NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Havel Hakimi theorem - Part 3

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Consider this degree sequence 5, 5, 3, 3, 2, 2, 2,
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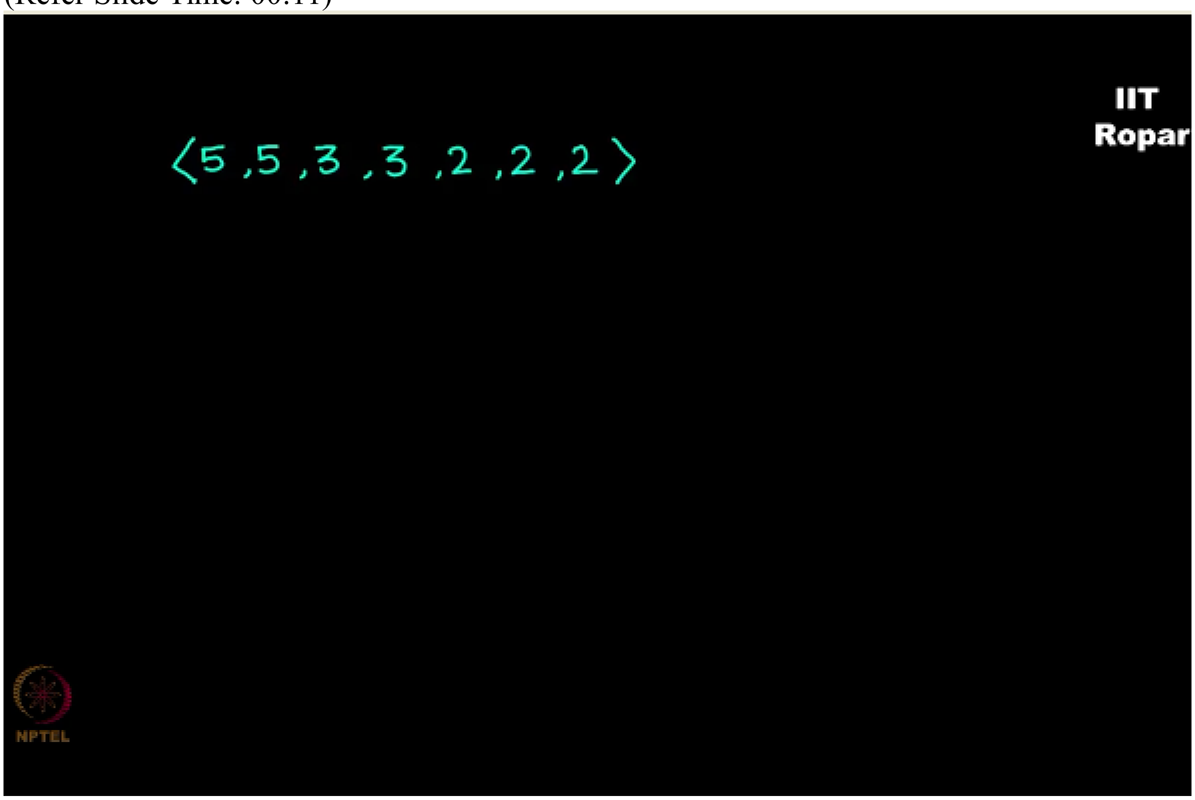

let us see if we can draws graph for the sequence, please note we had left the earlier video with the question, I had given a sequence there and told you to draw a graph, if you draw graph for this sequence, then answering that will become obvious.

Let us attempt drawing a graph for this sequence 5, 5, 3, 3, 2, 2, 2, how many vertices are there? There are 7 vertices, so let me name them A, B, C, D, E, and F,
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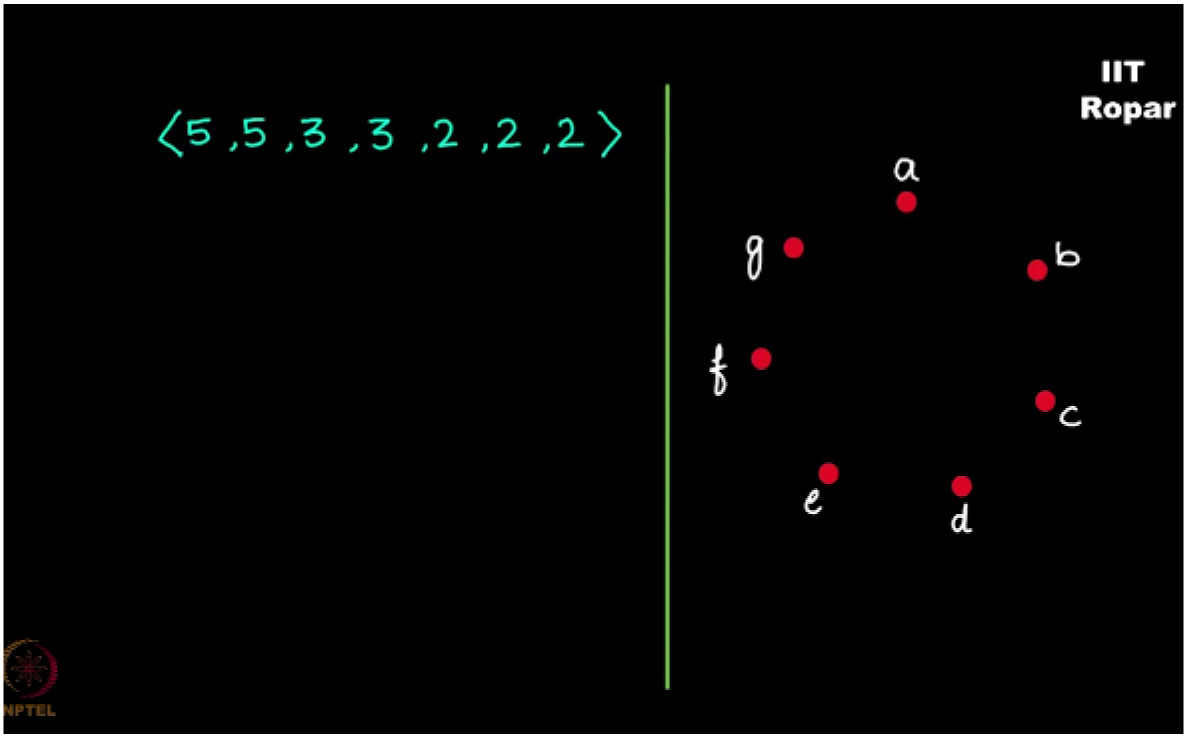

so these are the 7 vertices, now I'll write the graph here separately and I'll write the sequence here, so let me write it like this 5A, 5B, 3C, 3D, 2E, 2F and 2G

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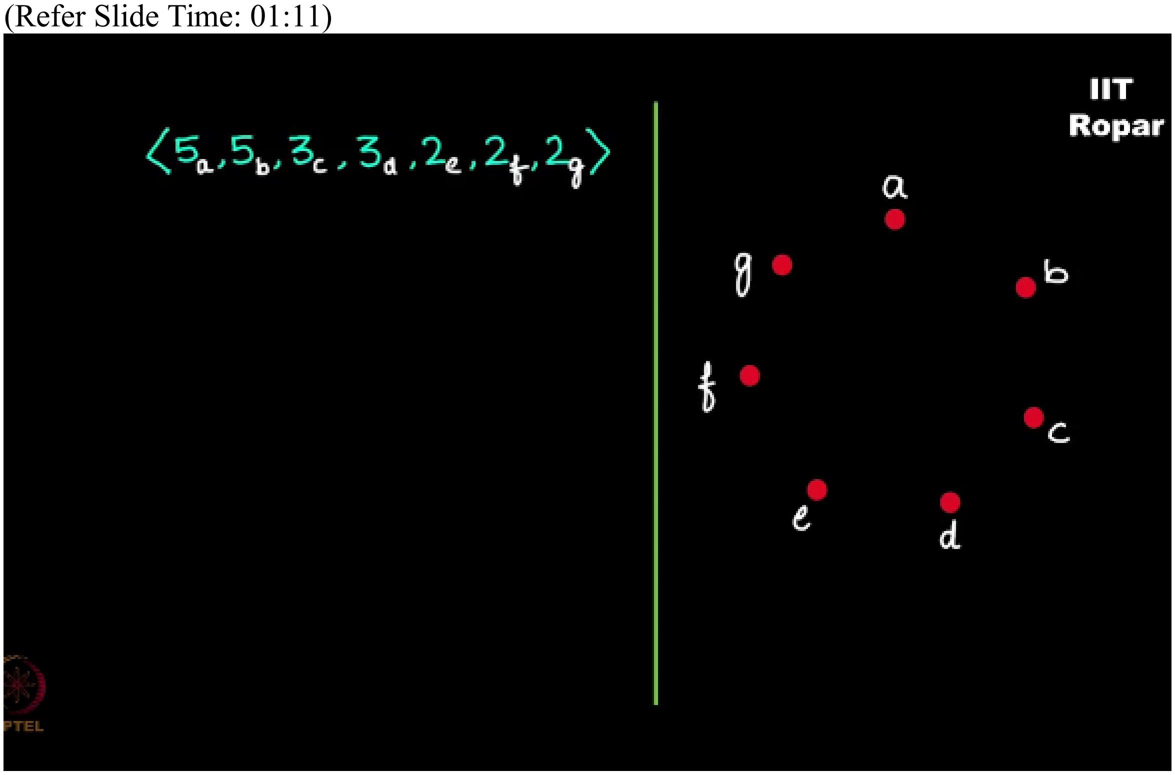

yes, now this is my sequence let me name it as S1.

Now what I'm going to do is observe the first vertex degree, that is observe the degree of the vertex A, it is 5, what does it mean? It means that A is connected to 5 other vertices, right,

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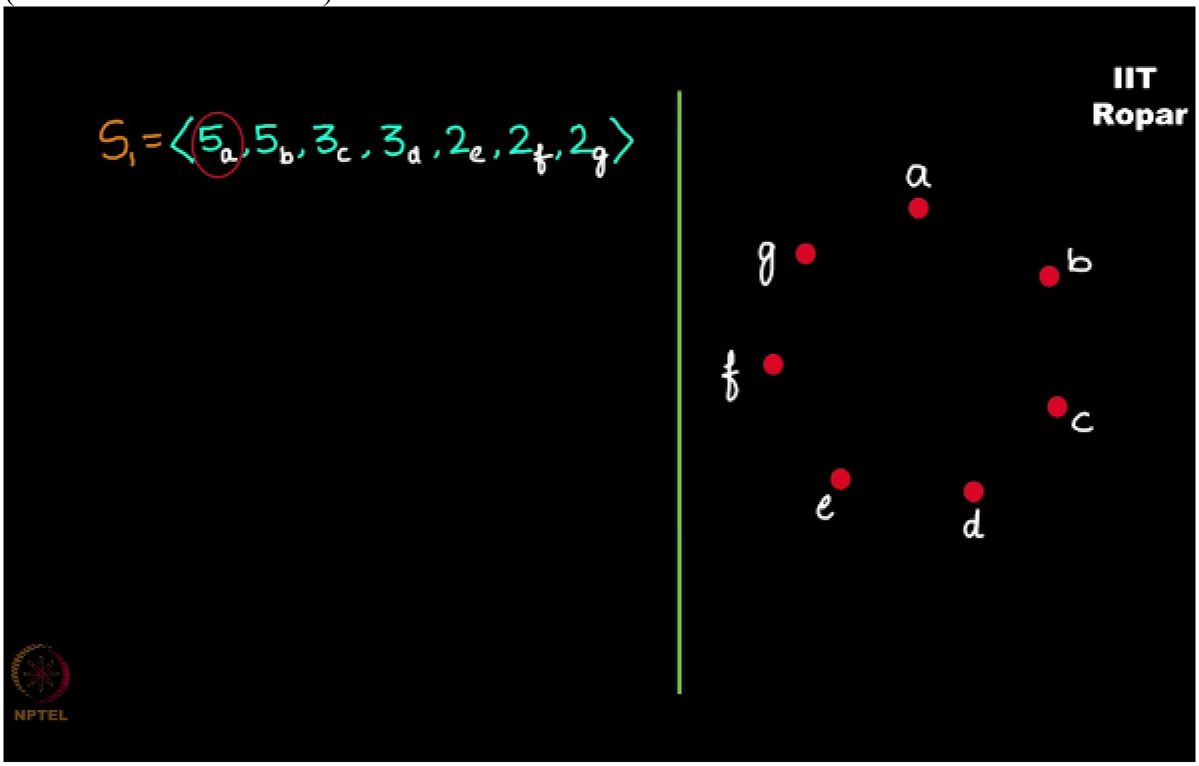

without loss of generality what I am going to draw here is, I draw 5 edges emanating from A, from A I'm going to draw 5 edges, so let me draw edges to B, C, D and E

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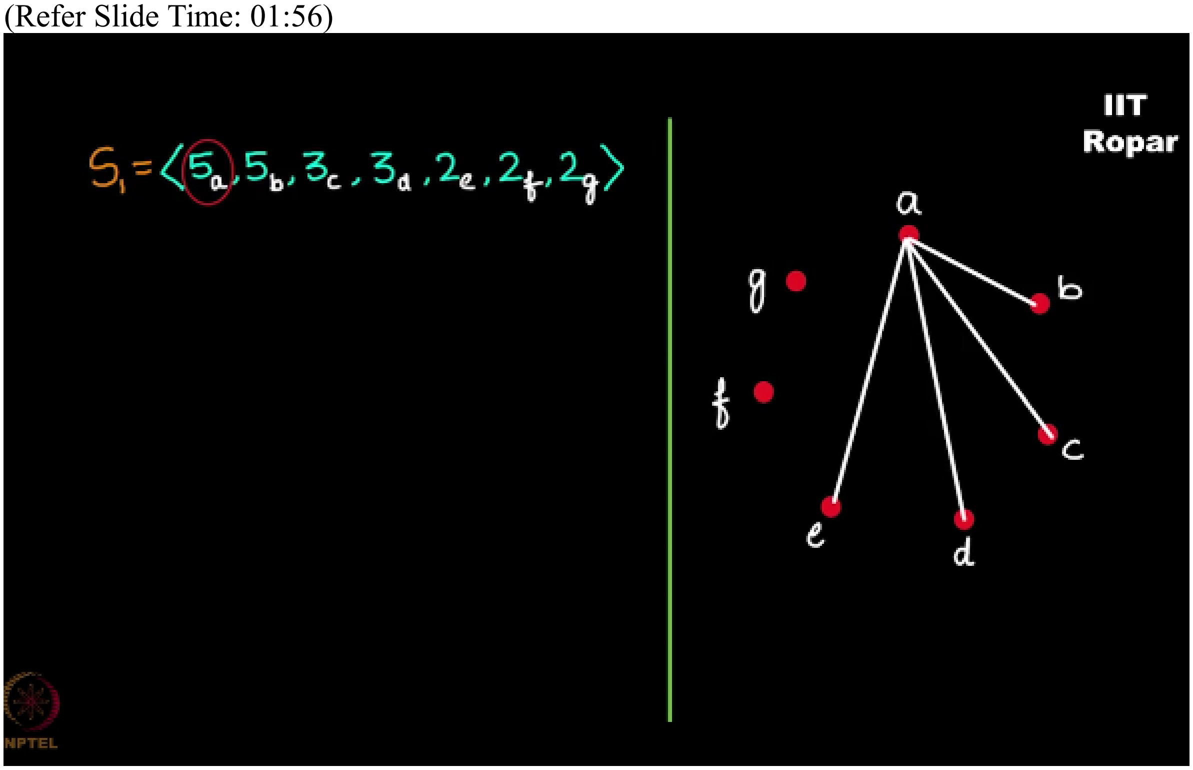

right, do you see the graph, see I'm constructing the graph this side so you must be able to visualize the graph as well as write the sequence here.

So now do you see that, okay, I had drawn 4 edges so this is another one, this is F,
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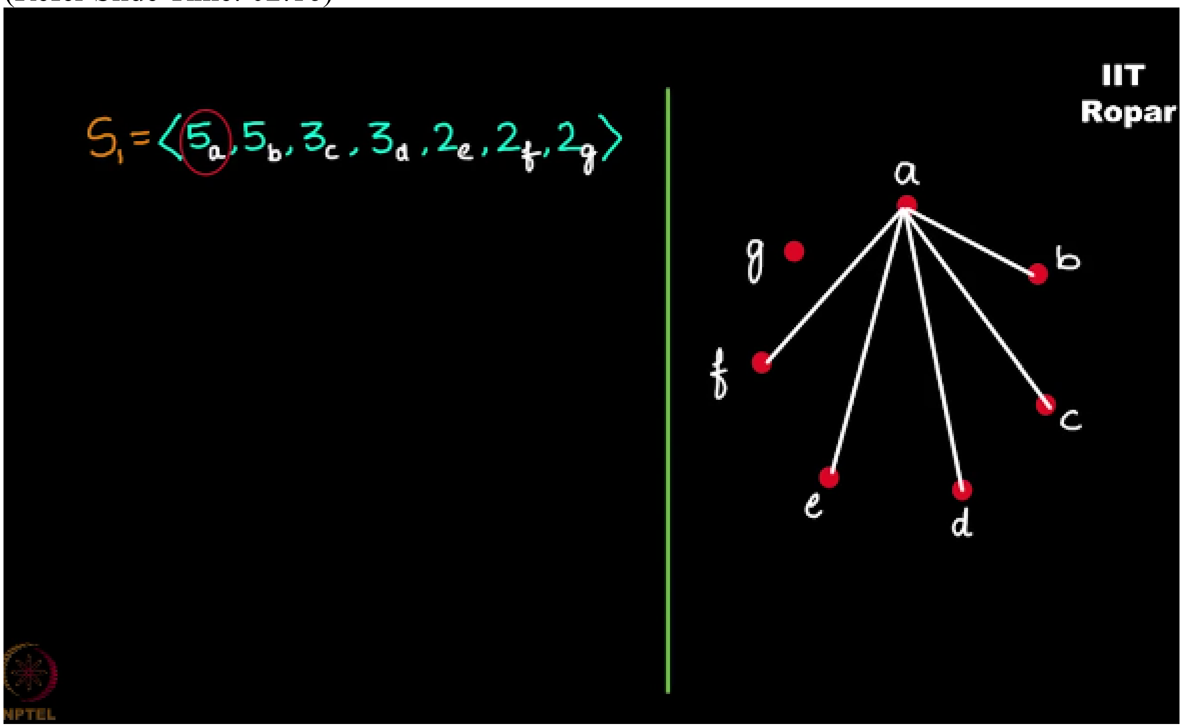

from A to F this is the last edge so we have 5 edges coming from A to B, C, D, E, F, respectively.

Now what has happened to the sequence let me modify it, I have finished drawing 5 edges of this vertex A,
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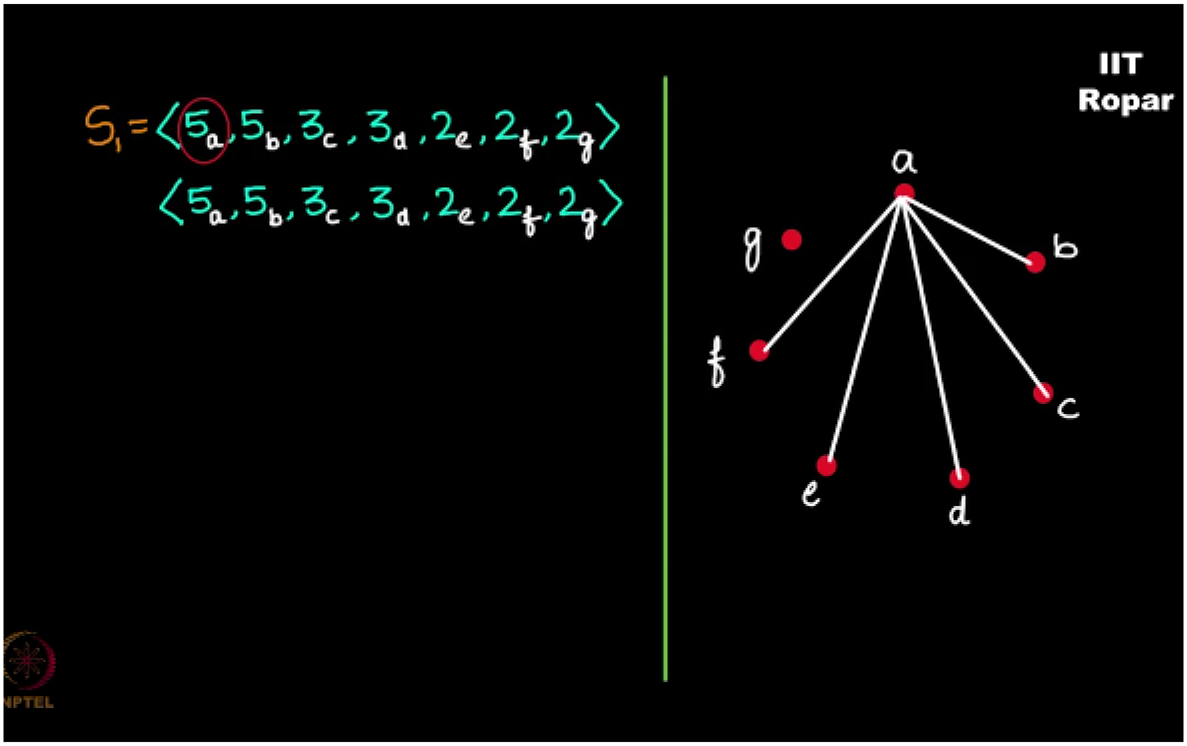

so A right now in the sequence instead of keeping it empty let me put a star here, so what does this mean? This star means that I am done with A,

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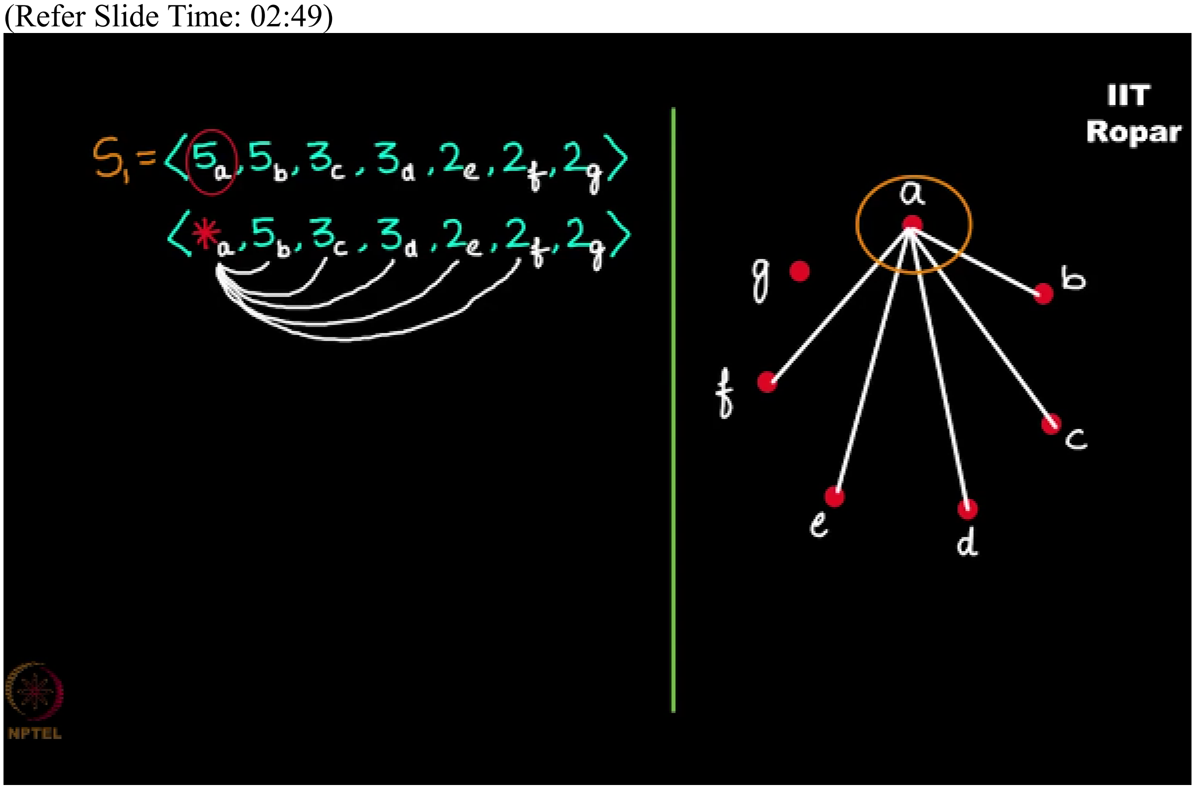

here I can go to the next vertex B, so star, now observe the graph here B has got 1 edge already, right, so from 5 the degree reduces to 4 now, and the same with C,

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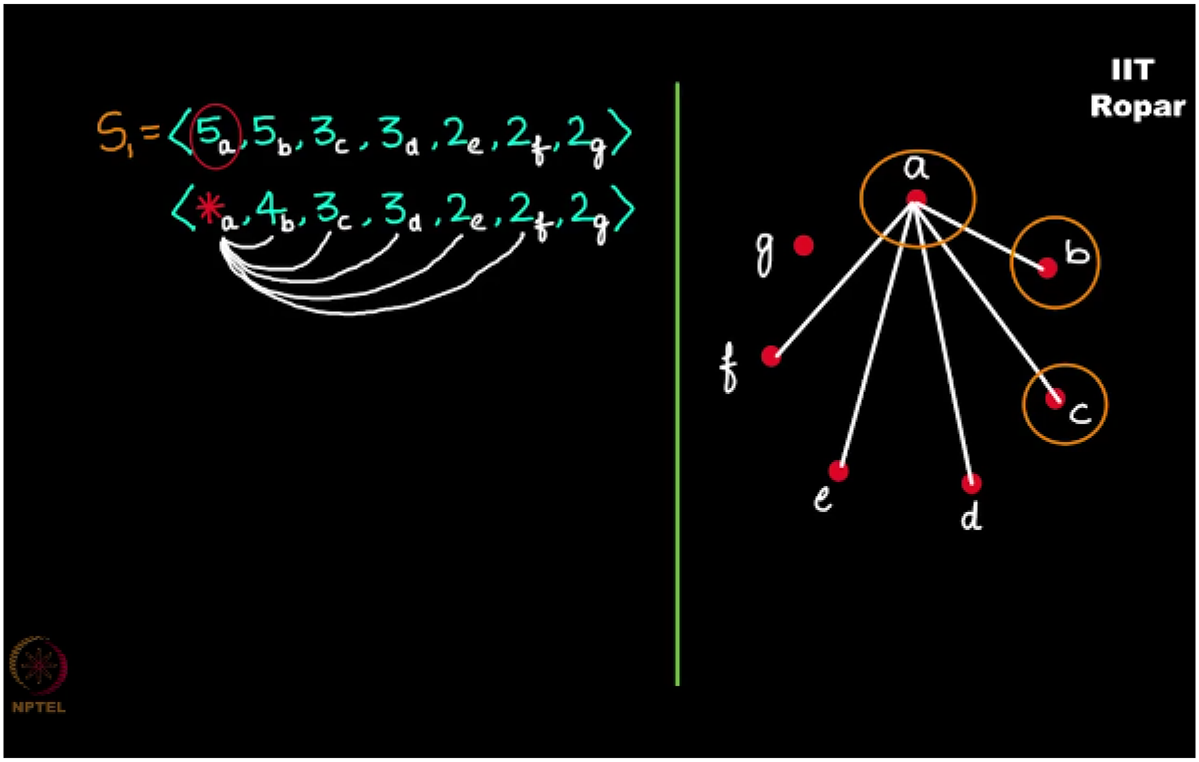

earlier the degree of C is 3, one we have completed so now it becomes 2. D earlier the degree was 3 now it has become 2, so what is it? It is star, 4B, 2C, 2D.

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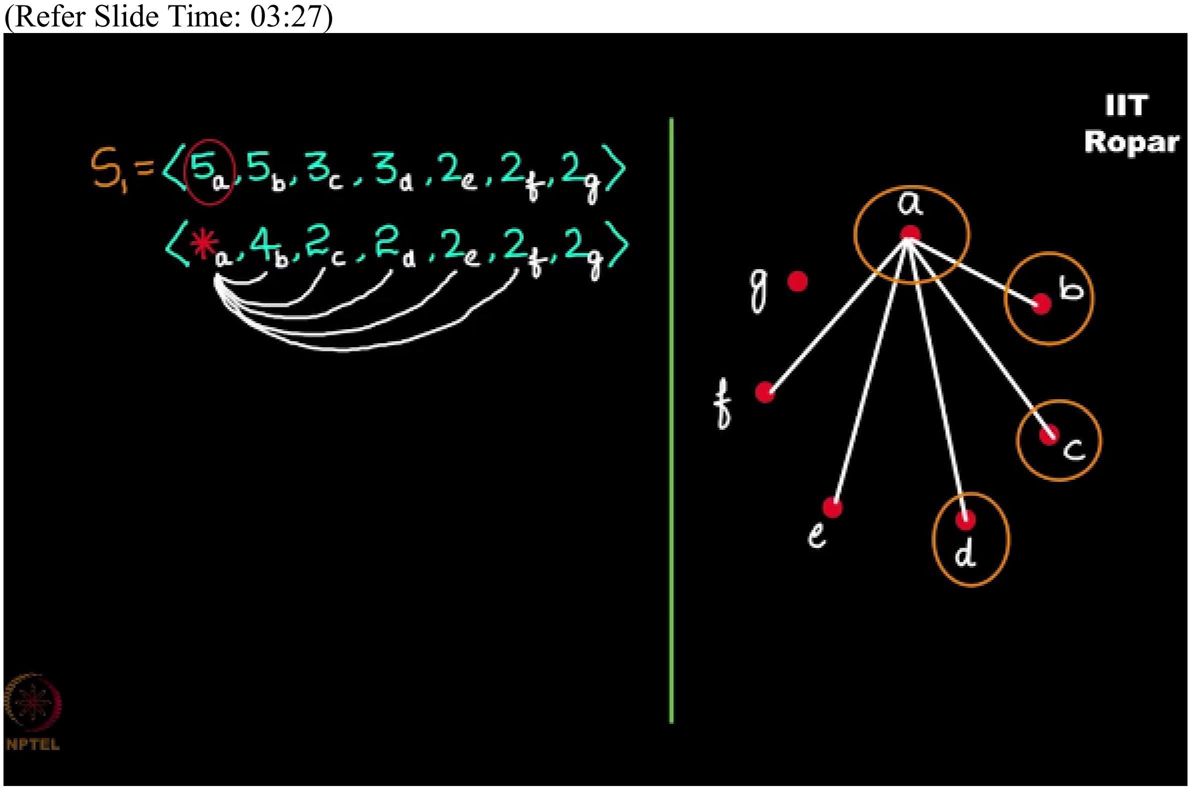

Now what is the case with E, E, F and G? The degree of E and F are two each, but one is already done here, and therefore the degree the modified degree becomes 1 for both of them, G we see that A is not connected to G, G remains as such, is it clear? I feel with the next step things will become more clear to you, so G remains as it is, the degree of G remains as it is so let me complete this, it has 1E, 1F, and 2G.
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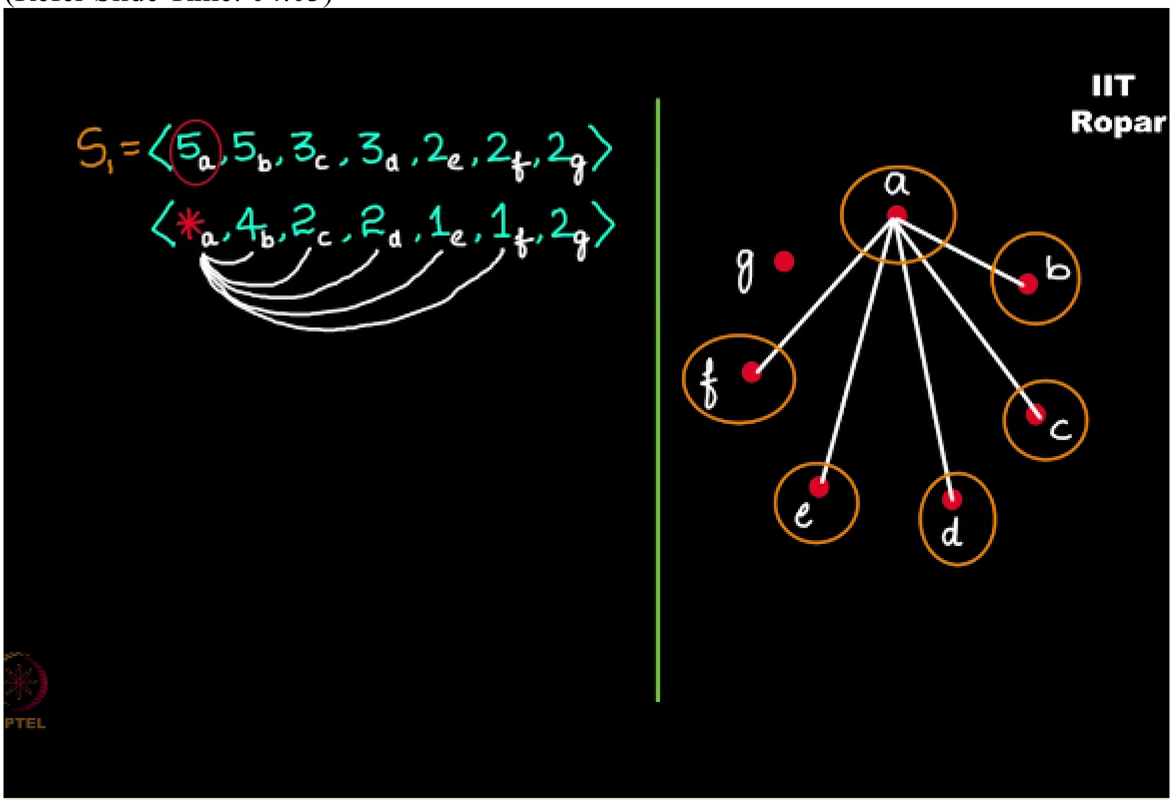

Now let me name this modified sequence as S2,
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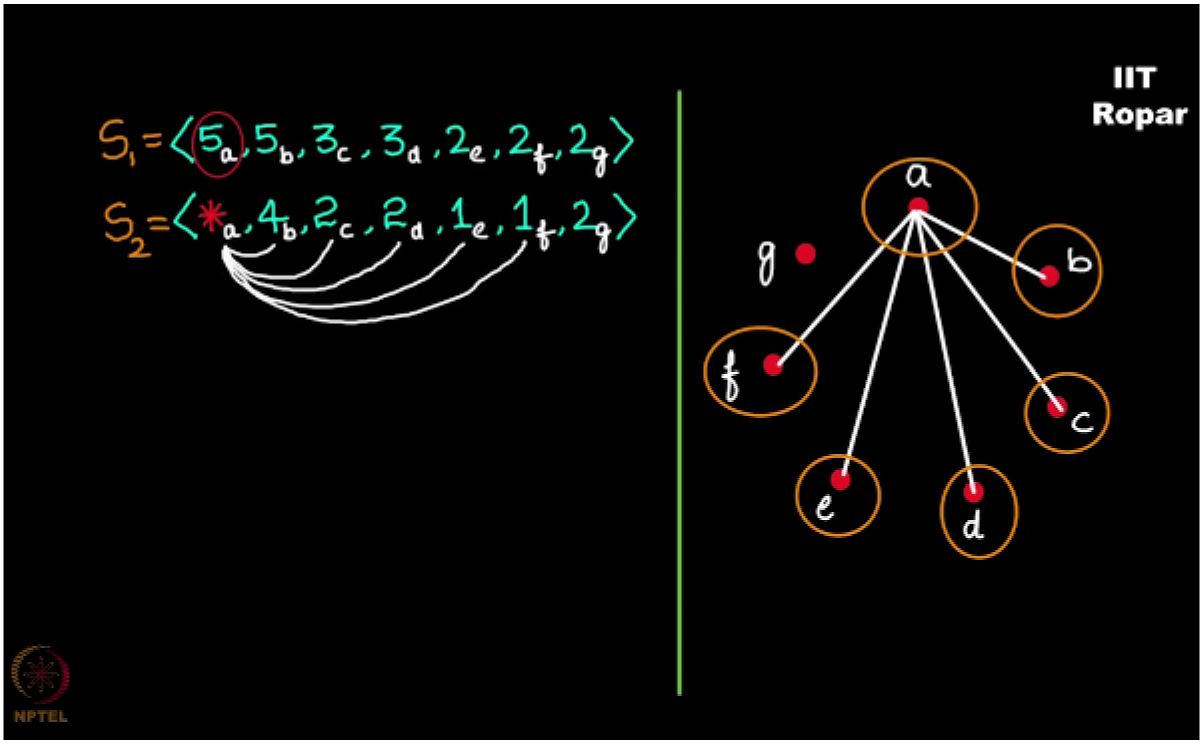

so this is S2, this is S1, now what am I going to do next is, do you see that we have 4, 2, 2, 1, 1, 2 here, so let me write that in an ordered way, what I will write is I'll just make some arrangement here, as star A, 4B, 2C, 2D

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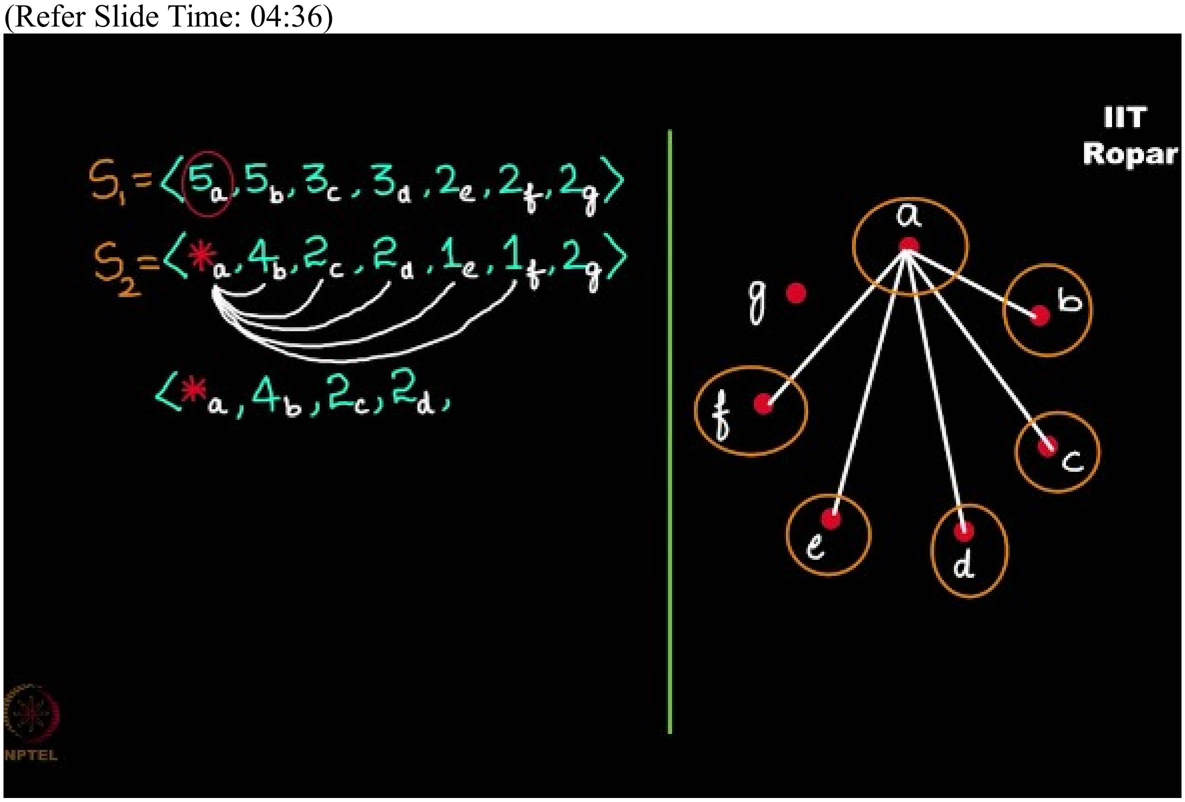

so it has remained unchanged, now 1E, 1F and 2G I will write it as 2G, 1E, 1F

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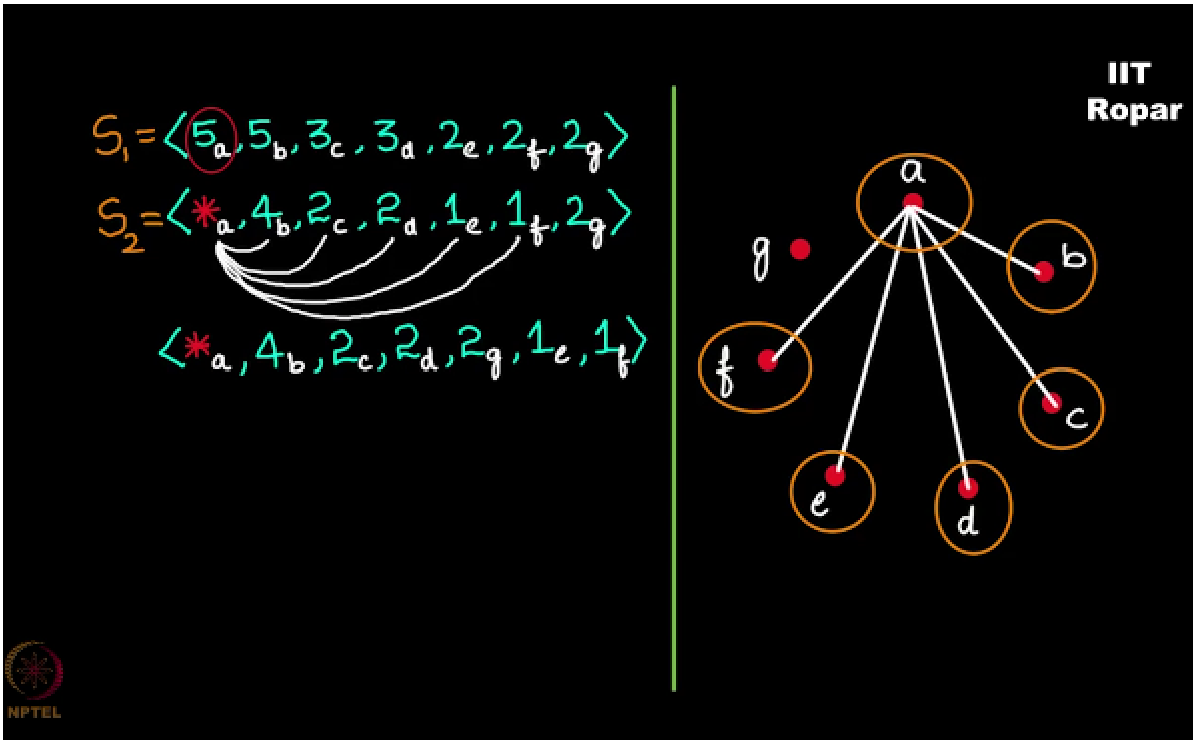

I just replaced a few vertices.

Now let me call this degree sequence as S2 dash, you see there is no difference between S2 and S2 dash, the only change I have made is that replaced or rearranged rather, rearranged the degrees here and there of E, F and G, that's it, so I am done with S2 dash.

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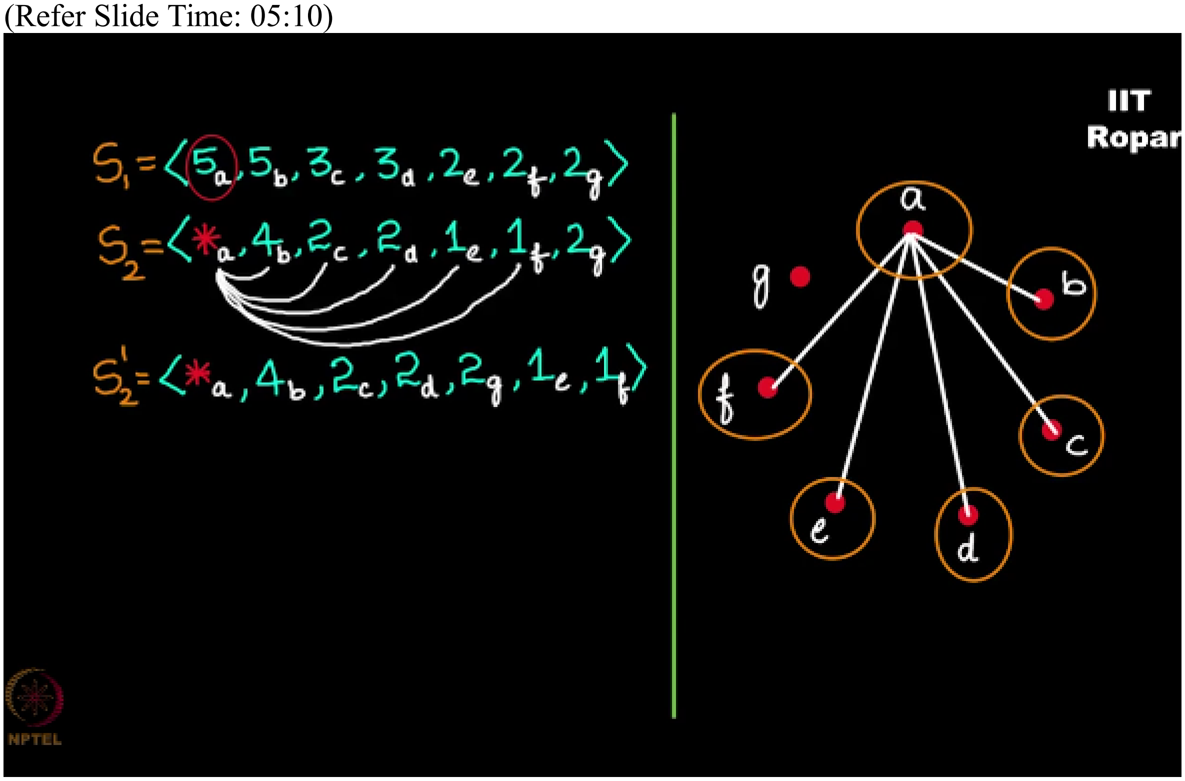

Now we were done with the vertex A, let me now go to the vertex B, the actual degree of vertex B is 5, but it has already got one from A, so what does it become? So from B I'll give one to C, one to D, and then observe

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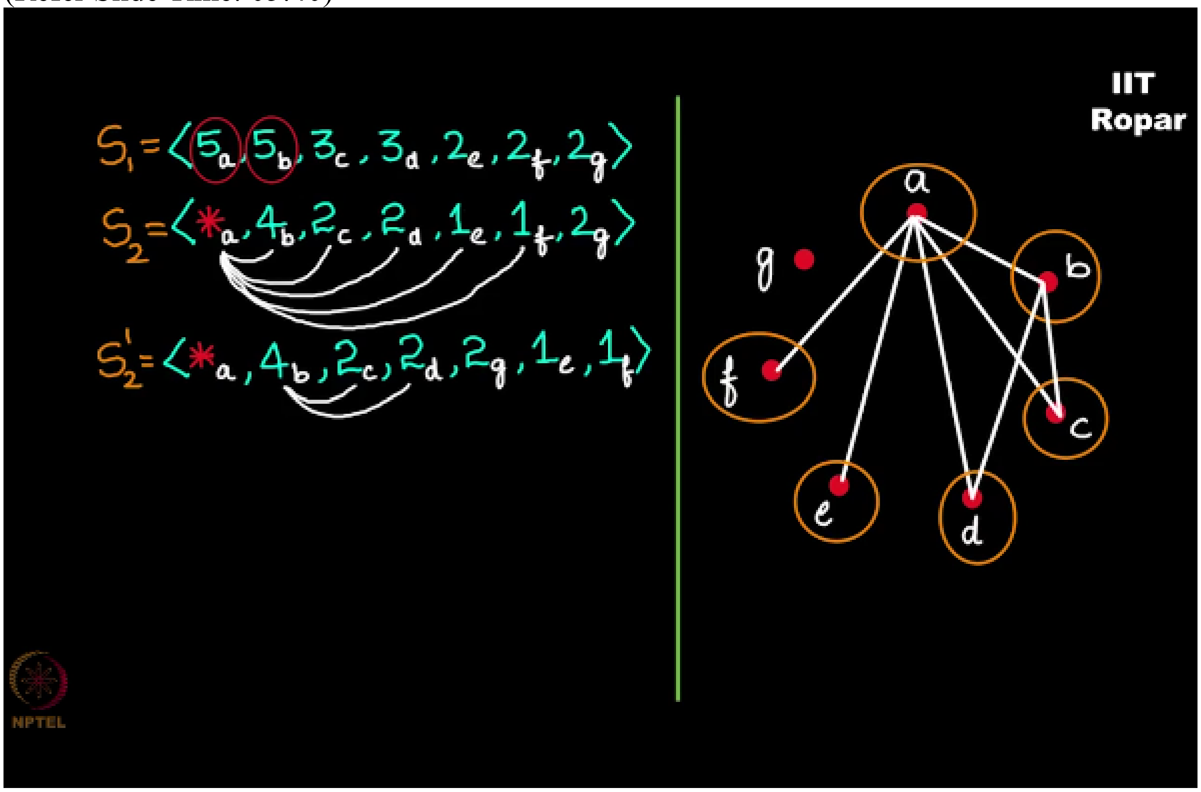

I am not going to go to E rather I give one from B to G,

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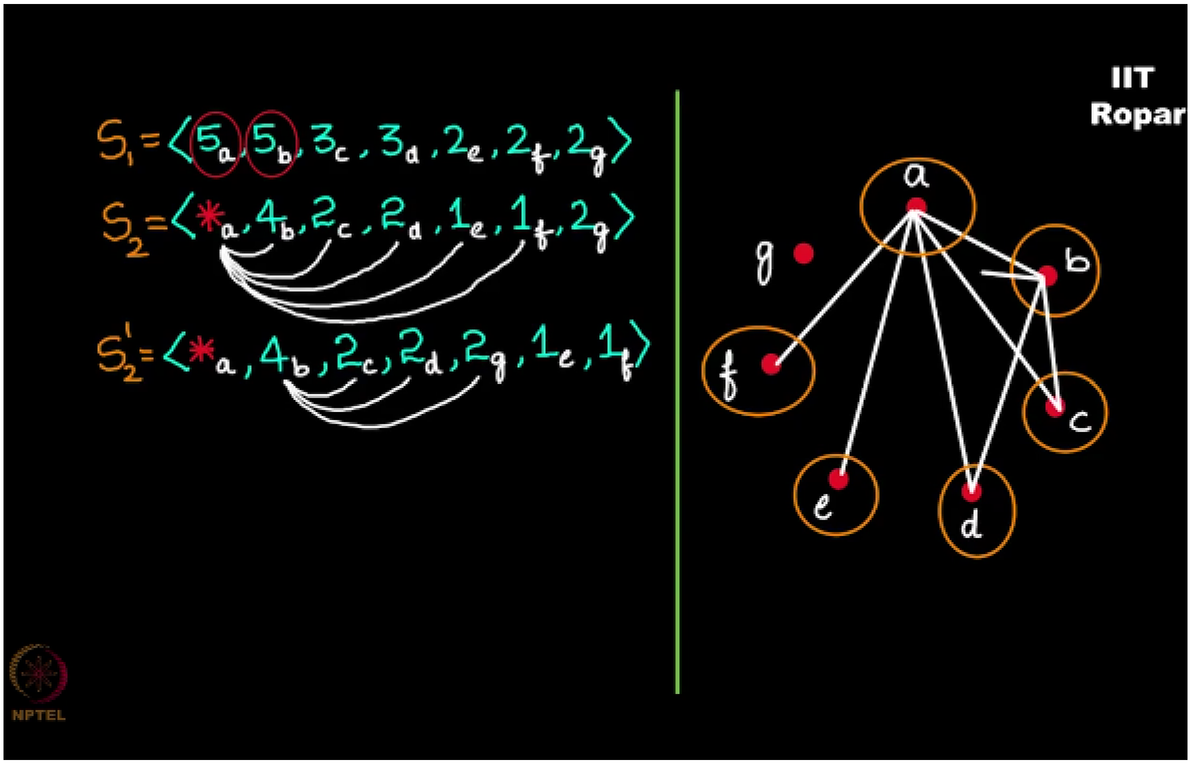

did you observe that? And then we have one more remaining I will give that to E now, so 1, 2, 3, 4, 5, yes

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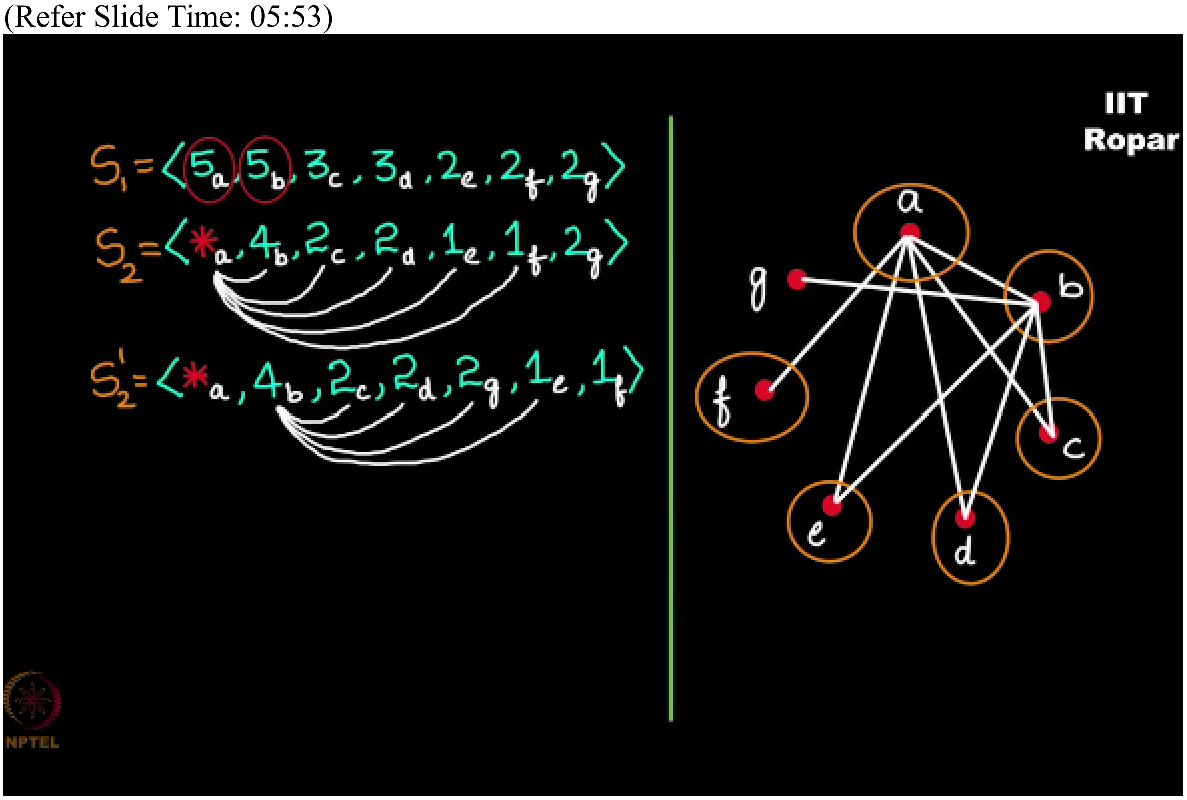

so B also has the degree 5, so A is done, B is done, and the other degrees have got modified.

So now let me write the sequence star as usual for A, and for B again I'm going to write the star, because 5 is done here, now C becomes one reduced, so it has 1C, D becomes one reduced so it is 1D, G again becomes one reduced so it is 1G,

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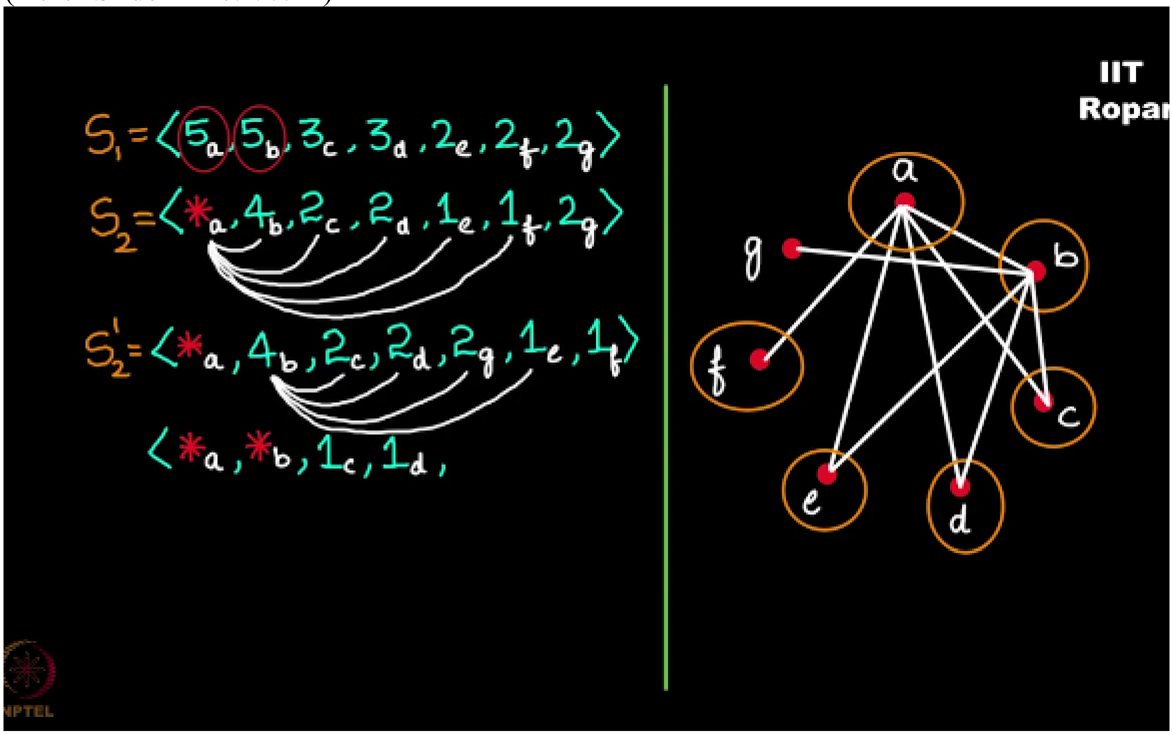

and I have given 1 to E2 from B there, there is 1 edge from B to E, and hence this becomes 0E, right, and F remains as it is, F is unchanged, did you observe that?

Now let me name the sequence as S3,

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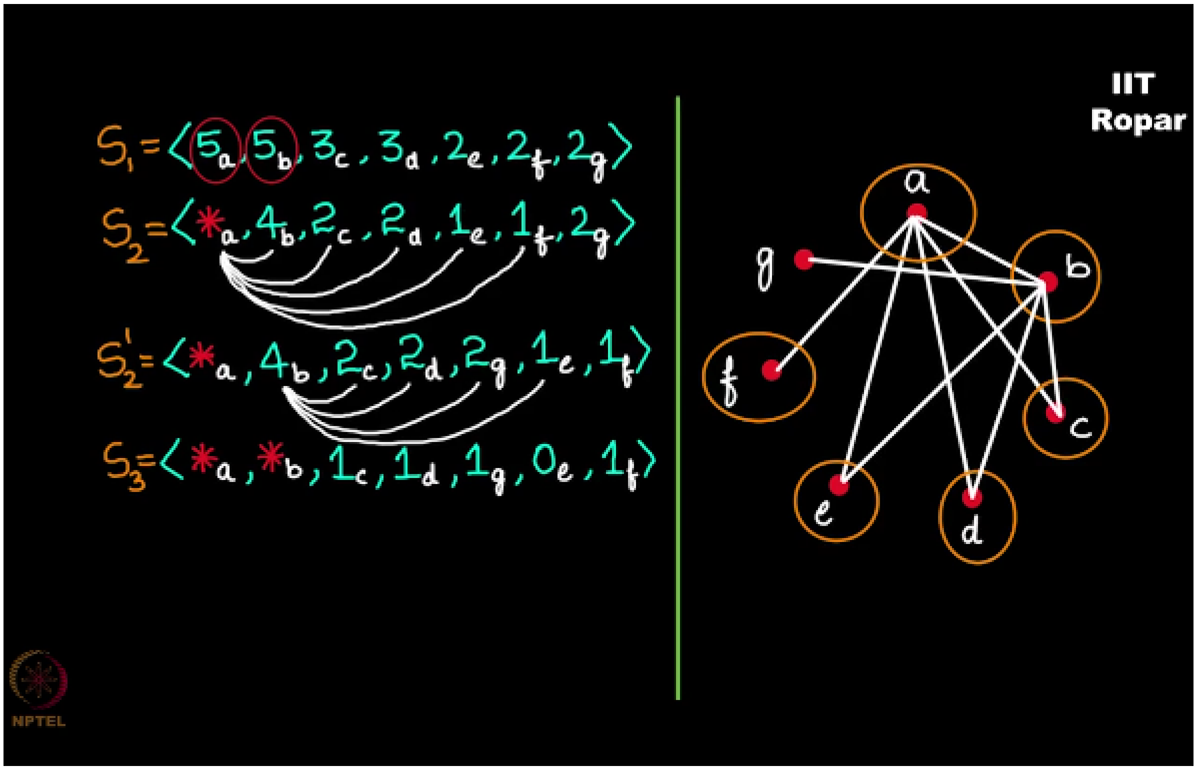

moving ahead what I did as S2 dash, I am going to repeat that procedure and name it as S3 dash, so what does it mean? I am going to rearrange the sequence S3 to form a sequence S3 dash, so what is S3 dash? Star, star, 1C, 1D, 1G, 1F, and 0E,

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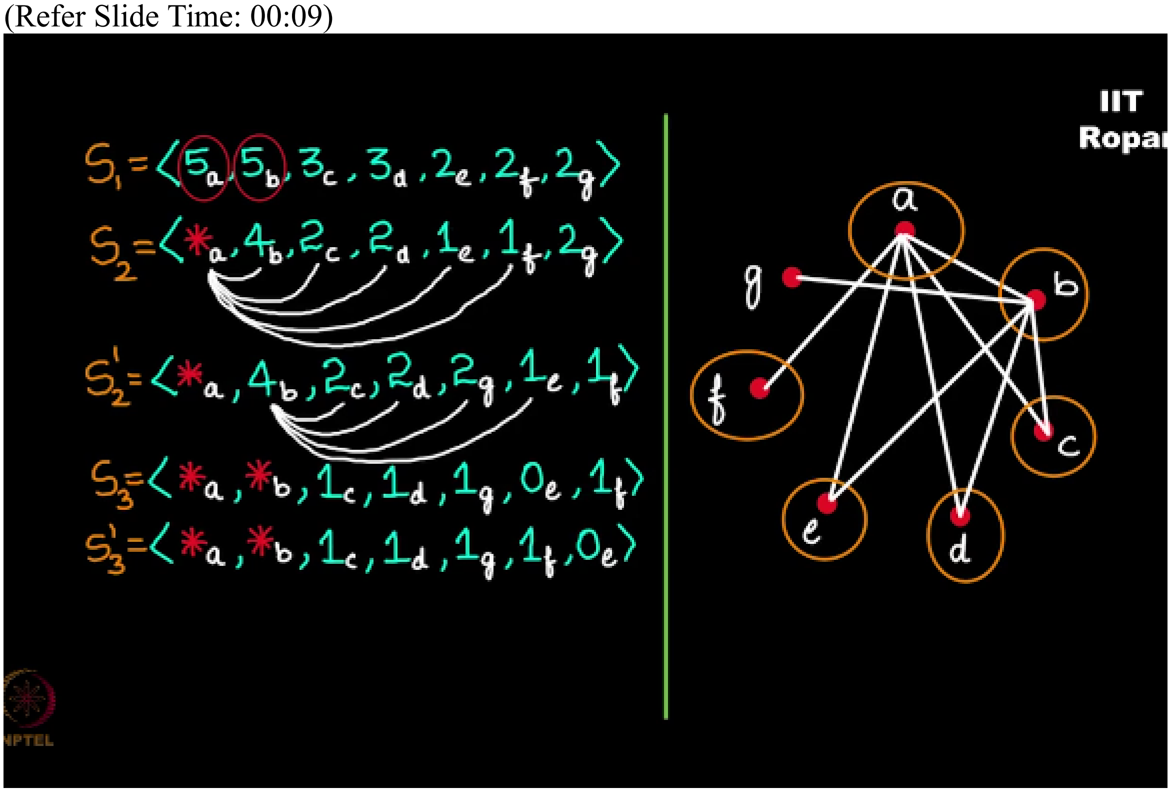

so what do you observe from this sequence S3 dash? You see that we are done with A, we are done with B, now let me go on to the vertex C, so C requires just one more degree, so what am I going to do, I am going to give C just observe see also needs one more, D also needs one more, so I'm going to connect C and D, did you observe that I'm done with C and I'm done with D as well.

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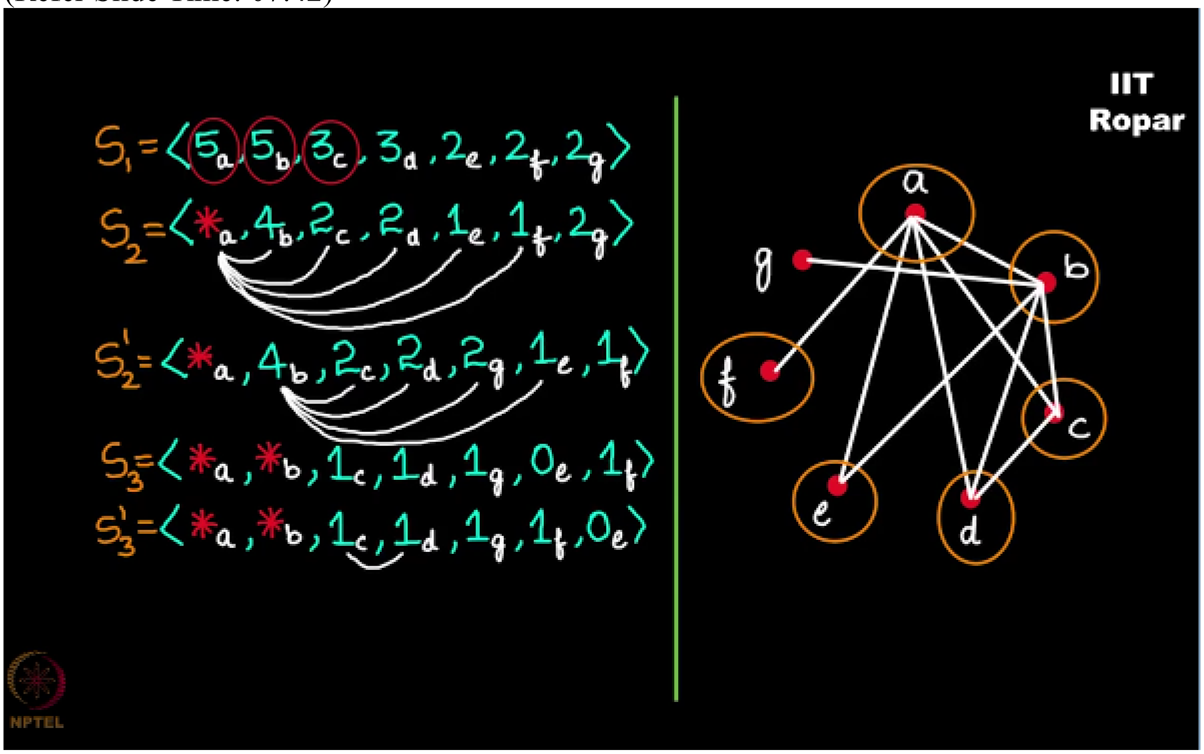

Now this is the sequence S4, I am not going to give any other edge from C to E, F or G because I am done, I just have to do one more edge, so what is the sequence S4? Star, star, and first C, again a star, D is 0, because we have satisfied the condition for D now, so let me put a star for D also,

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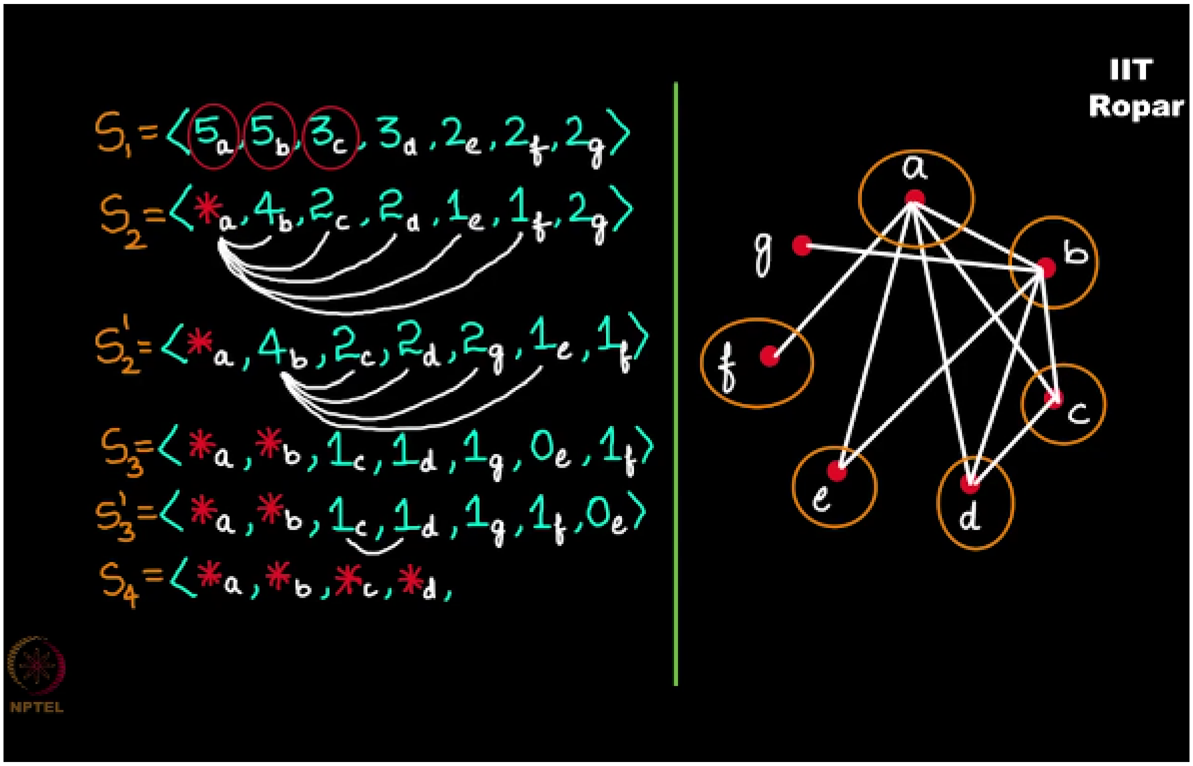

1G remains as it is, 1F remains as it is, and 0E remains as it is,

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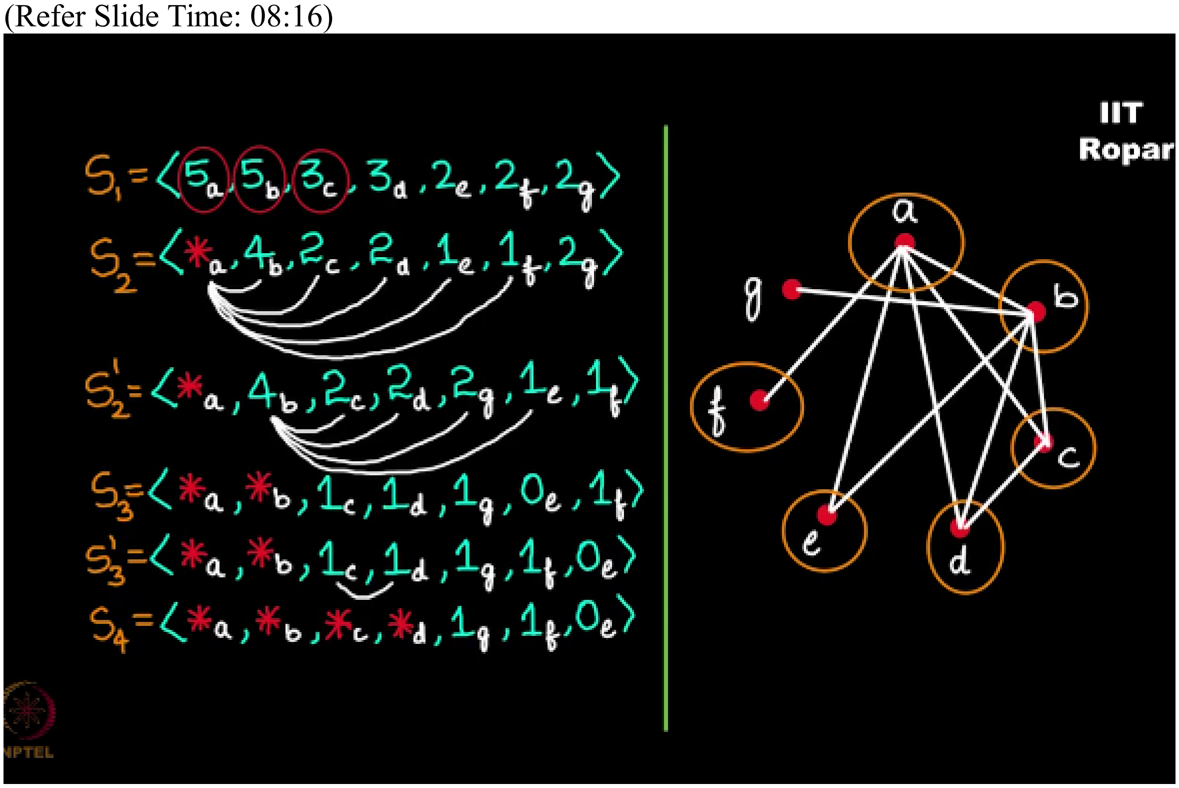

now this is a sequence S4, now and we don't need a step called S4 dash, we don't need such a sequence, why? Because it is already arranged we have 1G, 1F, and 0E so it has arranged.

Now you must be thinking that the next step will be trivial, because you observe here G requires one more degree, F requires one more degree, yes, so what are we going to do, we are going to connect G and F,

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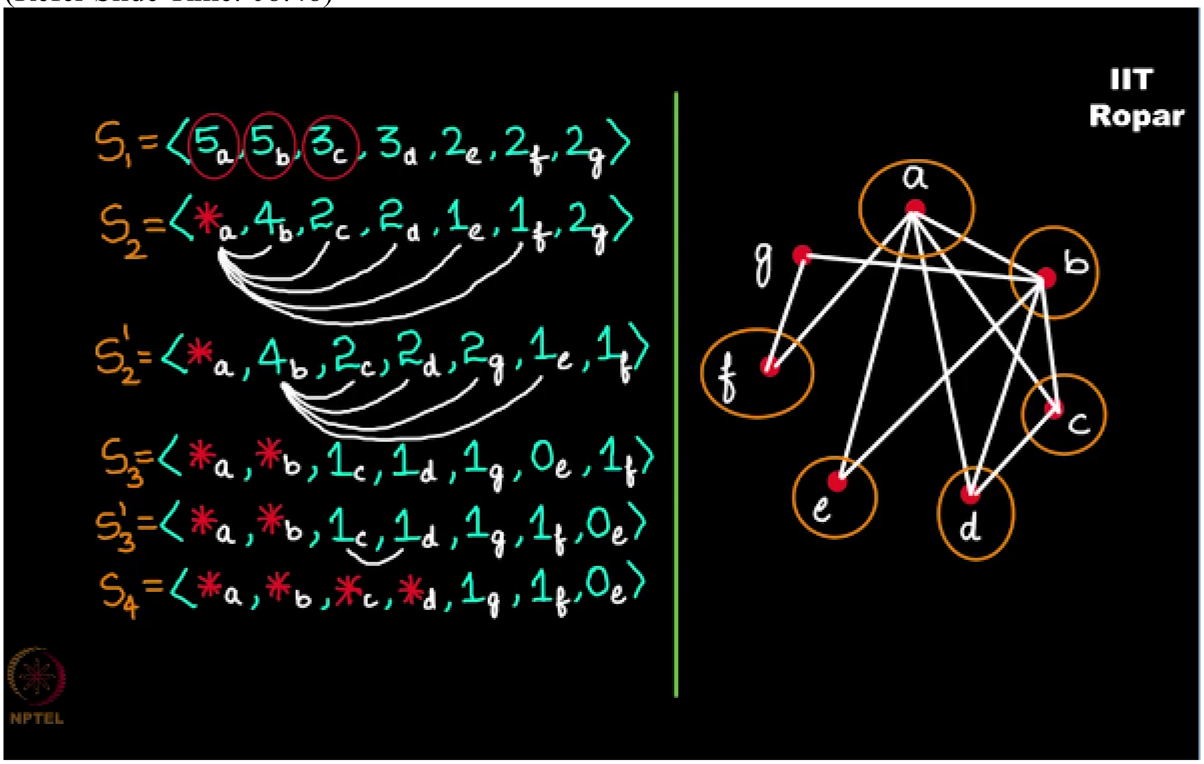

we are done because, now what is the sequence S5? S5 is star, star, star, star, yes G also has a star, because we are done with G, we are done with F also star, 0E

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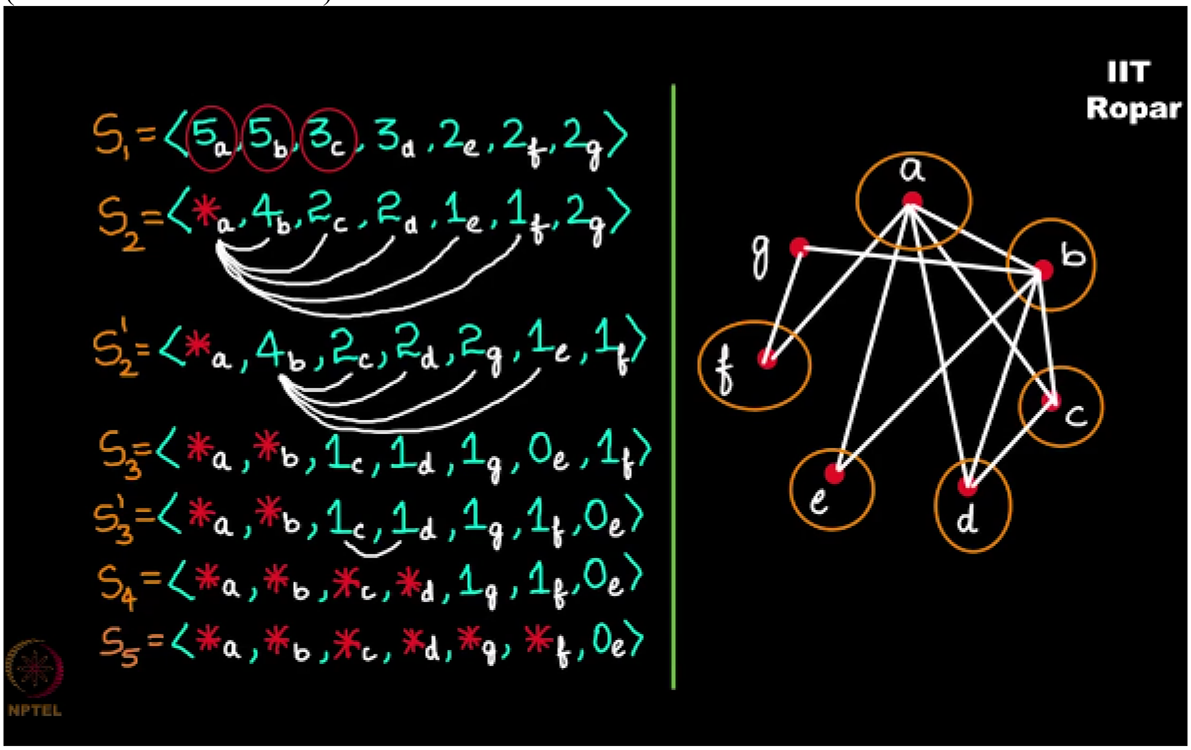

which means E does not need any more edges, rather anymore, its degree criteria is satisfied, yes this is a final graph.

Let us check if we have done it correctly, A has got degree 5, B has got degree 5, C has got degree 3, D has got 3, E 2, F 2 and G 2,

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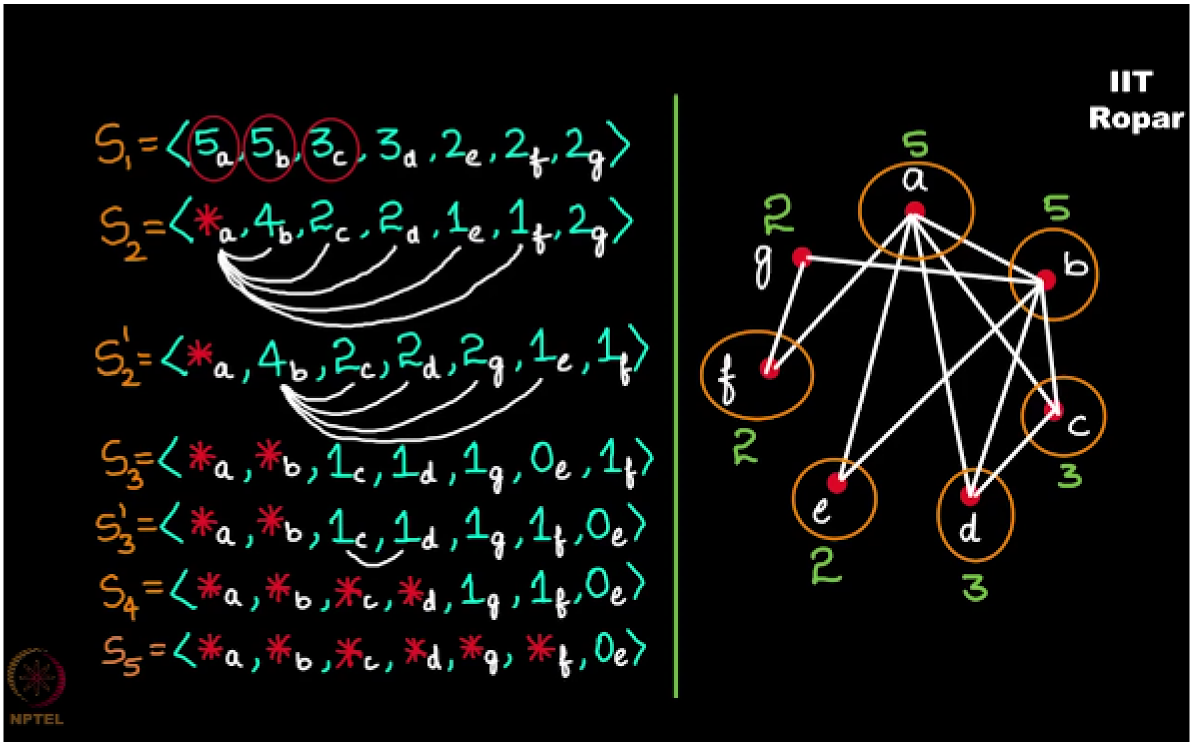

yes, and we are done.

Now can you do the same thing for the question which we left in the previous video, 5, 5, 5, 5, 2, 2, 2?

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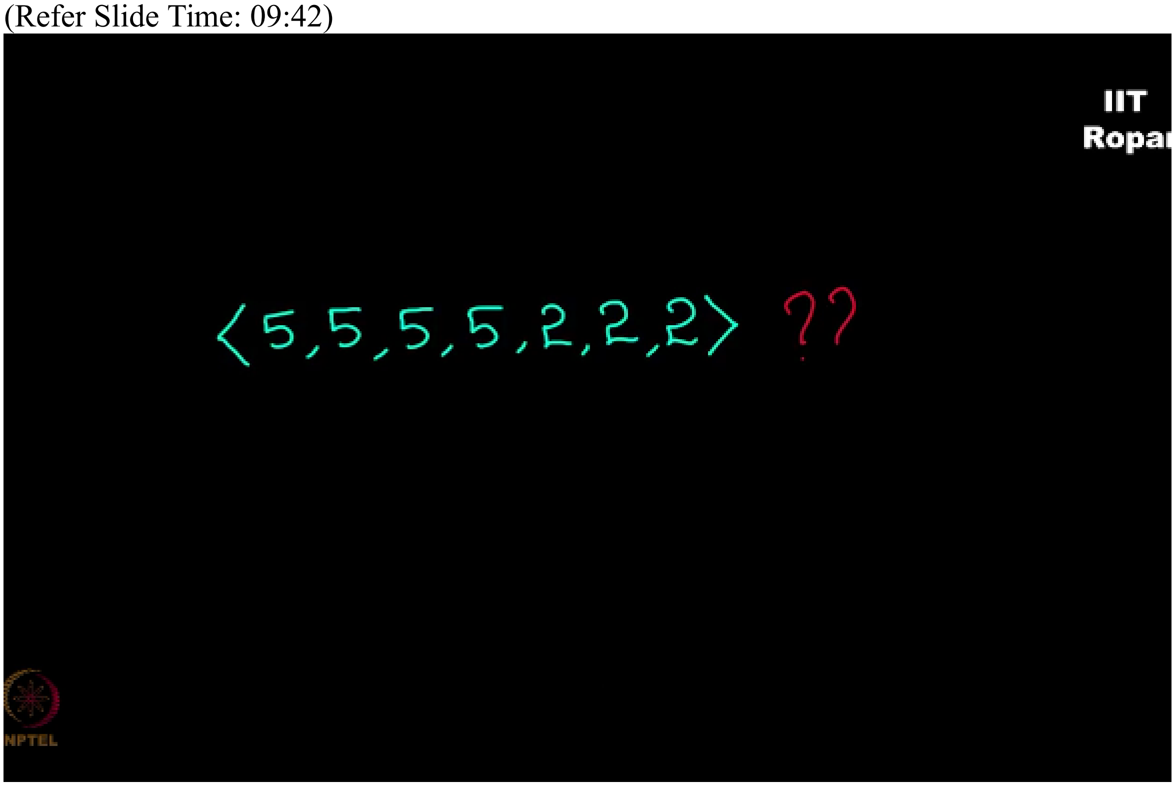

can you do this and obtain a graph for this sequence?

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