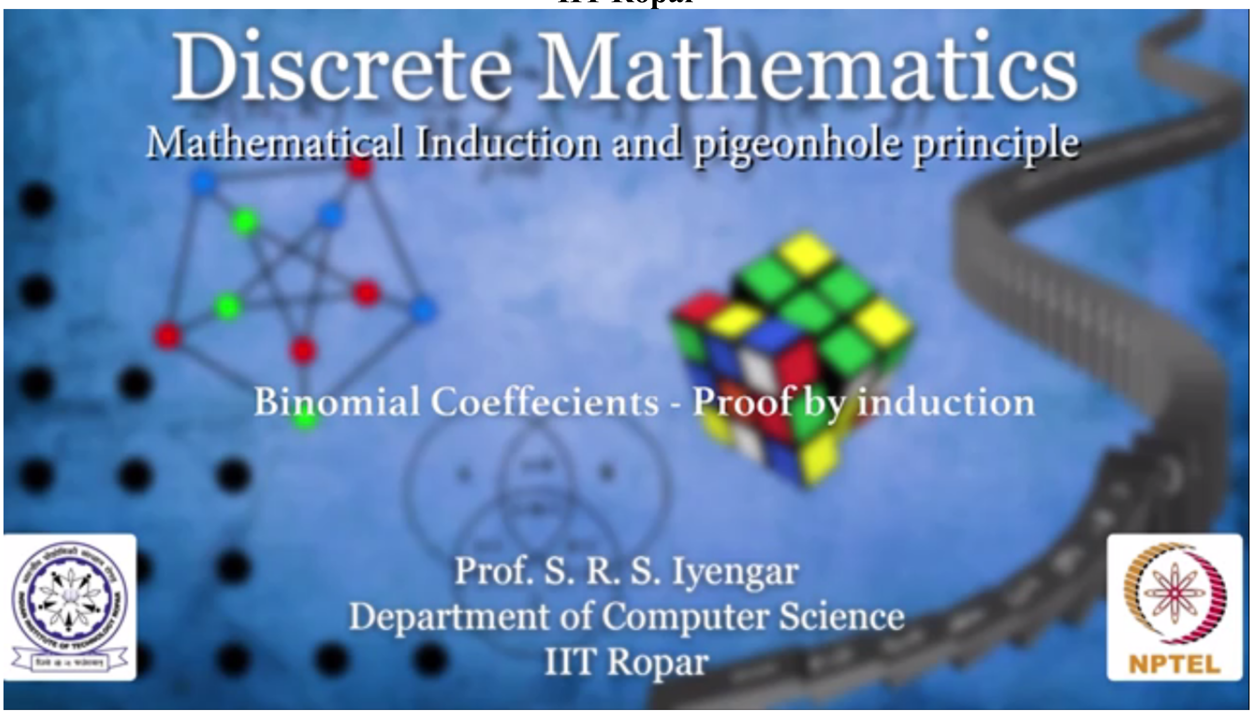
NPTEL
NPTEL ONLINE COURSE

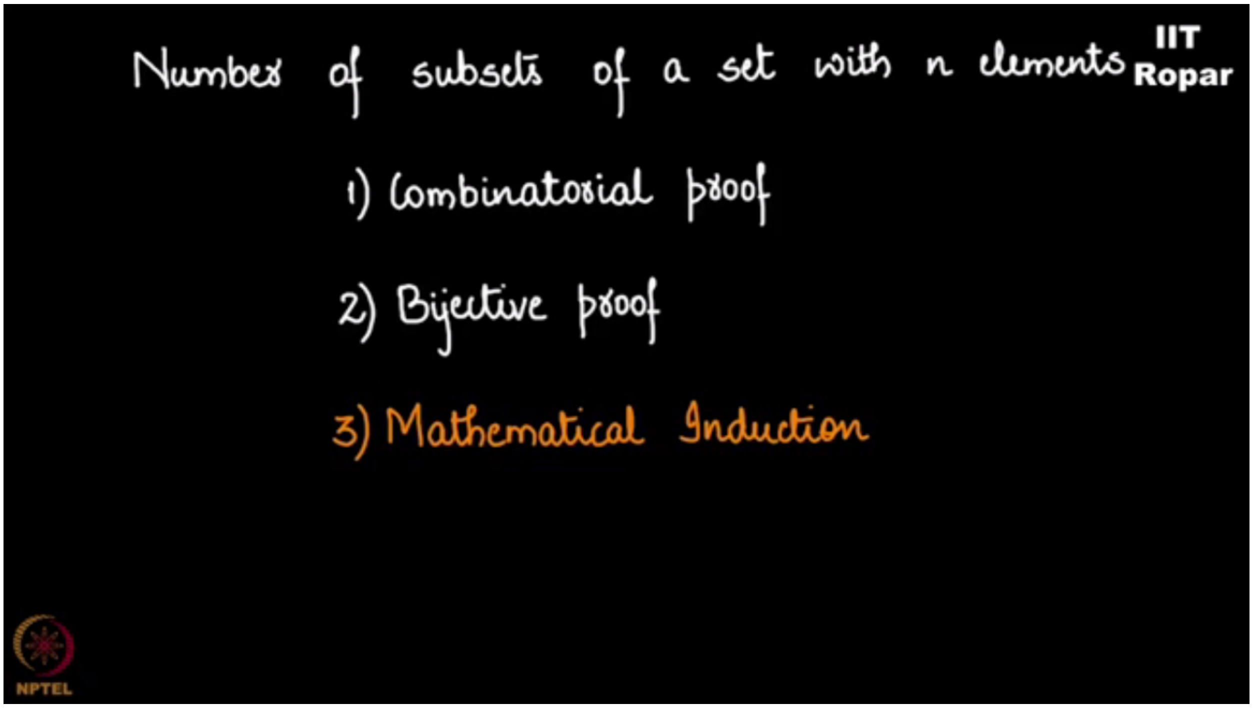
Discrete Mathematics
Mathematical Induction and pigeonhole principle

Binomial Coeffecients - Proof by induction

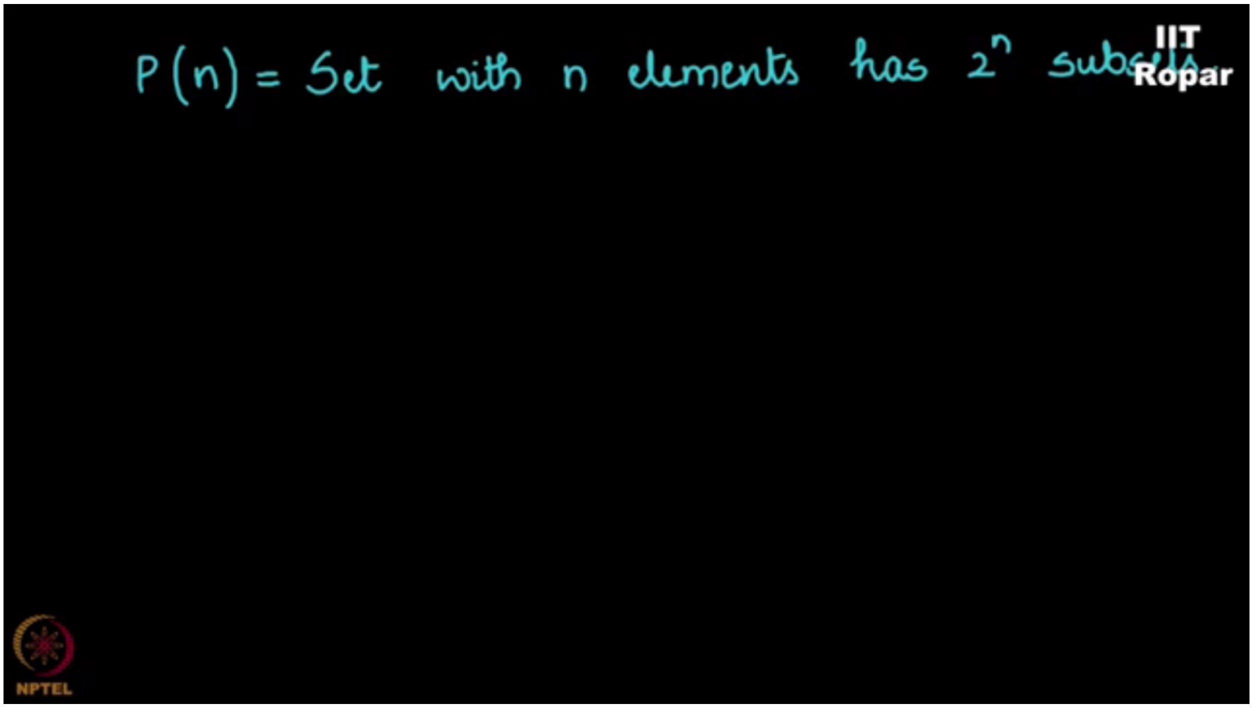
Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar



We have been seeing the proof of the number of subsets of a set of n elements from week one. We gave first a combinatorial proof for it using the binomial theorem and then we moved on to show a bijection using three digit binary numbers to all possible subsets. Well, now we'll move ahead and give a Mathematical Induction proof for showing that all possible subsets of a set of n elements are 2^n number.

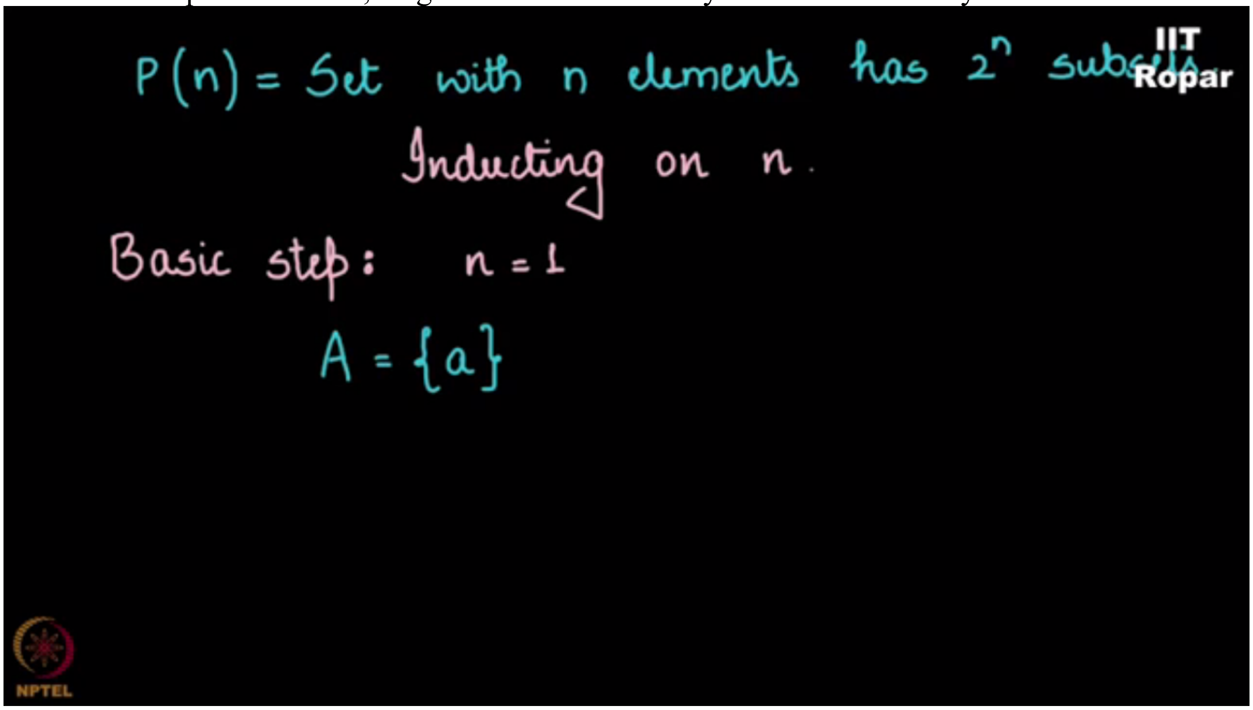


How do we proceed? As you all know, what is our statement given here? It is I will consider it to be P(n). It is set with n elements has 2^n subsets. So this is my statement. I use induction to prove this.

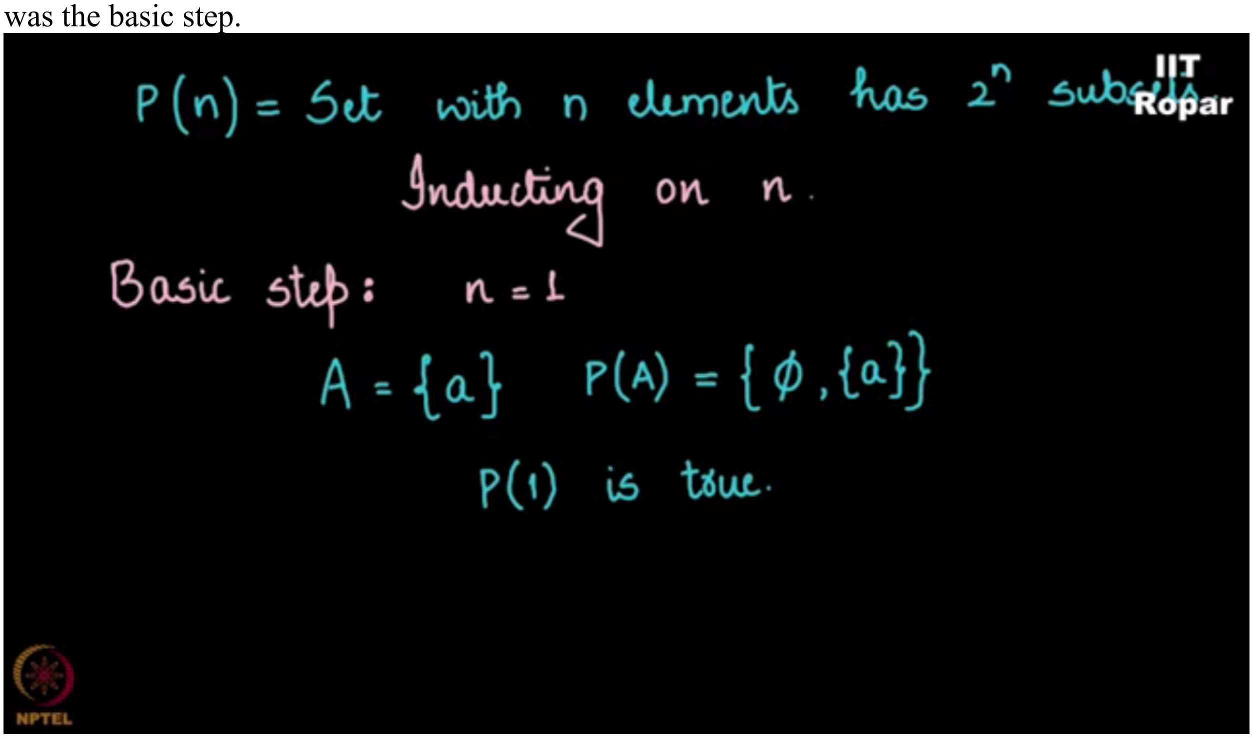


You must be familiar with how induction goes on. We will first consider basic step, and then we'll see what is the hypothesis and so on. If you are still not familiar, you will understand with this problem and the further problems.

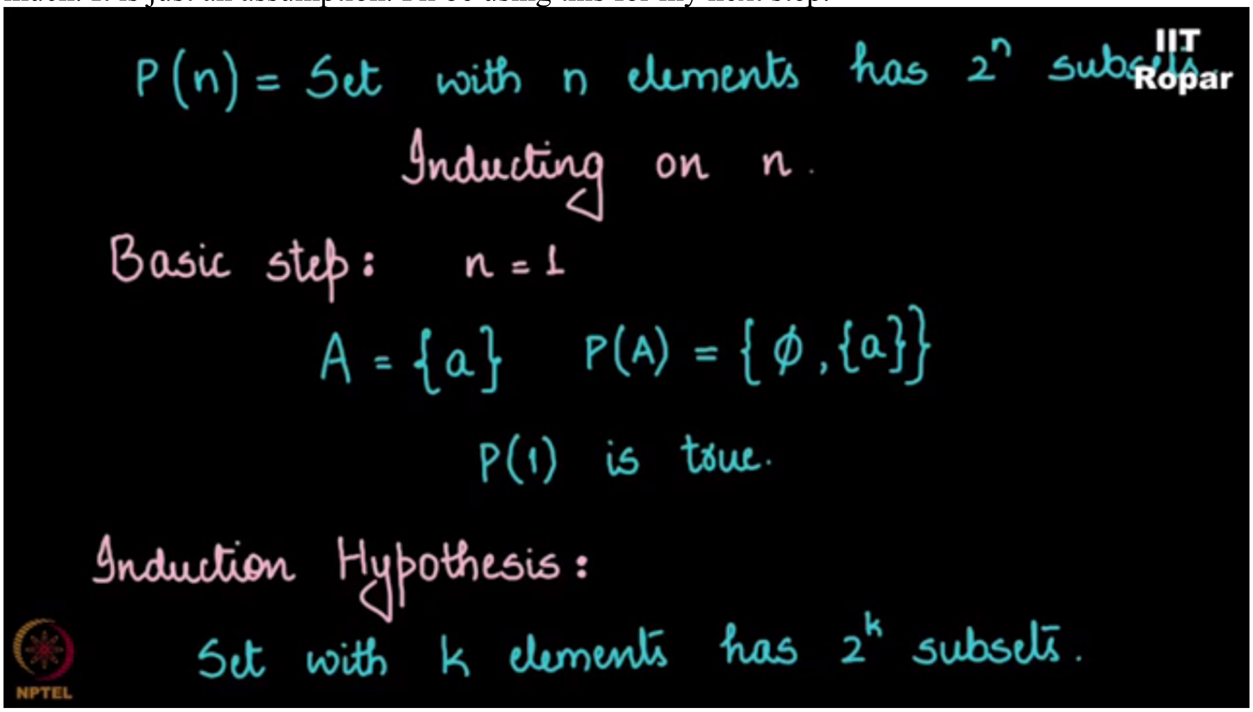
So what is the first step? I will consider a basic step where I will consider the number of elements in A to be 1. Please note what are we inducting on? We are inducting on n here. Okay. So I will write basic step n = 1. So what does n = 1 mean? The cardinality of A is 1. So I'll assume A is equal to this set, singleton set A. So a is my element here. Okay.



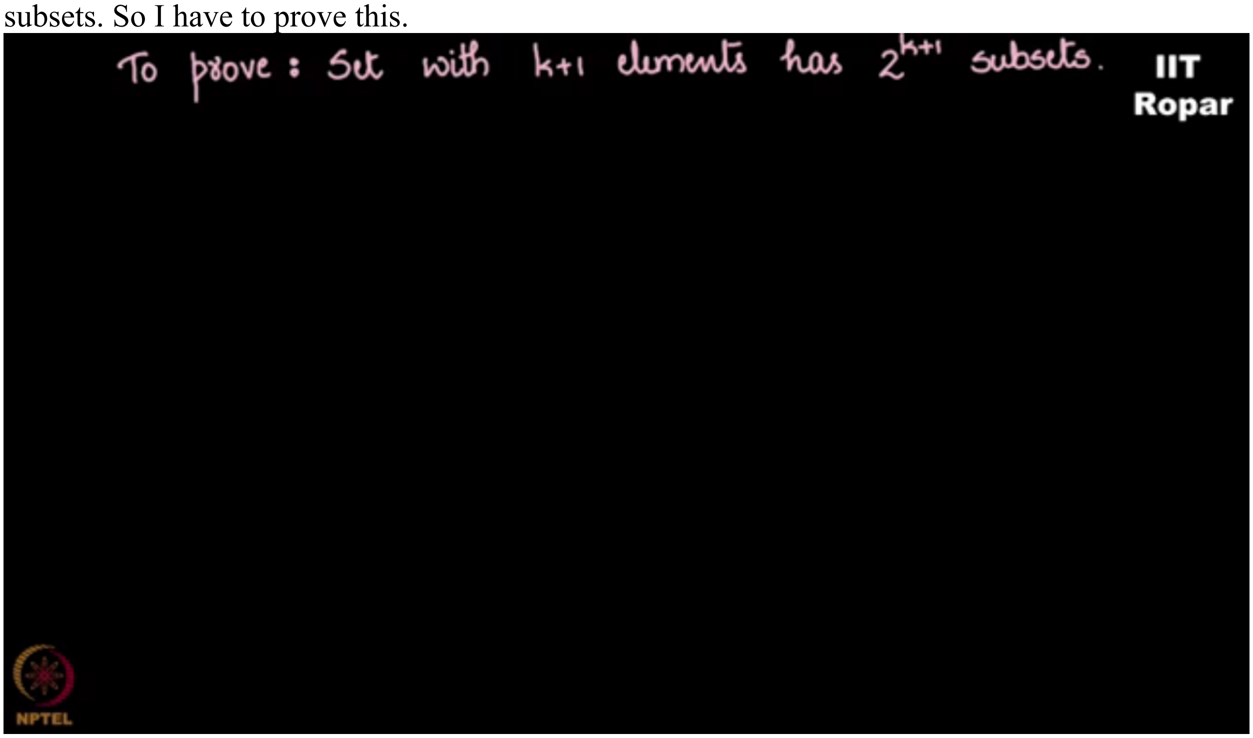
Now what are the power – what are the elements in the power set of A? Very obvious, phi and a itself. So for n = 1, the given statement holds true. For one element, we have two subsets. So this was the basic step.



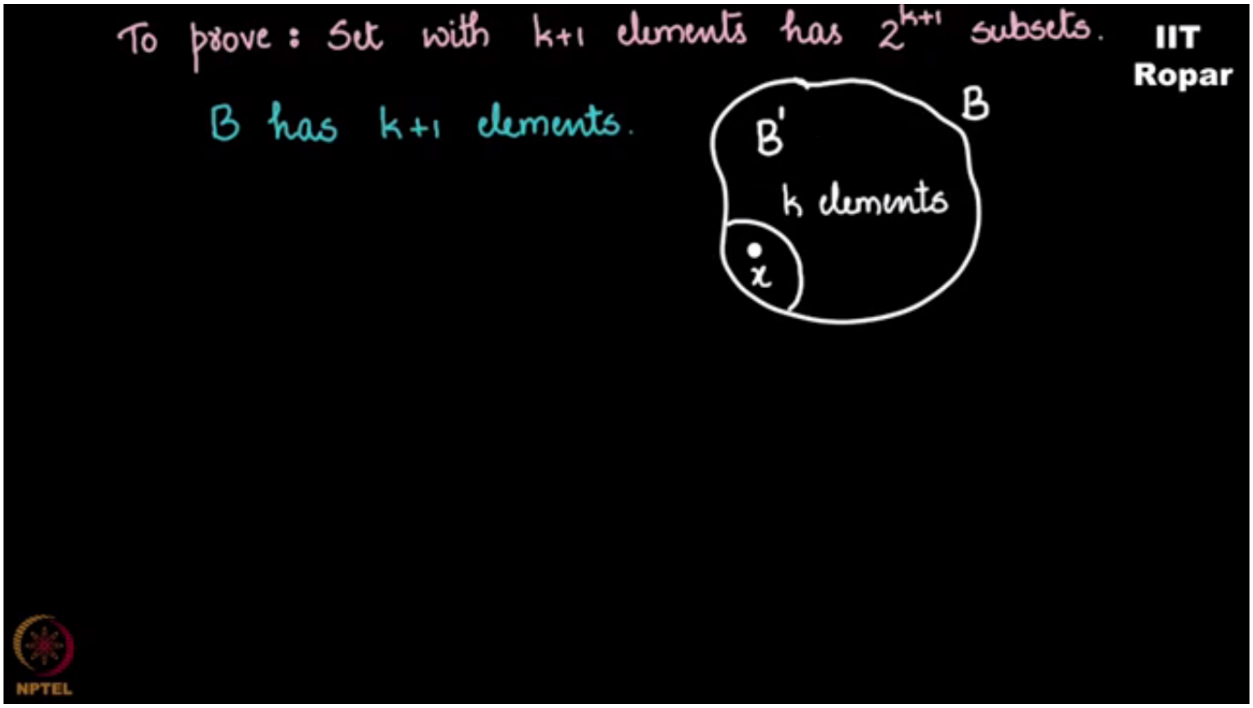
Now let us move on to the Induction Hypothesis. What do I mean here? I assume that this statement is true for some k elements. So this I will call it as Induction Hypothesis. Do not worry much. It is just an assumption. I'll be using this for my next step.



So Induction Hypothesis goes like this. I assume set with k elements has 2^k subsets. Now what do I have to prove next? I have to prove that, so to prove is set with k+1 elements has 2^{k+1} subsets. So I have to prove this.

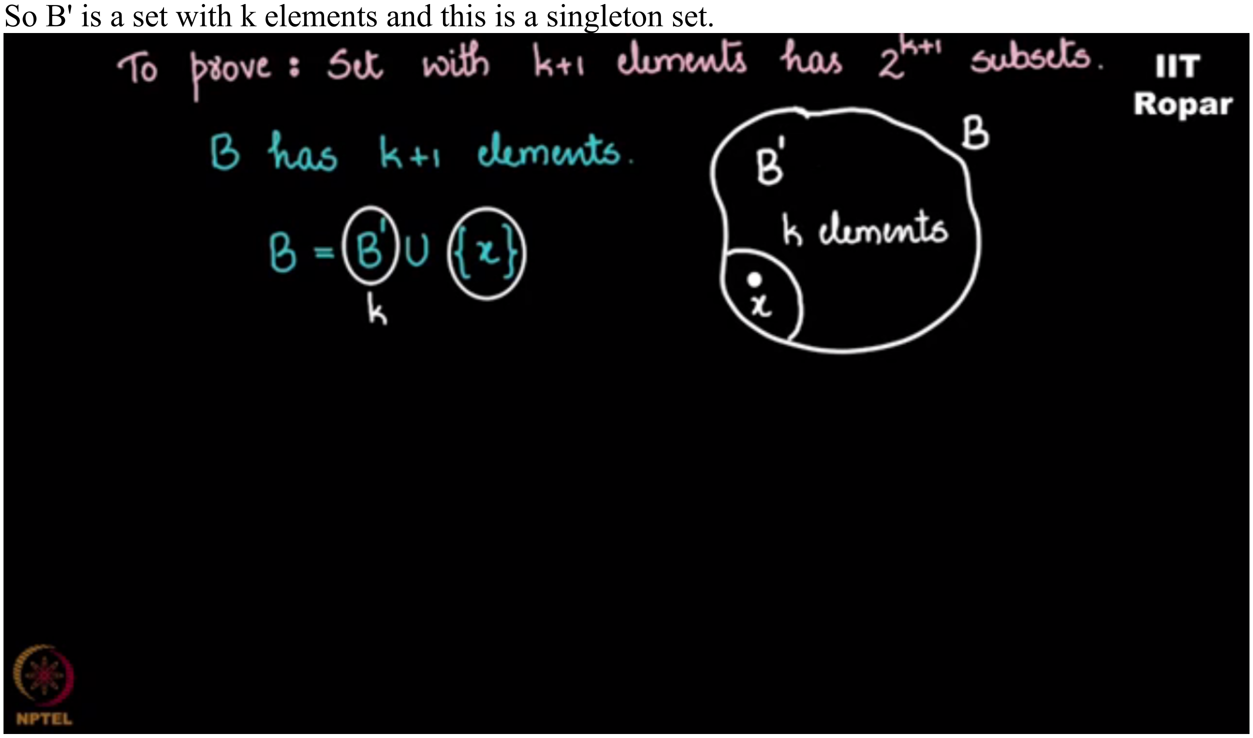


So what do we have to do now? I have this set B, which has k+1 elements. How am I going to proceed? I'll break this into two parts: one set having k elements and a singleton set. Okay.



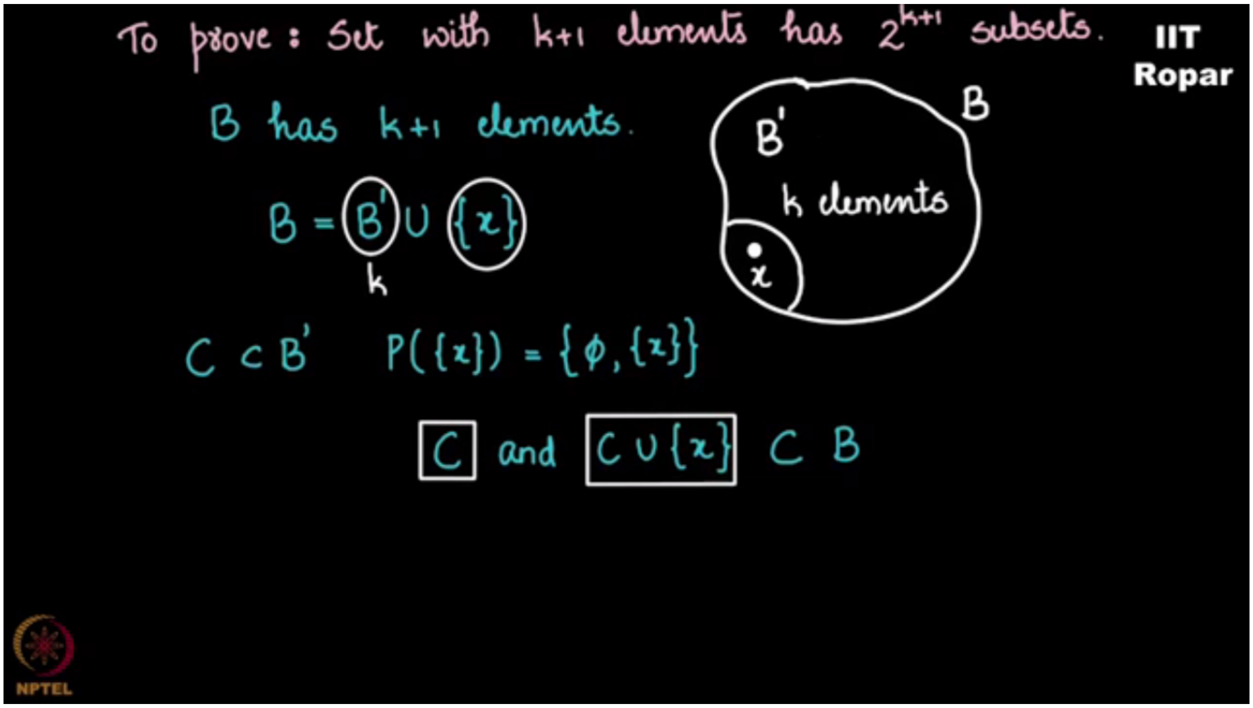
So you may want to pause here and go back to the Induction Hypothesis. Why? Because the statement we are assuming that the statement holds true for k elements which is the reason why I'm considering a set with k elements here.

So let me assume B' is such a subset of B which has k elements. So I'll write B to be B' ∪ {x}. So B' is a set with k elements and this is a singleton set.

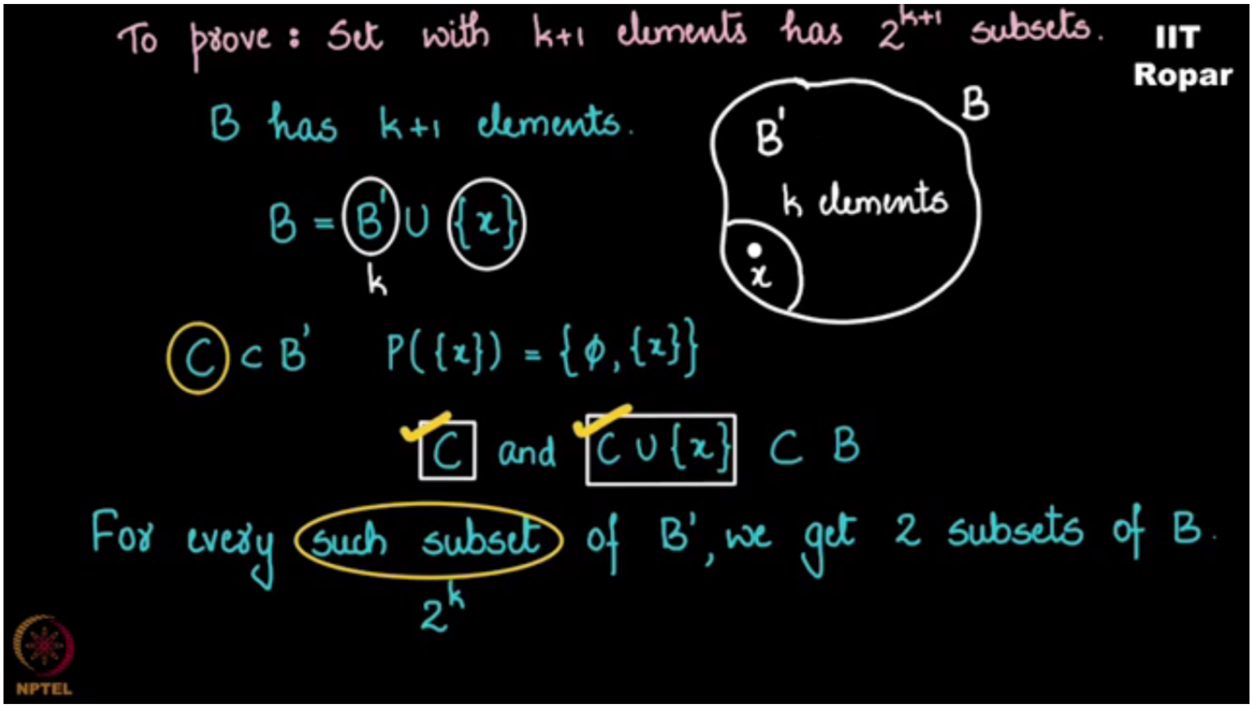


Now please listen. I'll consider a subset of B'. let me say it as C and how many elements are there in the power set of x? It is phi and {x} itself. Correct?

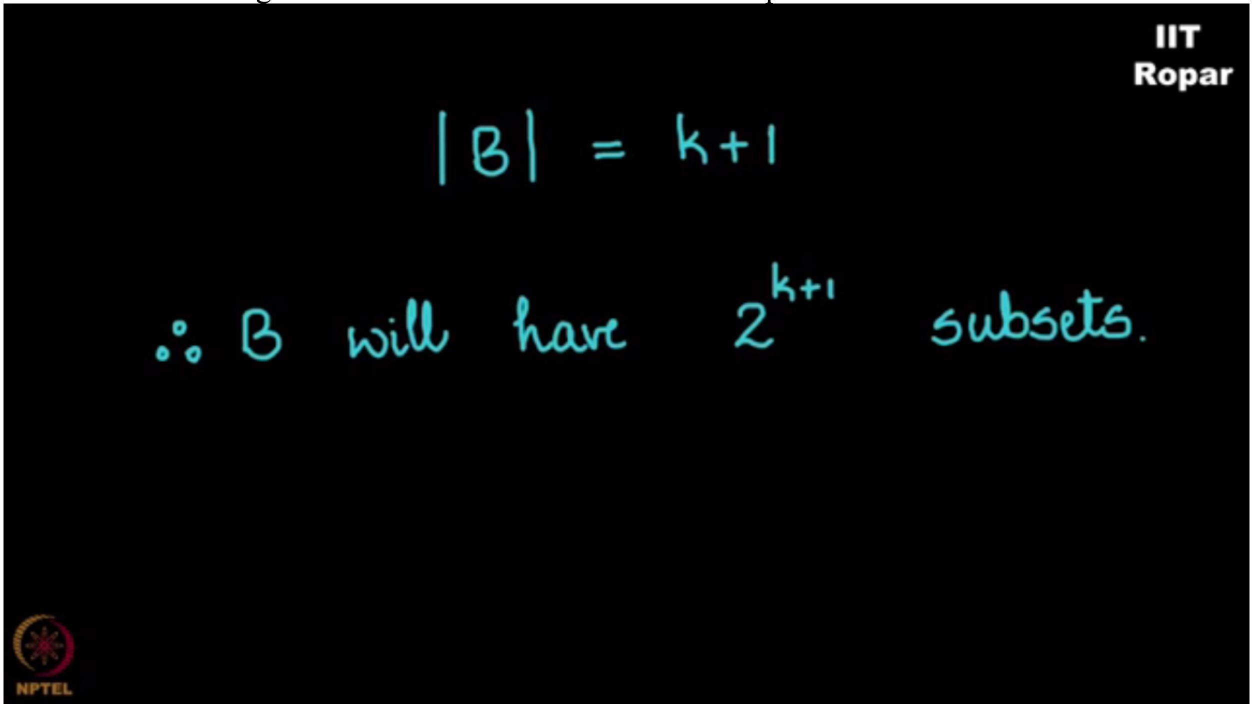
So now observe this. C ∪ null is C itself and C ∪ {x}, these two sets are subsets of my set B. Is this point clear? So I'm getting two subsets here. One is C and one is C ∪ {x}. This singleton set x.



So where did I initially start? I considered a subset of B' and I ended up having two subsets of the set B. So what can I conclude? For every such subset of B', C, T, S whatever, for every such subset of B', I am going to end up having two subsets of B.



Now how many such subsets are there? 2^k. How did I get this? From the Induction Hypothesis. So B is a set having k+1 elements and I am getting every time for every subset of B' I'm getting two subsets. Hence B will have 2 x 2^k subsets, which is 2^{k+1}. Hence I have concluded that B, which is a set having k +1 elements has 2^{k+1} elements in its power set.



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