NPTEL
NPTEL ONLINE COURSE

Discrete Mathematics
Mathematical Induction and pigeonhole principle

Group of n people

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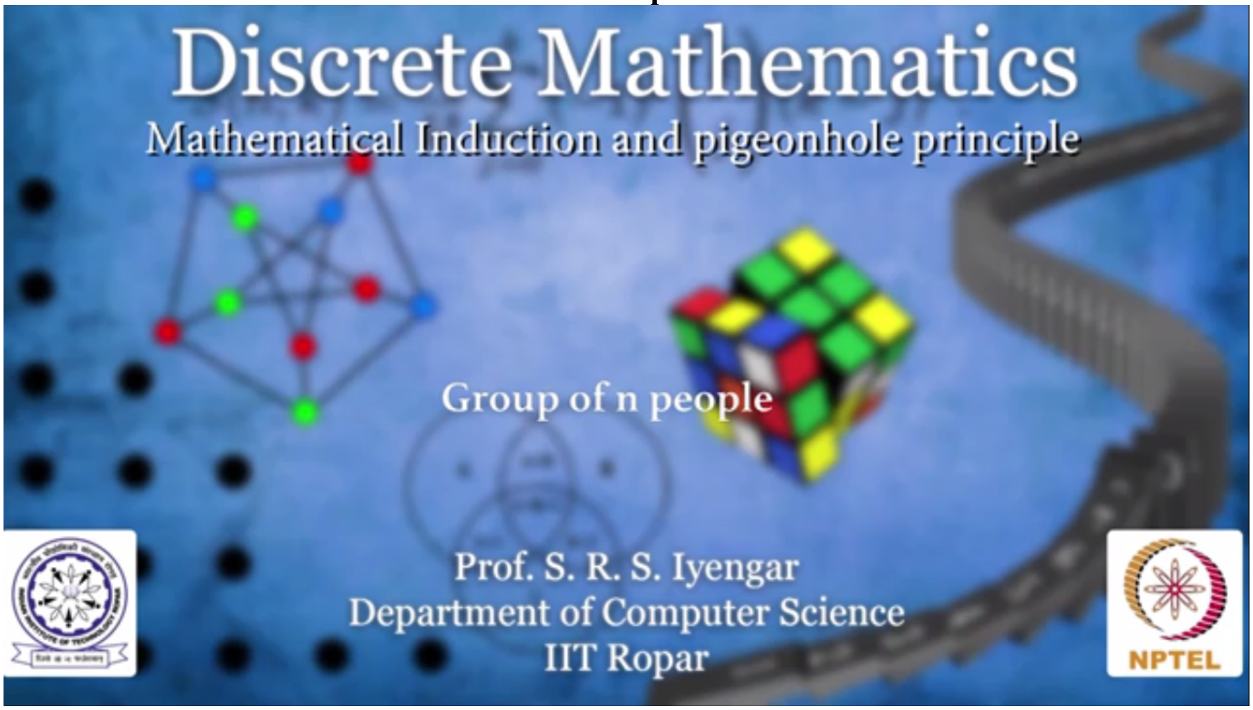
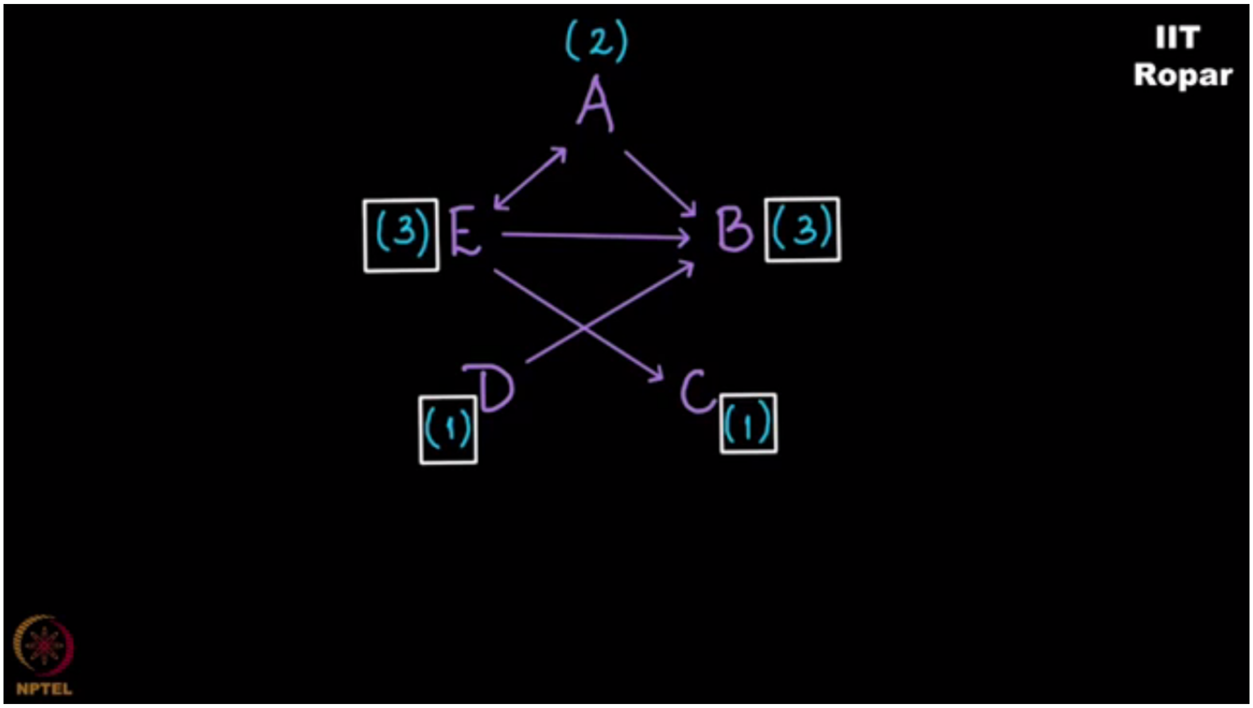
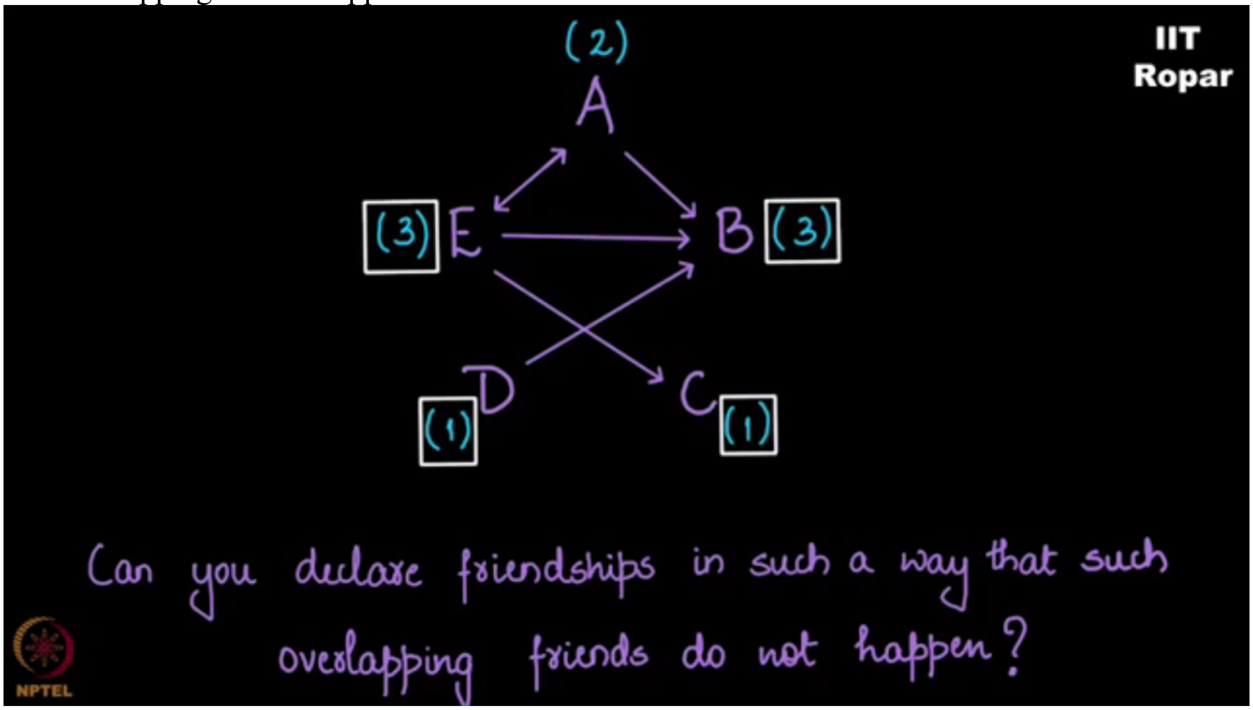


Assume there are 5 friends, okay, and each one of them, let me name them as A, B, C, D and E is friends with the others. A knows B and E. E knows A, B, and let's say C. D knows let's say B alone. Now you see the number of friends of A is 2; number of friends of B is 3; E is 3; D is 1; and C is 1.

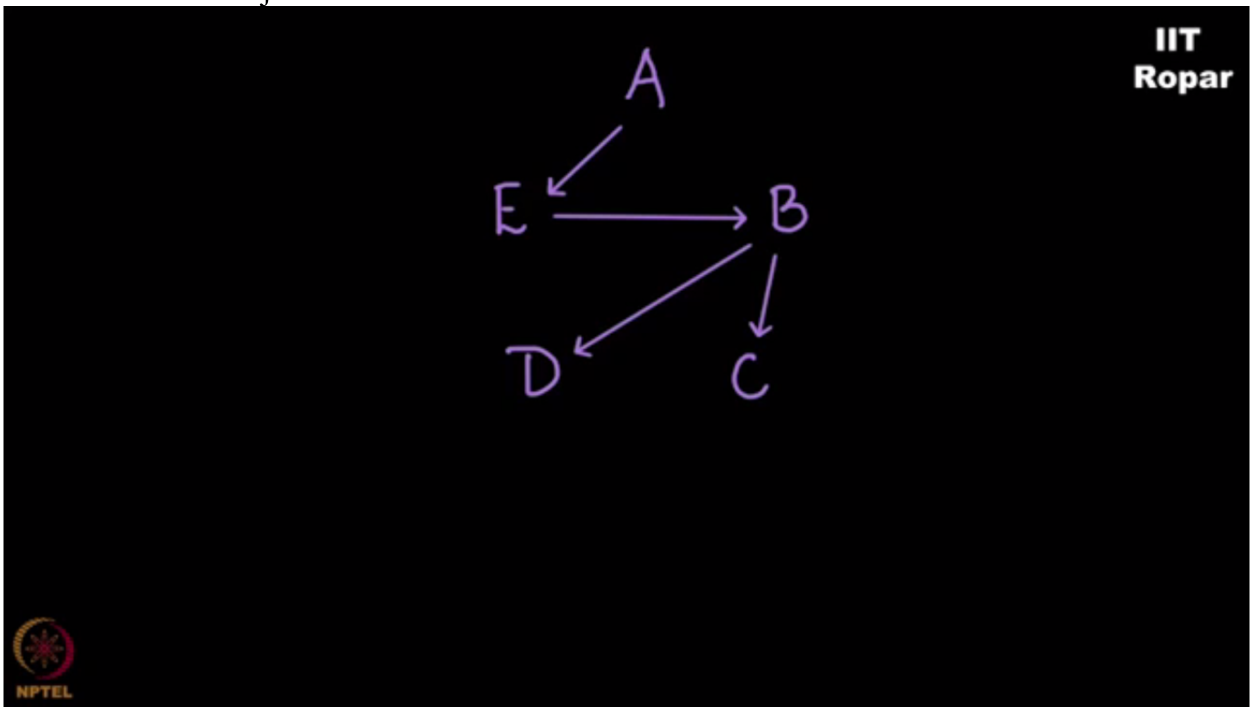


Do you see D and C have the same number of friends? So do B and E.

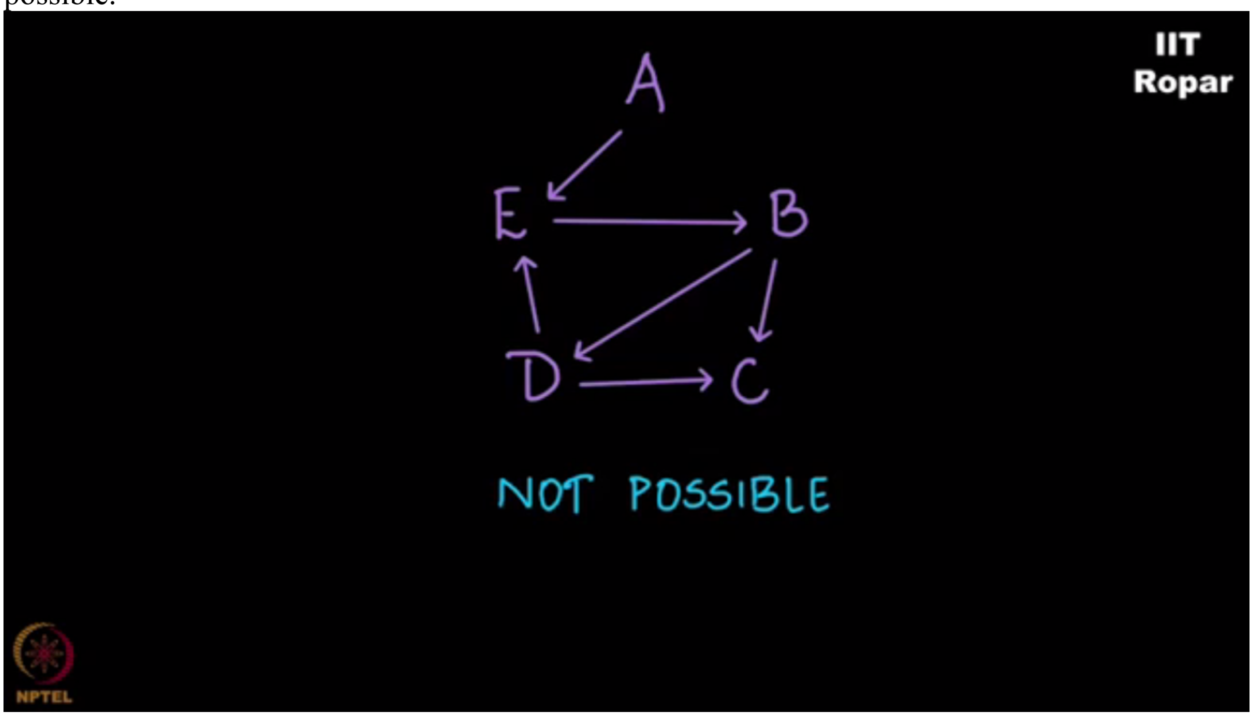
Now can you try to write these five dots A, B, C, D and E such that you declare friendships in such a way that such overlapping number of friends do not happen? Right? How do we ensure that overlapping doesn't happen?



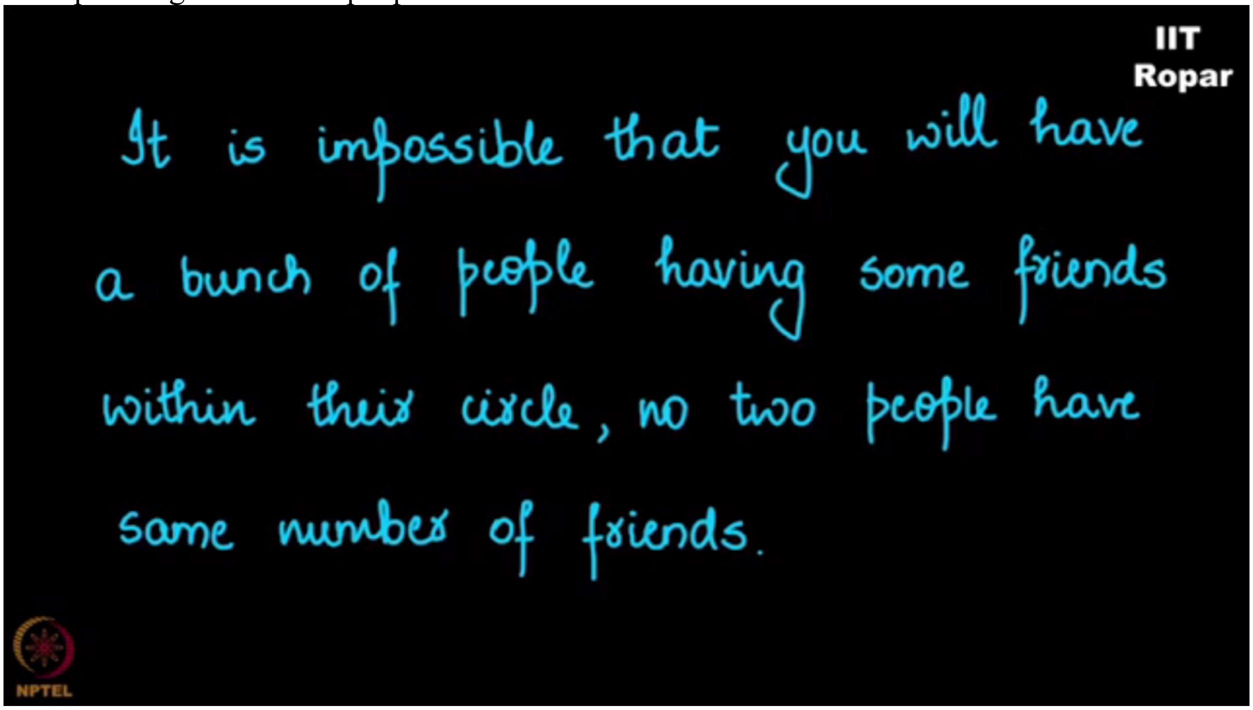
So I will try to ensure that A is friends with only one person. E is friends with 2 people like this and B is friends with 3 people let's say like this, 1, 2 and B is 3. So D must be 4 now. How do I make this 4? I must join him to E and that will disturb E.



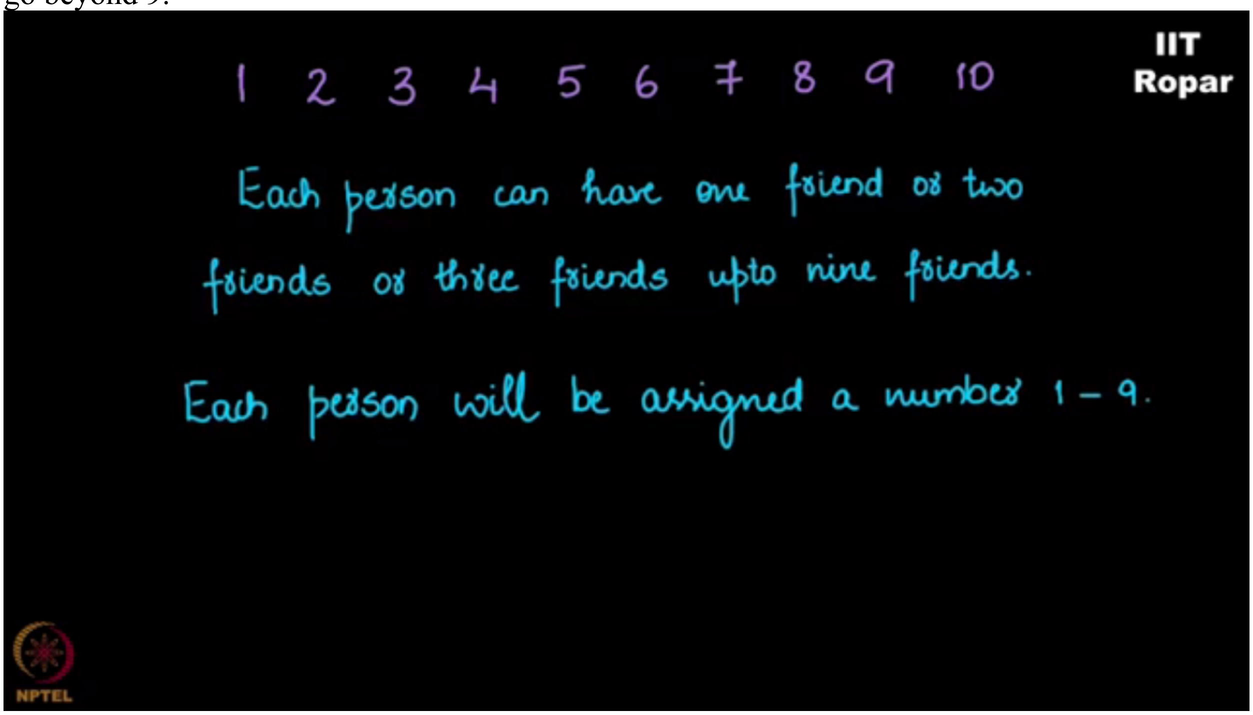
Now it looks like you keep trying, you keep trying. You will realize that this is not really possible.



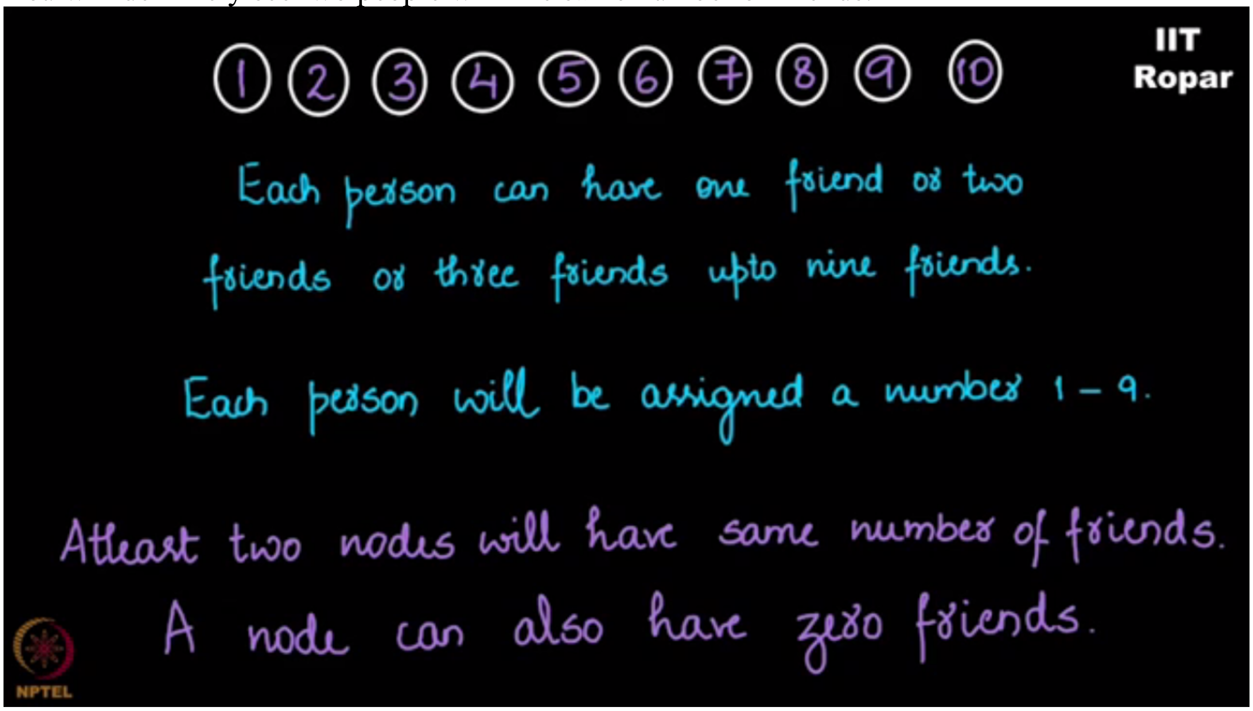
Not just on 5 friends. You take any number of friends. This is plain impossible. What is impossible? It is impossible that you will have a bunch of people having some friends within their circle such that no two people have the same number of friends. It's impossible and you will end up having at least two people with the same number of friends. How is this true?



Pigeonhole Principle to our rescue once again. You see every person will have either – assume we have 100 such people. Okay. Let’s say 10 such people for simplicity sake I will say. I will take 10 people. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 10 such people. Each person can have either one friend, two friends, or three friends up to nine friends. You see if there are 10 people, you cannot go beyond 9.



So each person will basically be assigned a number from 1 to 9. There are actually 9 – there are 9 possible assignments for a node, but there are 10 nodes and hence you will find at least two nodes with the same number of friends. Correct? And that shows that take any number of friends. You will definitely see two people with the same number of friends.



But then there is a small problem here. I will state the problems. You should fix it. A node can also have zero number of friends. That way it becomes 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This isn't a problem. Tell me why? Think about it.

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