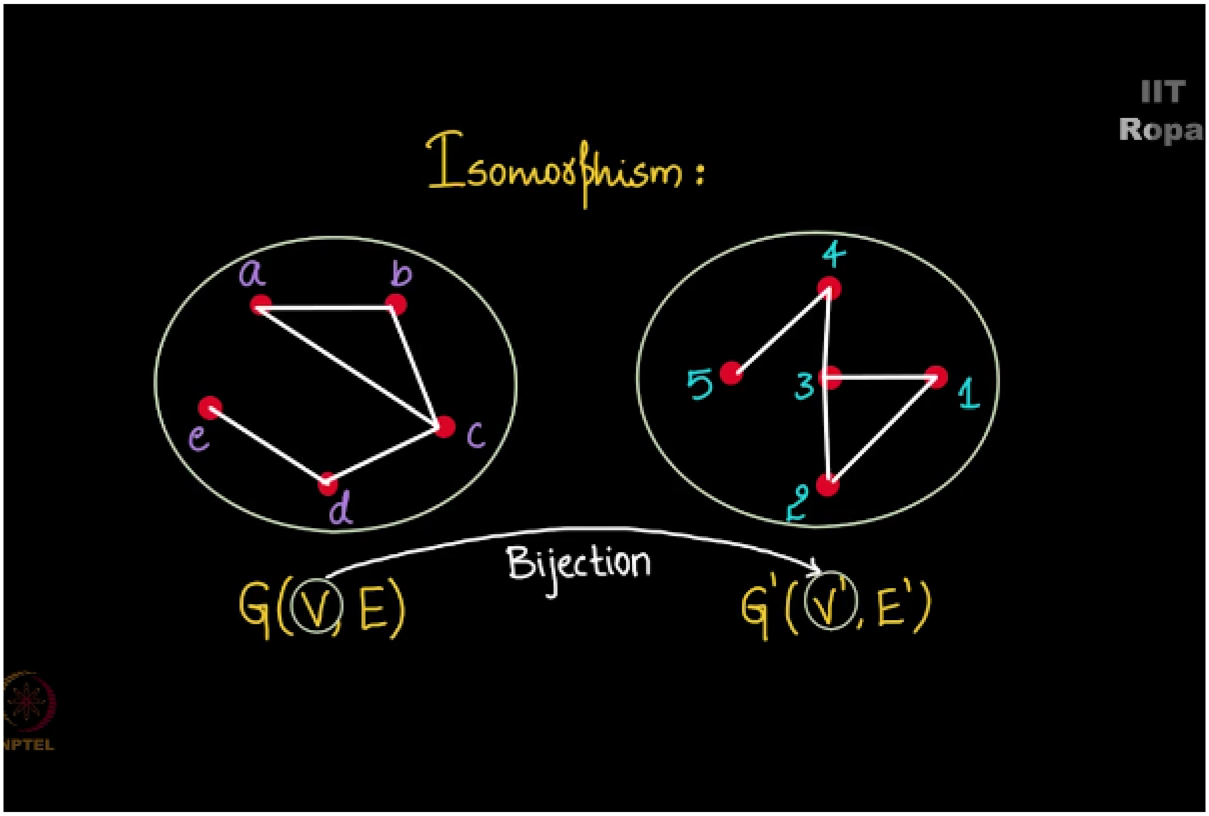

So a bijection from V to V dash which preserves the edge, this is the place where we get confused, where did the world preserve come here, what does it even mean, it's very simple,
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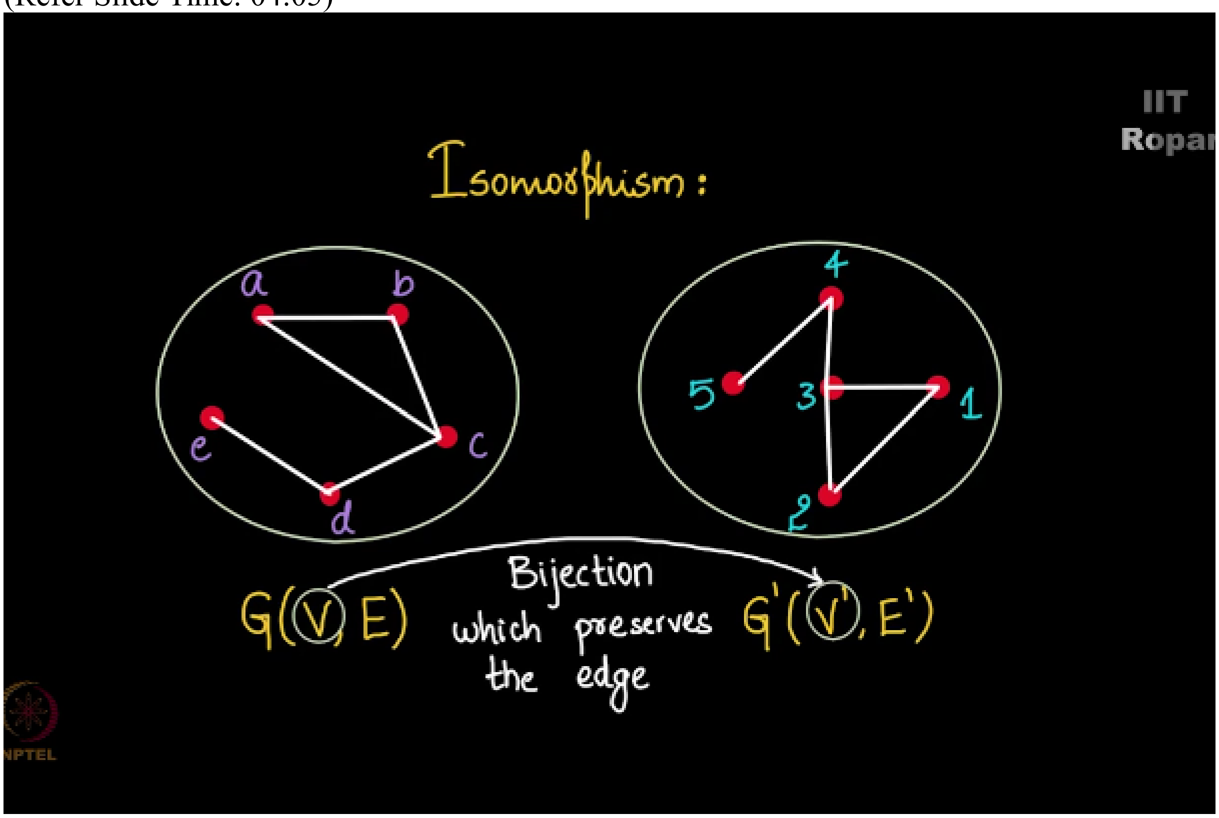

all that we are trying to say is if A and B are friends, look at this example, if A and B are friends then the mapping whatever it is A is going to 1, and B is going to 2, 1and 2 will be friends, that's what we mean by the word preserve,

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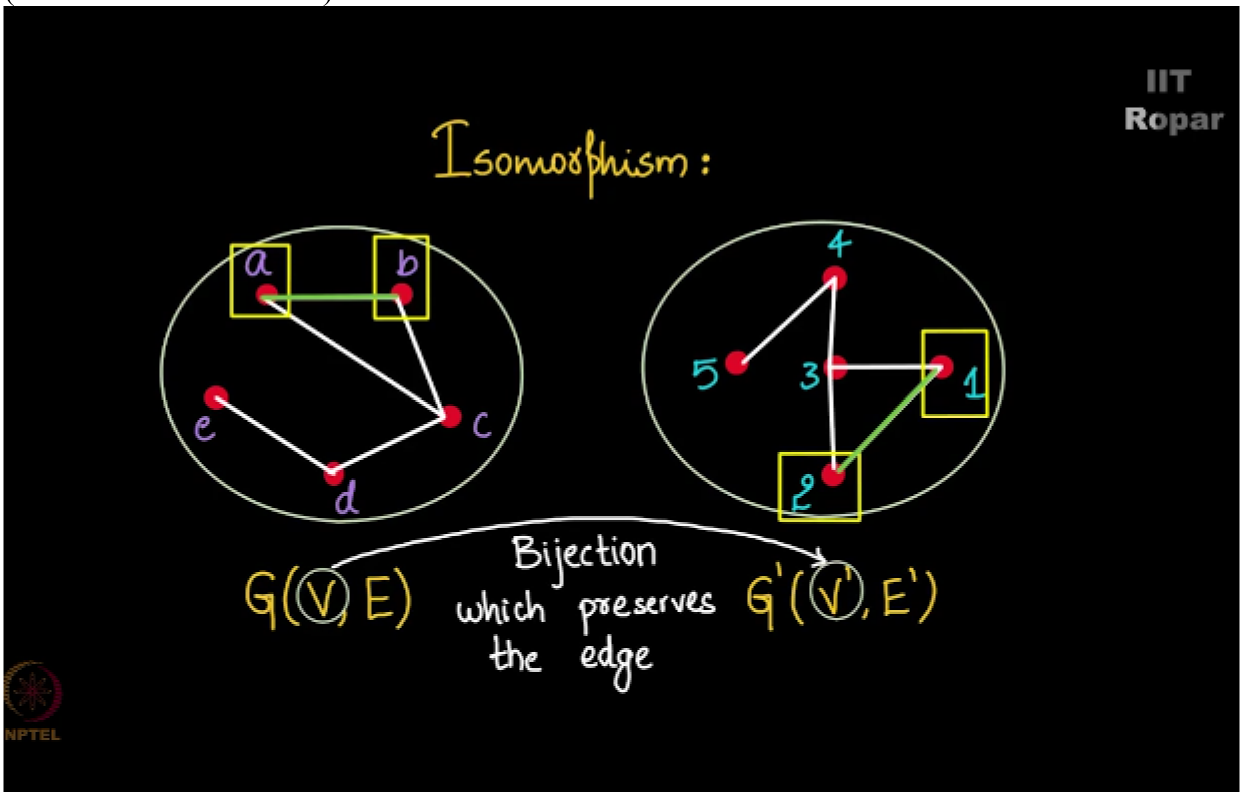

so let me rewind back and then restate the definition, isomorphism between 2 graphs is nothing but a function F which is a bijection from V to V dash which preserves the adjacency, if A and B are friends here, F(a) and F(b), namely 1 and 2 will be friends here, C and D are friends here, then F(c) and F(d), 3 and 4 will be friends here,

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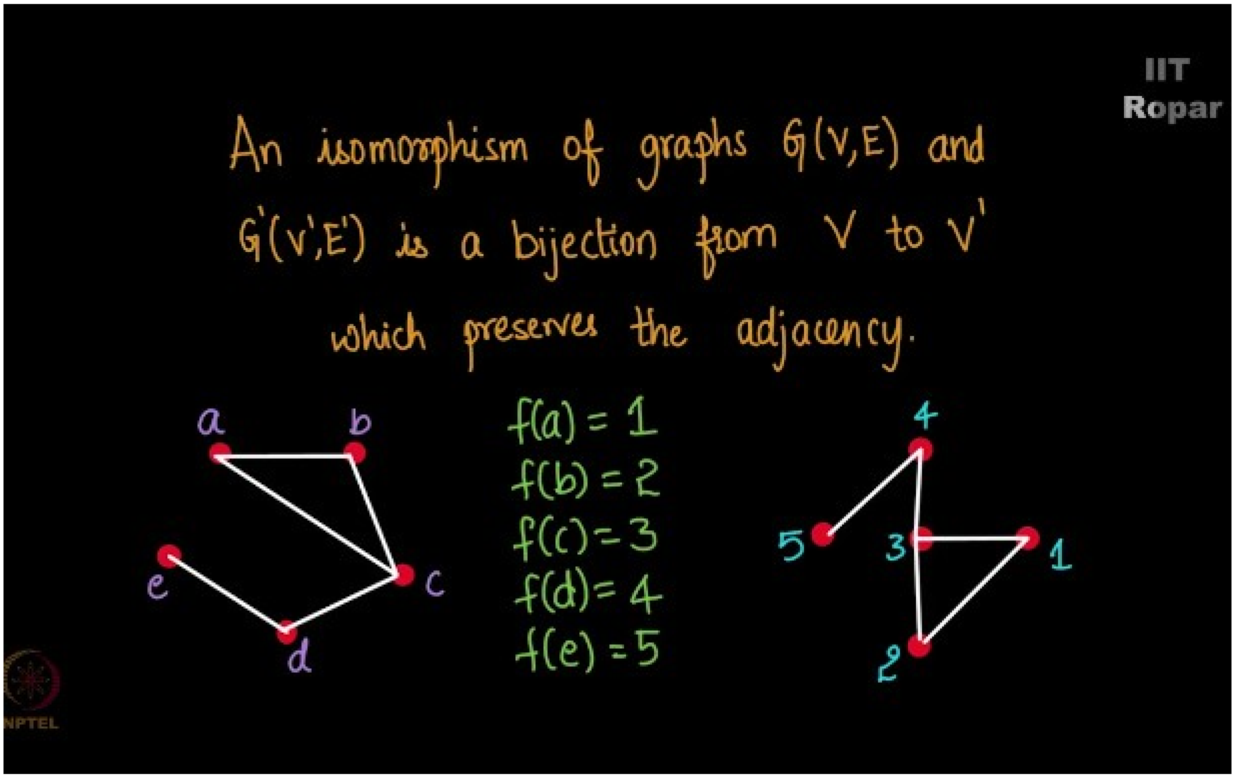

I'm sure things are very clear to you, if not please watch this video once more, otherwise you will not be able to proceed further.

Now look at this graph, here is a graph with 4 nodes
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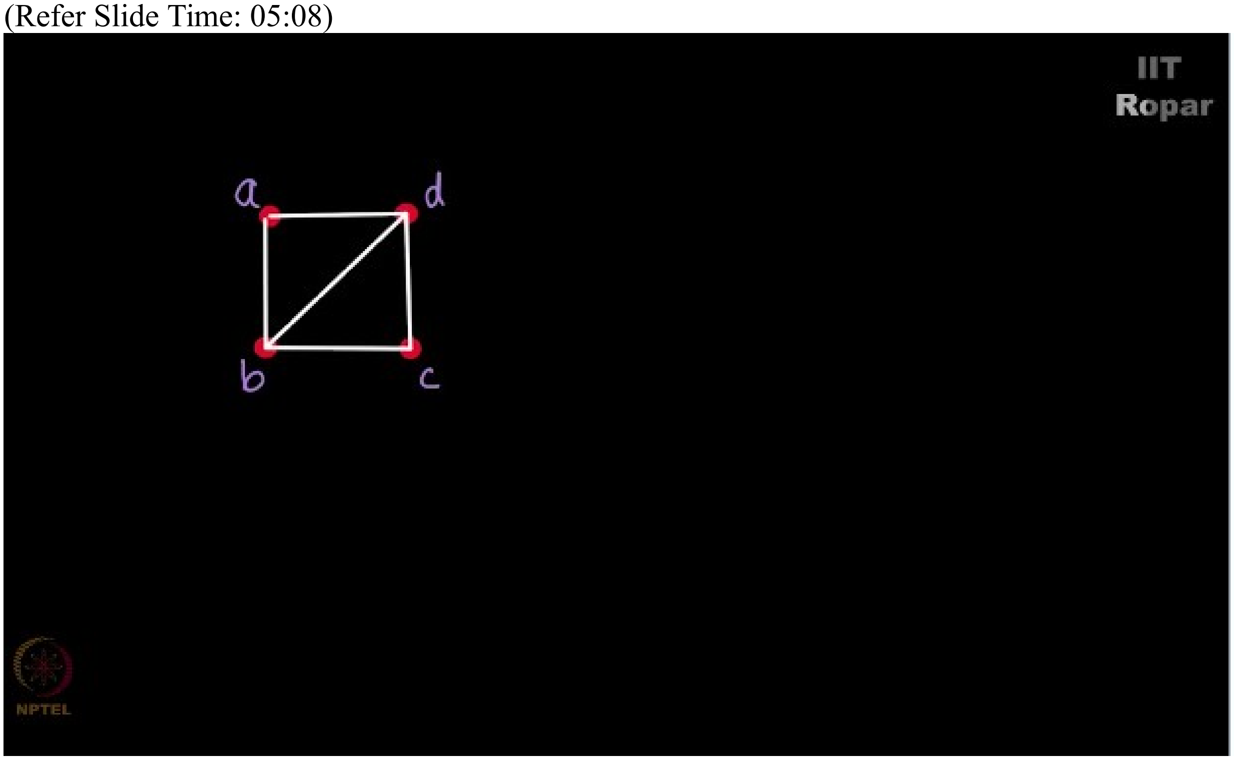

but with a diagonal, look at this another graph, right, 1, 2, 3, and 4, do you think these two are isomorphic,
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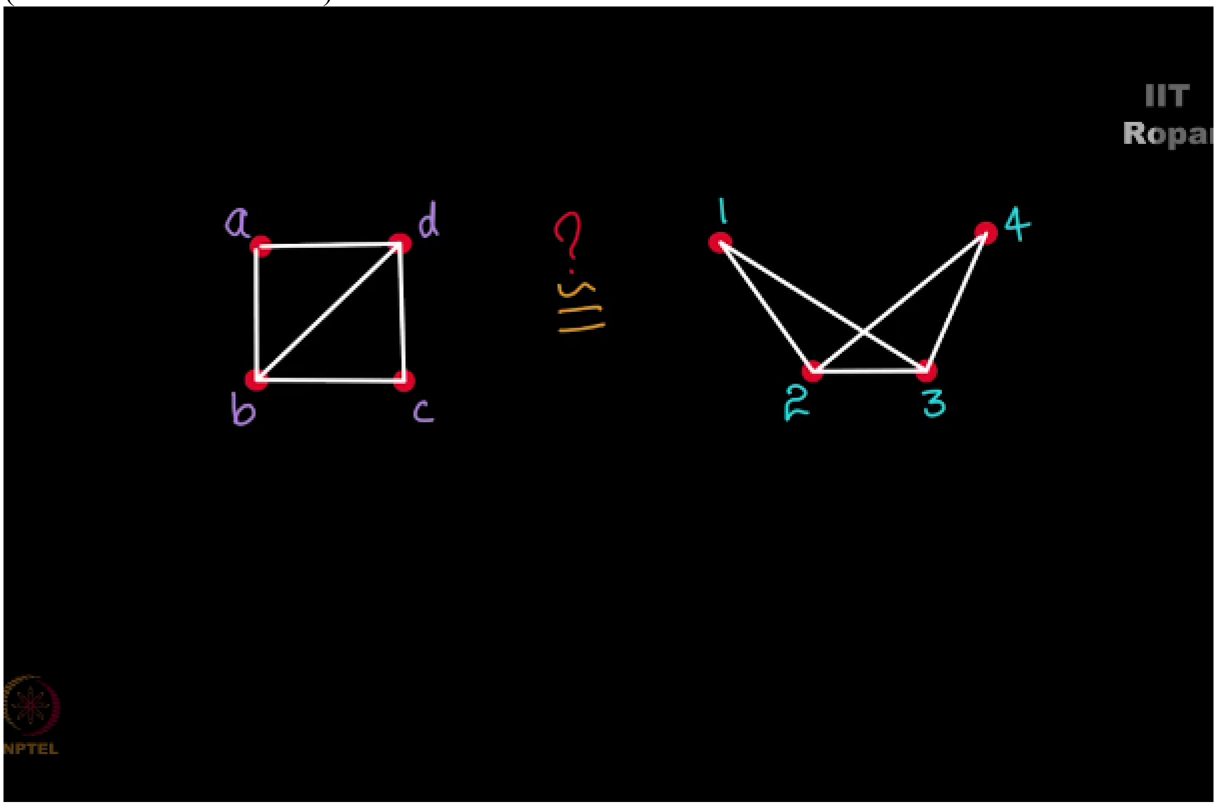

whenever you are asked to show something is isomorphic all that you need to do is exhibit a bijection from the vertex set here and a vertex set here, look at this boom,
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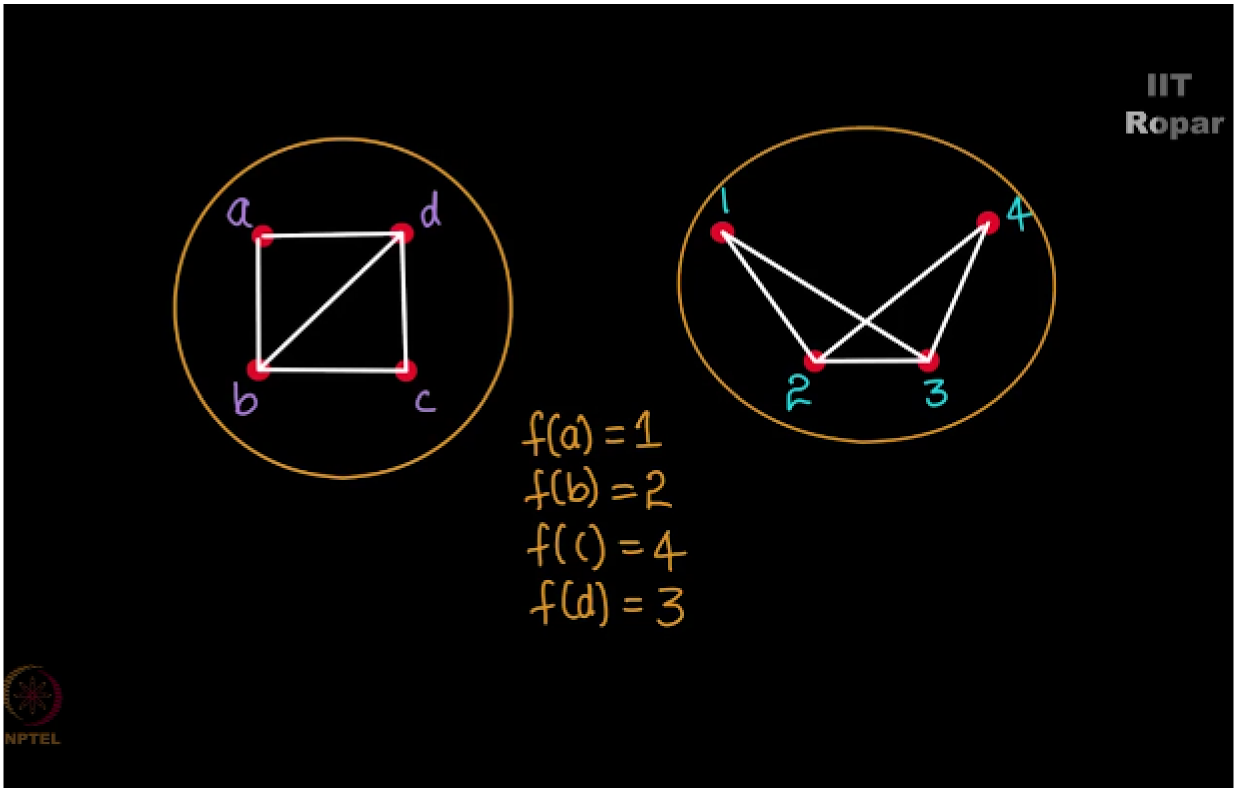

this is a bijection and it does preserve adjacencies whenever 2 nodes here are adjacent the corresponding 2 nodes on this side is also adjacent and hence these two graphs are indeed isomorphic, now you see, look at this second graph you basically can twist and turn the second graph and make it become the first graph you see, here is how you do it, right, this is another way to quickly intuitively see

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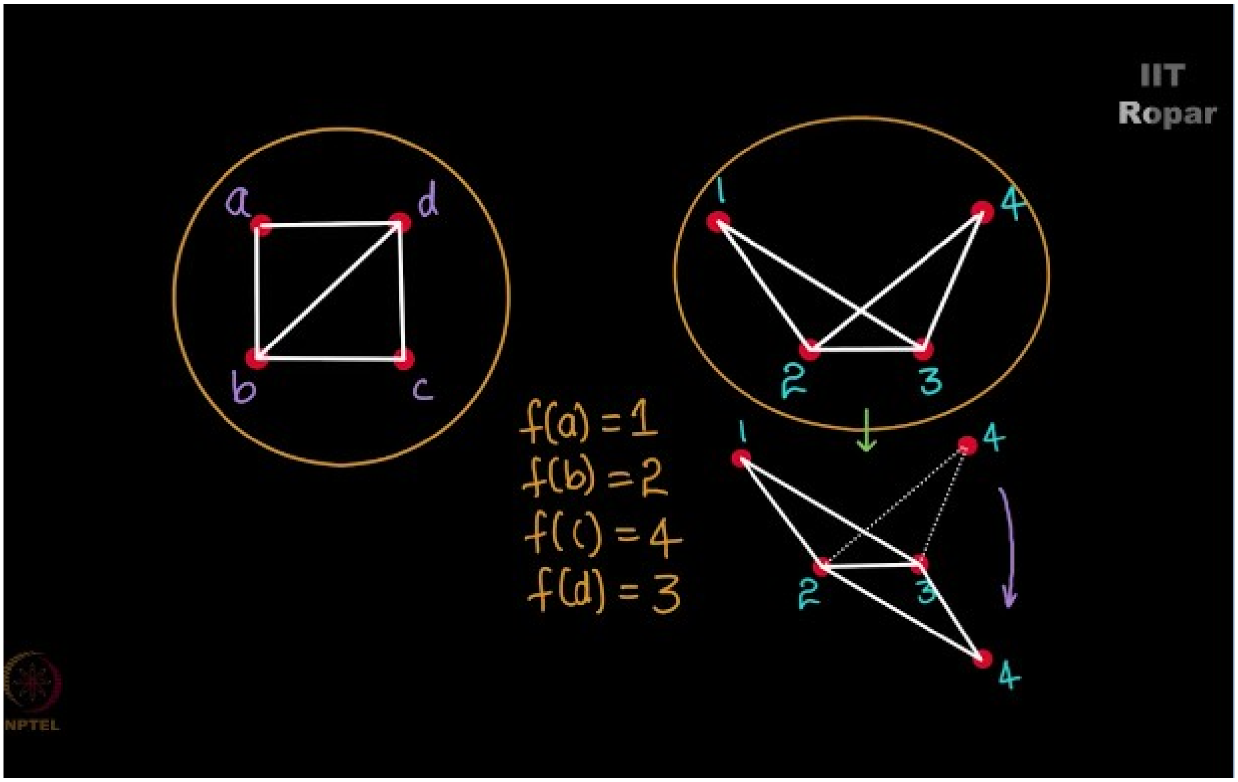

that 2 graphs are isomorphic, I hope things are very clear to you right now, we will now see a little more examples on isomorphism.

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