

**NPTEL**

**NPTEL ONLINE COURSE**

**Discrete Mathematics**

**Let Us Count**

**Combination with Repetition - Part 2**

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So the solution is actually very short and sweet. Let me explain it properly. Look at this vanilla chocolate and mango are the three flavors. A possibility is two vanilla, three chocolate so four chocolates and four mango amounting to ten, correct. Or let's say one vanilla, one chocolate and eight mangoes right or maybe all vanillas. No chocolate and no mangoes. All these are valid ways of selling ten chocolates three flavors. Now what I will do is I will visualize this very thing in a particular way that makes my counting easy. What do I do? I will take one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, slots and two sticks. I put a stick here and maybe here which denotes that always the first cell denotes vanilla ice cream zone. Second cell is the total number of chocolate ice-cream that he sells and the last one denotes the total number of mango flavored ice-cream that the shopkeeper sells.

So here in this instance two plus four plus four, right, is basically the same as putting a stick in the third slot and eighth slot, correct. So that corresponds to my first possibility. Second possibility is putting a stick in the second slot and fourth slot and finally this last thing where he sells all vanilla is putting the sticks in the last two slots denoting that between the first stick and the second stick there is nothing which means he doesn't sell any chocolate flavored ice-creams and after the last stick there is empty space which means he doesn't sell any mango flavored ice-creams and so the total number of ways in which the shopkeeper can sell 10 ice-creams of three different flavors is simply the same question can be converted the question of in how many ways can you put two sticks in twelve containers separating ten containers into three different categories. Correct? You have to observe this deeply to understand what I am saying. This is one very important problem and it appears in different forms in different places. You should quickly recognize the question and you should also know the answer. So what's the answer? The answer

is 10 ice-creams plus three flavors resulting two separators. So it is three minus one. You see what happened here three minus one, observe carefully, you had three ice-cream flavors and hence you should put two sticks so that you get three categories. If there were to be five ice-cream flavors you should put four sticks. So it is ten plus three minus one and from that you should choose two placeholders where you can put the stick. So the answer is  ${}^{12}C_2$  the question and in general if you were to sell  $r$  ice-cream flavors okay and then if you need to sell it to  $n$  kids namely  $n$  ice-creams and the number of flavors is  $r$  the total number of ways in which you can sell this happens to be  $n + r - 1 C_{r - 1}$ . This concept goes by the name combinations with repetition. A very important one. We'll see more examples of this in the next video.

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