Discrete Mathematics

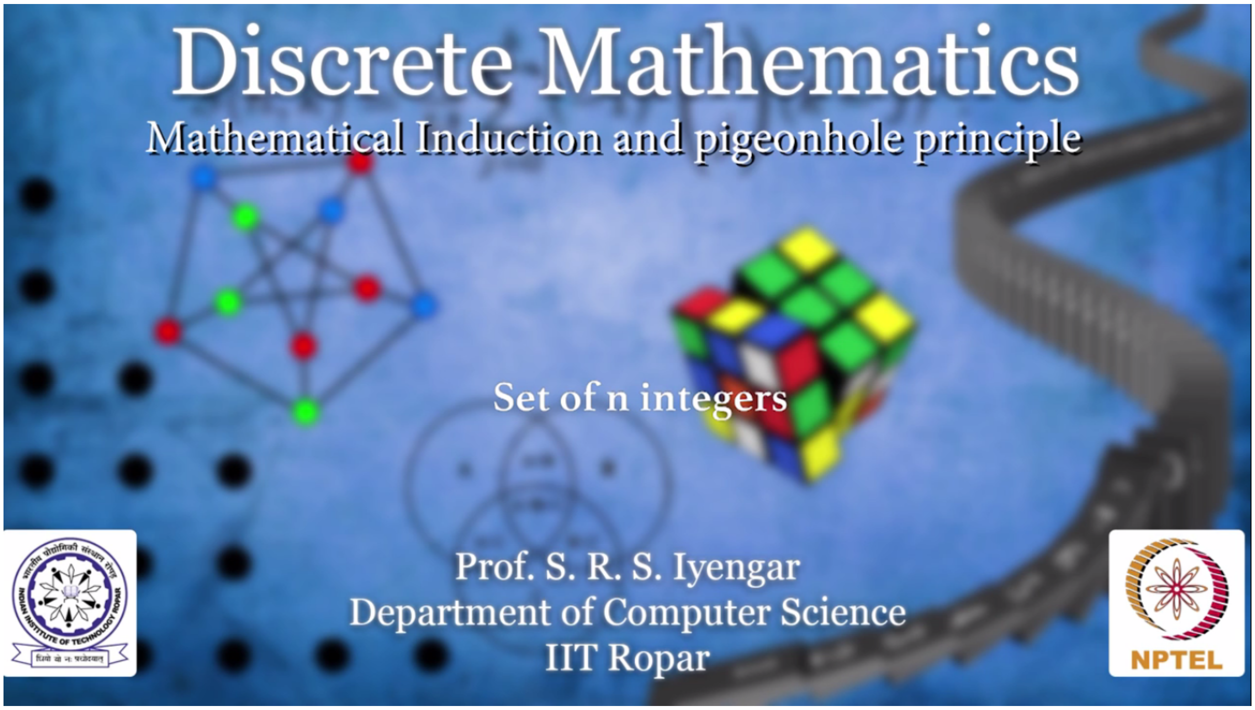
Functions

Set of n integgers

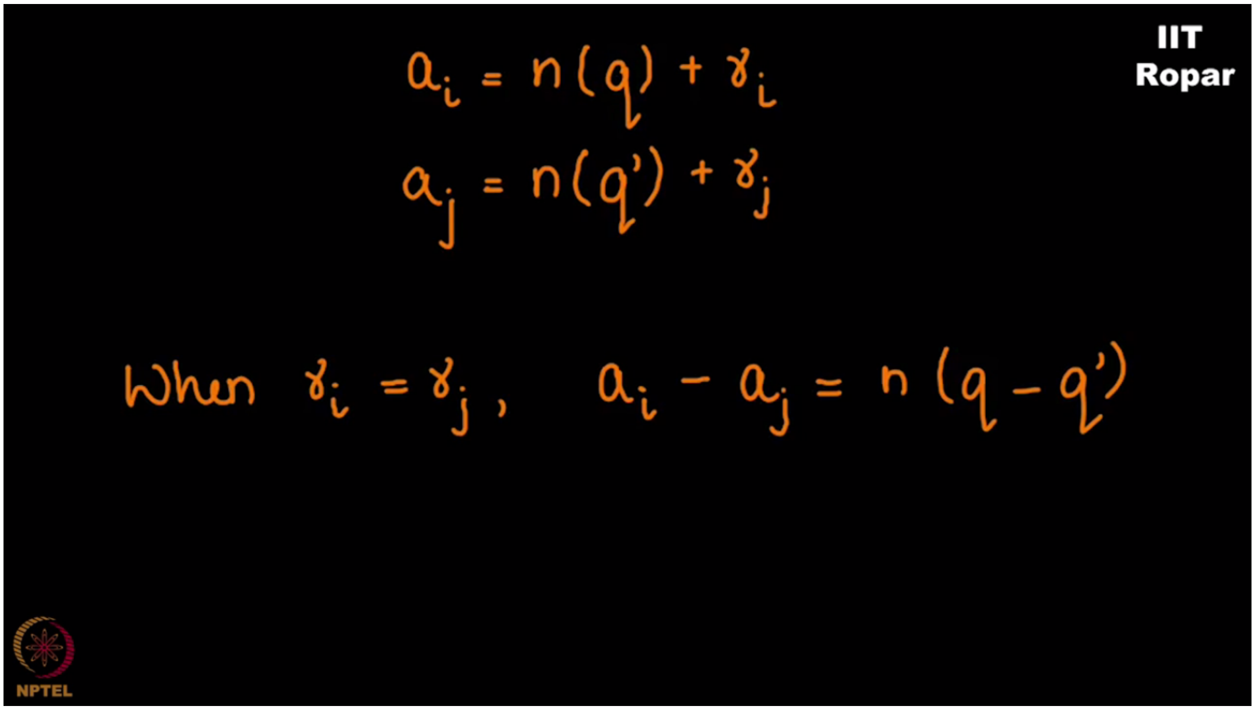
Prof S.R.S. Iyengar

Department of Computer Science

IIT Ropar



Slightly abstract problem. Consider any n+1 numbers. Let me call them a1, a2, upto an+1. My claim is you will always find two numbers such that their difference let's say ai and aj their difference is a multiple of n. That sounds very weird. What are we even stating? Let us see with an example. Take some five numbers of your choice, let's say 24, 29 and 3 and then 64 and then maybe 100. I am saying that when you take five numbers n+1 numbers, n will divide some two numbers difference. Let me see what is that difference. 24 and 29 difference is 5, four digit divide 5. 24 and 3 difference is 21, no it doesn't. 29 and 3 and is 26, No. 24 and 64 yes look at that. The difference is 40. 64-24 is 40 and 4 divides 40. When you take any five numbers you will find at least one such pair such that it's difference is divisible by 4. Now how is that true? Let us try to show it in general. So you take n+1 numbers, our claim is that you will always have two numbers whose difference is divisible by n. How is that? You observe something. When you take n+1 numbers each number when divided by n will leave reminder 0, 1, 2, upto n-1. So each one of these a1, a2, upto an+1 each one of them will leave the remainder let's say r1, r2, respectively rn+1 when divided by n and each of these ris should be number between 0,1 and -1. So there are n possible remainders 0 to n-1 is n, note that and there are n+1 numbers which means some ri will be equal to rj which means there is a ai and aj so that they leave the same remainder when divided by n. When you take two numbers which leave the same remainder when divided by n then n divides ai-aj you tell me why. I am skipping a step here. I am giving you a small exercise to solve.



Whenever you have two numbers having the same remainder their difference will be divisible by n. Let's see this with an example. Let's say take a number 23 and another number let's say 48 and then look at 5 being n so basically you see that 5 and divide 23 when divided by 5 leaves 3 as remainder. 48 when divided by leaves 3 as remainder. It's the same remainder so their difference will always be a multiple of 5. Rather it's very obvious to see that. Why? You check it out. So any number let's say ai can be written as n times the quotient plus some remainder ri. Correct. Aj can be written as n times some other quotient q' plus rj. When ri is equal to rj, ai-aj will be equal to n times q-q' plus ri-rj which you know is 0 so ai-aj as you can see on the right hand side is a multiple of n. So it's as easy as that.