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Discrete Mathematics

Let Us Count

Applications of Binomial theorem

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So we have seen what the binomial theorem states. Let us now see some beautiful applications of it. You all must have heard of 1 plus 1 by n whole power n. When I apply binomial theorem on this let us see what the terms are; 1 plus 1 by n whole to the n becomes nC0 1 power n into 1 by n whole power 0 plus nC1 1 power n minus 1 into 1 by n whole power 1 plus nC2 1 power n minus 2 into 1 by n whole square plus nC3 1 power n minus 3 into 1 by n whole cube and so on when I expand this I will get nC0 to be 1, 1 power n is again 1 into 1 plus nC1 is 1 plus again 1 to the n minus 1 becomes 1 into 1 nC1 becomes n into 1 into 1 by n plus nC2 can be written as n into n minus 1 by 2. I have written this by simplifying nC2 into 1 into 1 by n square plus nC3 can be written as n factorial by 3 factorial into n minus 3 factorial into 1 by n cube and so on. This becomes 1 plus 1 the ends get canceled here plus I'll cancel 1n from the denominator and the numerator and what remains is n minus 2 by 2 into n and now simplifying the next term we get n into n minus 1 into n minus 2 by 3 factorial into 1 by n cube plus so on.

Now this becomes 1 plus 1 plus let me take a n common from the numerator here and this becomes n into 1 minus 1 by n by 2 into n plus we have n here and n cube here so after simplification what remains is n minus 1 into n minus 2 by n square into 3 factorial and so on. Now this one I'll write it as 0 factorial and this I will write it as 1 by 1 factorial plus. You will understand why I'm writing like this, plus I cancel these ends here and I get 1 minus 1 by n by 2 is same as 2 factorial and hence I'll write this as 2 factorial plus again as I did earlier I'll take out ends common in these two terms so I will get n into 1 minus 1 by n into n into n1 minus 2 by n. I hope the step was clear, by n square into 3 factorial plus so on.

Now I'll cancel these ends with n square and what remains is zero factorial as we have seen earlier is one so it does not make a difference if I write 1 by 0 factorial plus 1 by 1 factorial plus 1 minus 2 by n by 2 factorial plus 1 minus 1 by n into 1 minus 2 by n by 3 factorial and so on. Now what happens to these terms when n becomes sufficiently large? What I mean by sufficiently large is we have to choose n to be a very large number. In case n happens to be a very large number then 1 by n or 2 by n or any small number by n becomes very close to 0 and hence this term can be written as 1 by 0 factorial plus 1 by 1 factorial plus I am approximating this 2 by n to be a number very small very close to 0 and hence this becomes 1 by 2 factorial.

I'm neglecting the terms which have denominator as n and hence in all the further terms what will I get? It will become 1 by 3 factorial plus 1 by 4 factorial and so on. So this becomes summation 1 by K factorial K from 0 to infinity. Now this is called the famous Euler's number. I'm sure you must have heard of e and this we have arrived at this by starting from expanding 1 plus 1 by n whole to the n by using binomial theorem. So this was one application of binomial theorem.

Now let us see the next one. The derivative of X to the n involves binomial theorem. Now derivative is a concept from calculus and I am talking of binomial theorem. So how is it related? The proof involves certain technicalities from calculus and I leave it to you guys to go and check it. I am not giving the proof of the derivative of X to the n here but let me state it. The derivative of X to the n is n into X to the power n minus 1 and this involves the usage of binomial theorem. Let us move on to the next one. We know that binomial theorem states that x plus y whole to the n is summation K from 0 to n, nCK into x bar n minus K in the y power K. What happens when I plug in x and y as 1? So it becomes 1 plus 1 whole power n equals summation K from 0 to n nCK. So in place of x and you I am writing it as 1 and hence this becomes 1 to the n minus K into 1 to the K. Now you must be wondering why I am doing this because I'm just plugging in 1 for x and y but let me tell you this is 2 power n equals summation K from 0 to n nCK. So did you observe what happened? We have as a summation only the binomial coefficients and the sum of all these binomial coefficients turns out to be 2 power n. What does this actually mean? If we have n objects each time we choose K objects from these n objects like I'm choosing 0 from n objects, I'm choosing one object from n objects, two objects from n objects and so on and I add up all this so what it becomes, it results in 2 power n. Did you see the beautiful connection between binomial theorem and choosing objects from n objects and 2 power n.

So these were a few applications of binomial theorem. Next we'll move on to the generalization of binomial theorem and solve a lot of problems.

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