NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Properties of Binomial theorem

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Let us now see some interesting properties of binomial theorem. Let me again recall what the binomial theorem states. x plus y whole power n is summation K from 0 to n nCK into x power n minus K y to the K. Now what if I replace x by 1 and y by x. So I basically I want the expansion of 1 plus x whole power n. So this is a particular case what we are going to see. So what is the expansion of 1 plus x whole to the n be, summation K from 0 to n, nCK 1 to the n minus K into x to the K. So this happens to be summation K from 0 to n, nCK into x to the K because the rest of the terms here is 1 and hence we have nCK into x to the K. So whenever we have something like 1 plus x whole square or 1 plus x whole cube or any n for that matter you need not basically expand everything. You can directly write it as nCK into x to the K. The terms will be these.

The next property. When I expand x plus y whole power n I have these terms nC0 x power n y to the 0 plus nC1 x to the n minus 1 into y plus nC2 x to the n minus 2 into y square and so on and the last term would be nCn x to the 0 into y to the n. Now what is the first term? It is nC0 into x to the n, y to the 0 is 1 and nC0 is again 1 and therefore I have only x to the n as the first term. The second term is nC1 into x to the n minus 1 into y. The third term is nC2 into x to the n minus 2 into y square and so on. Now when are we getting to? Let me generalize. The r^{th} term will be any general r term will be what? nCr minus 1 into x power n minus r plus 1 into y to the r minus 1. This might seem to be complicated but let me just tell you observe the patterns in the terms here. Consider the third term we had nC2. The third term has nC2 x to the n minus 2 so if I consider r to be 3 n minus 3 plus 1, minus 3 plus 1 is minus 2 and hence n minus 2 we have and y square your r was 3 so you have a 2 here.

So this is very helpful in solving problems. So the r^{th} term is given to be nCr minus 1 into x power n minus r plus 1 into y to the r minus 1. So you can have this formula and use it to find any term in the expansion.

The next property. In your expansion you might have several terms but how do we find the middle term? Supposing say n is very large like 15 or 20 you really cannot keep counting the terms one by one to find out the middle term. So let us use a nice formula here. Say n is even then what? Then there is only one middle term which is the n plus 2 by the 2th term. So the n plus 2 by 2 this term will be your middle term in the entire expression. What if n is odd? Then you will have two middle terms. You can consider the example of x plus y the whole cube. This is x cube plus y cube plus 3x square y plus 3y square x.

You can rearrange this and get it as x + 3x square y + 3y square x + y cube. You have two middle terms. You cannot decide this as a middle term or this as middle term. So you consider both of them and therefore if n is odd we have n plus 1 by 2th term and n plus 1 by 2 plus 1, this term is also the middle term and hence we have two middle terms if n is odd.

I hope I am clear till now. The largest coefficient in the expansion of x plus y whole power n will be the coefficient of the middle term. This must be quite intuitive because you see in the expansion n choose 0 and n choose n both turn out to be 1. you see the coefficients keep increasing till a point and then go on decreasing. So the point where it is highest is the coefficient of the middle term. So this was some of the nice properties.

Now let us solve some examples.

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