NPTEL
NPTEL ONLINE COURSE

Discrete Mathematics
Mathematical Induction and pigeonhole principle

Mathematical Induction - Example 9

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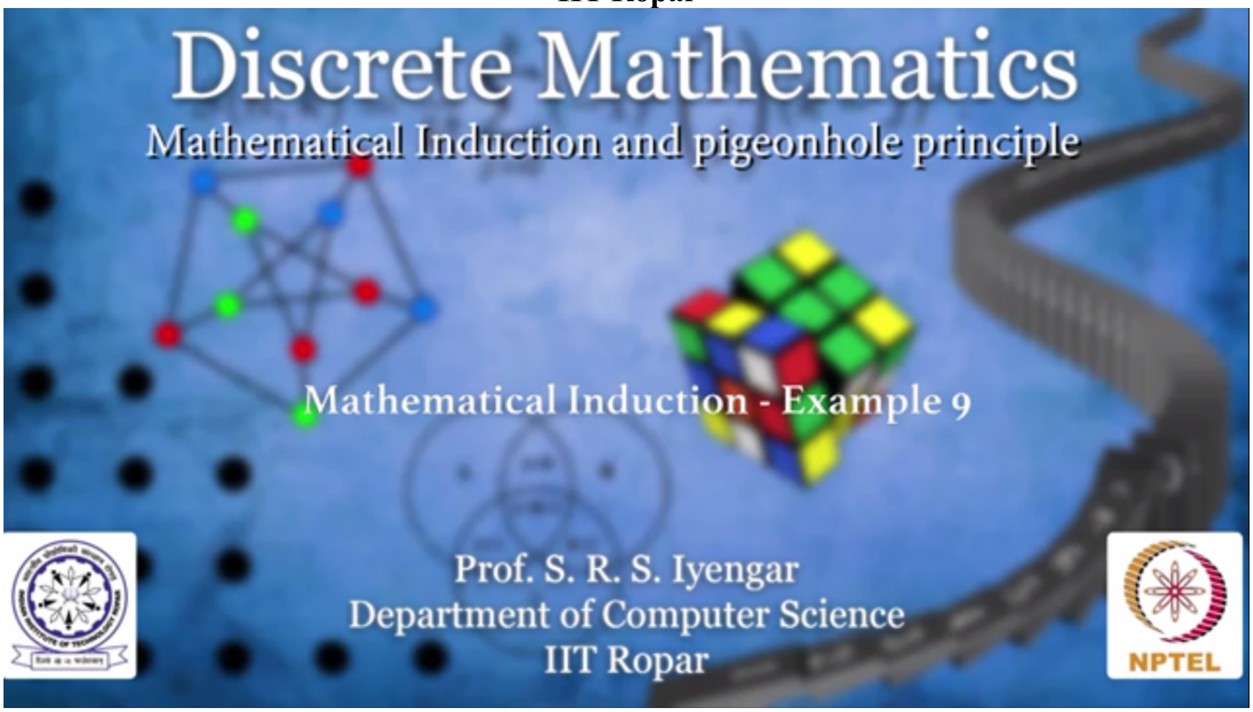
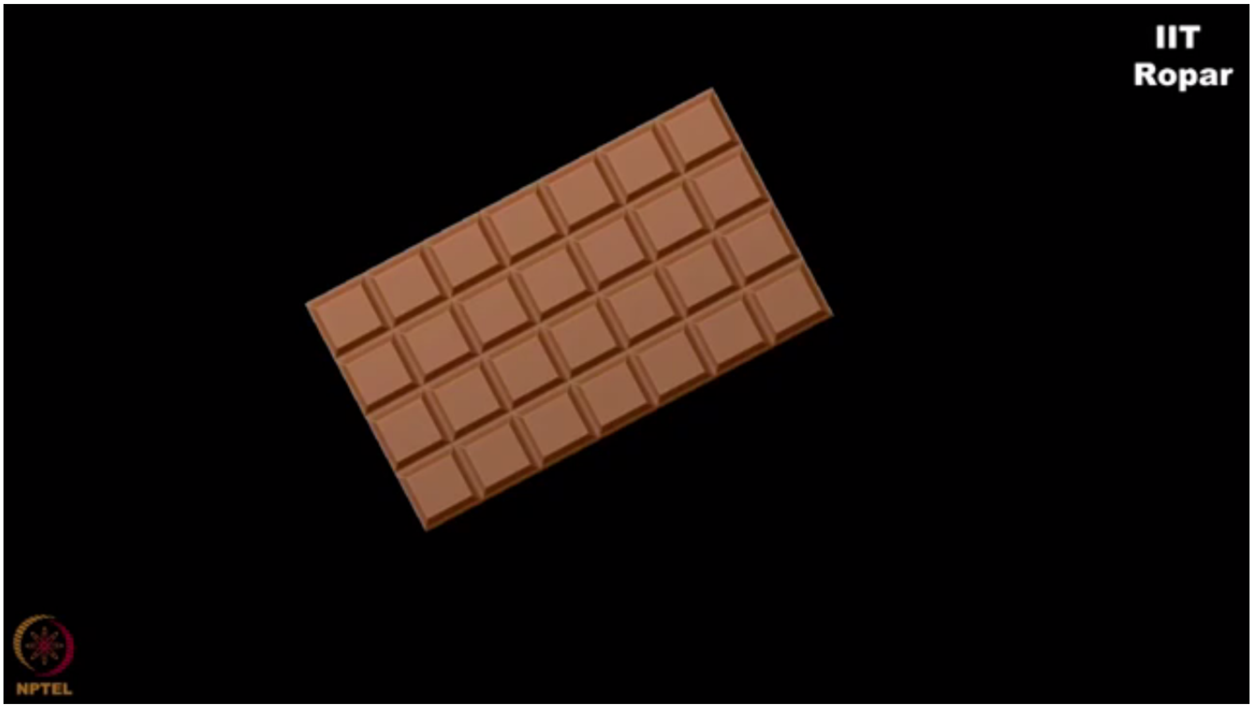
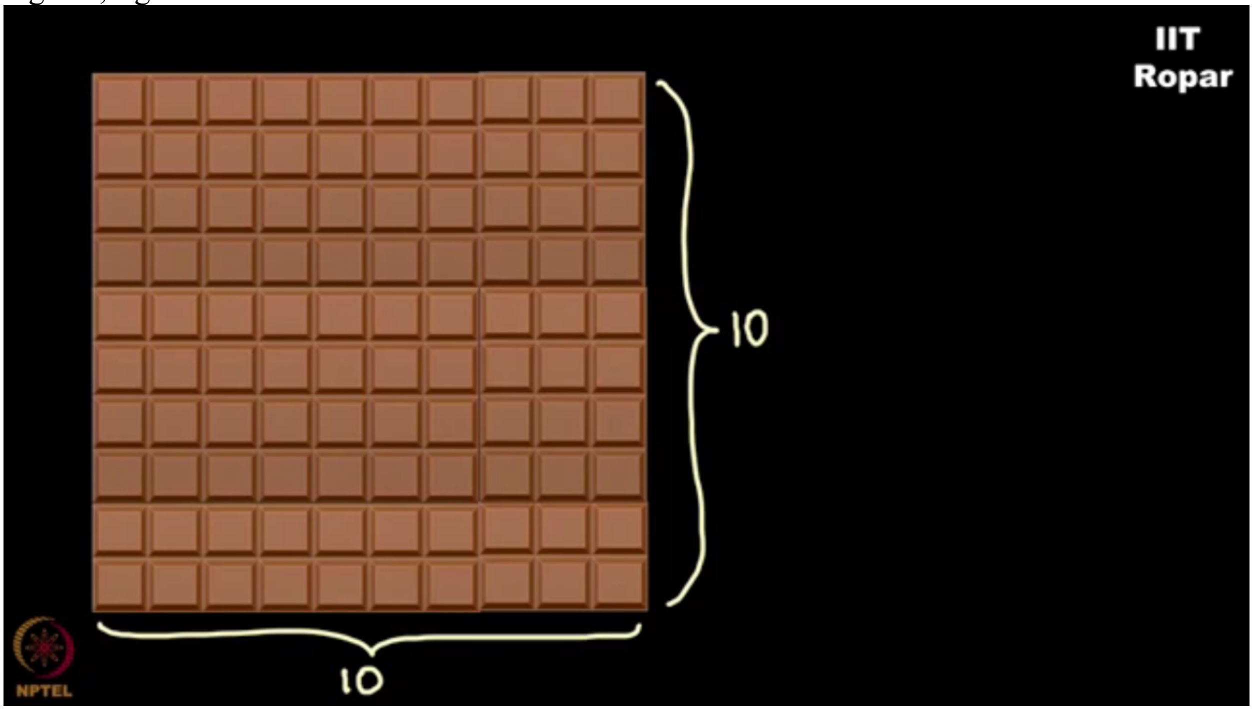


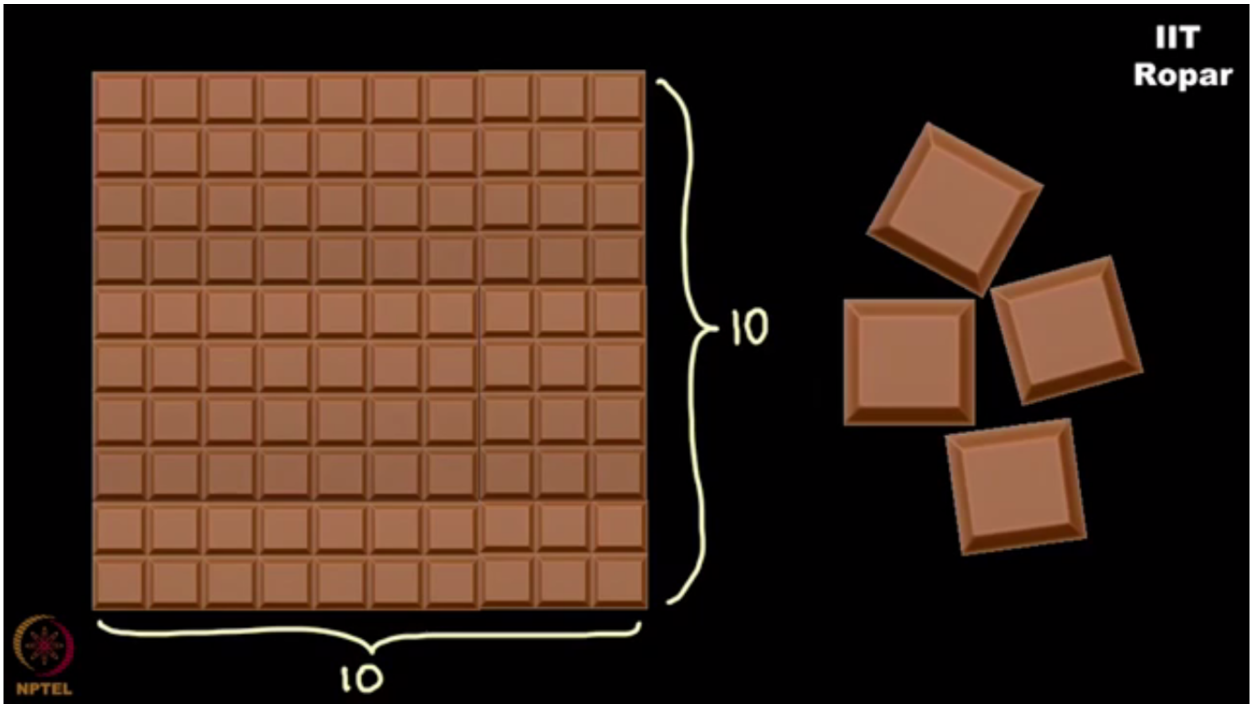
Consider this problem of taking a chocolate bar. How is a chocolate bar? Of course, different chocolates are made differently, but I'm talking about this kind of a chocolate bar.



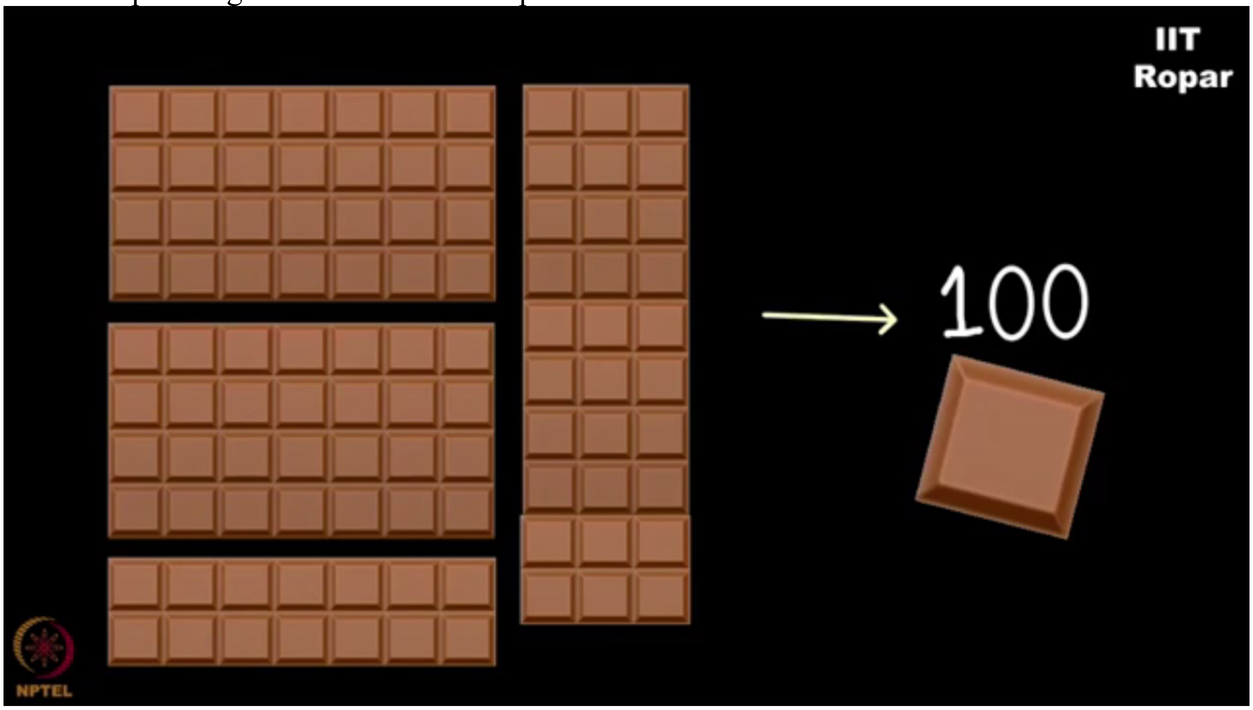
You see, it is made up of several squares glued with each other. Here is a 10 x 10 chocolate bar, a big one, right?



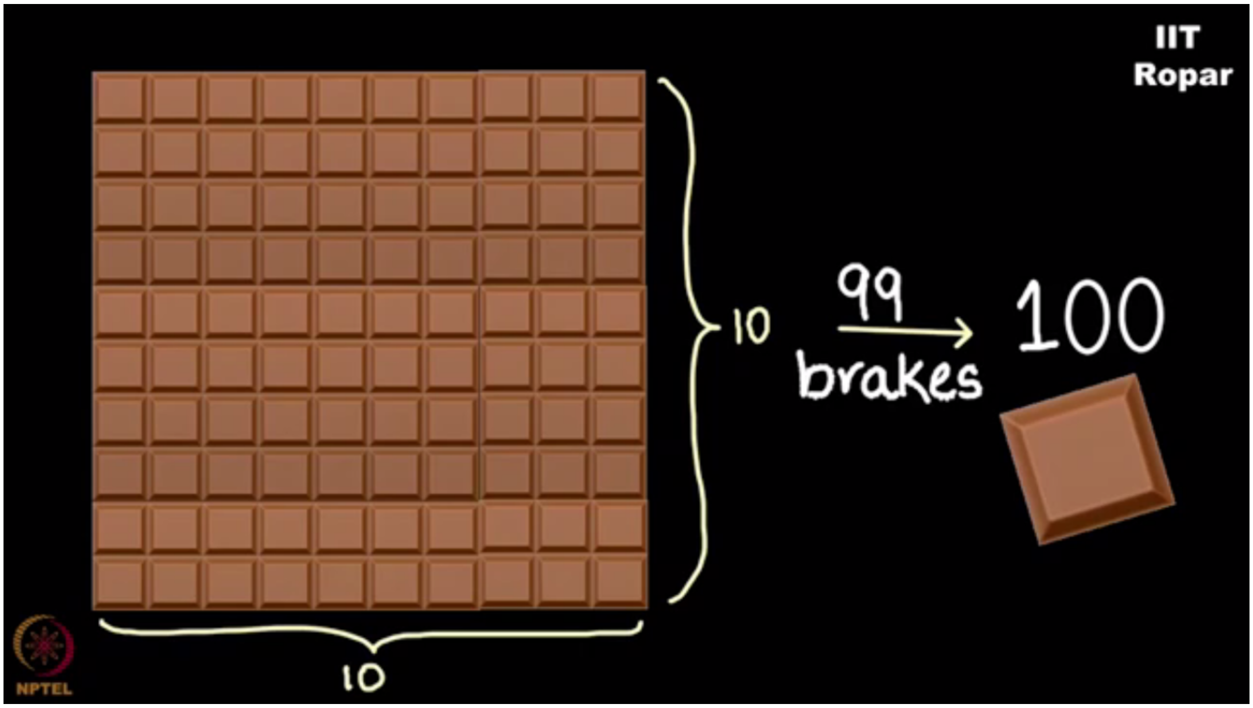
Now I want you to slice this up so that you collect the individual chocolates like this.



How will you do it? You can simply keep breaking them at different places and then finally you will end up having individual chocolate pieces.

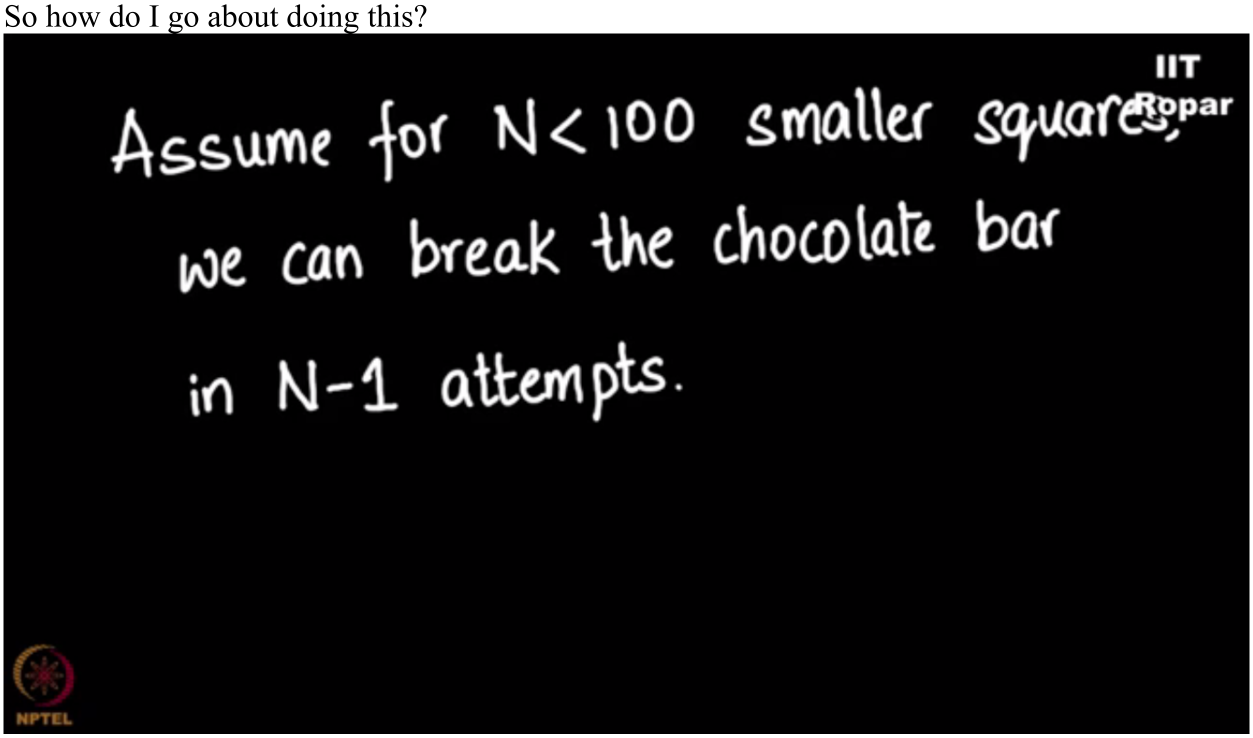


Now I say given this chocolate bar with 10 x 10 chocolates, which is 100 chocolates, with 99 breaks you will be able to get all the hundred pieces like this. Okay.

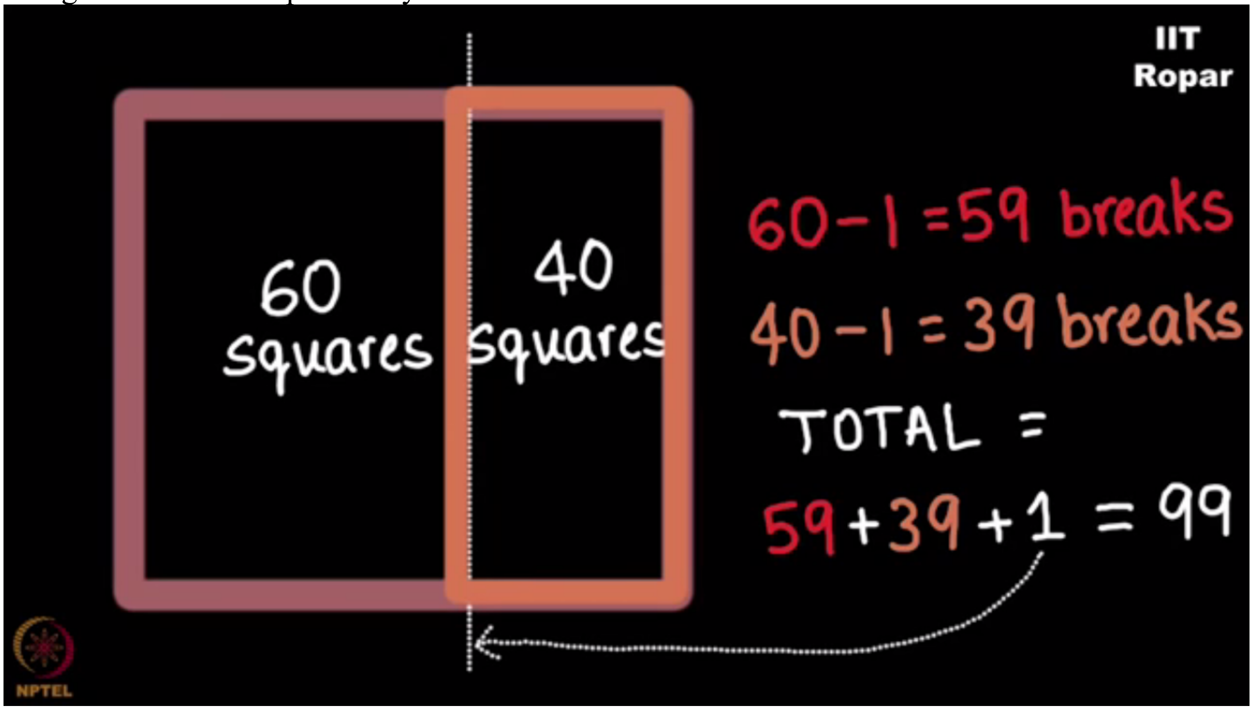


How can you do it?

Now let me assume that if a chocolate bar is less than 100 in size, this is doable. For anything less than 100, this is doable. Let me see with that if I can conclude that for 100 it is doable. Okay. So how do I go about doing this?

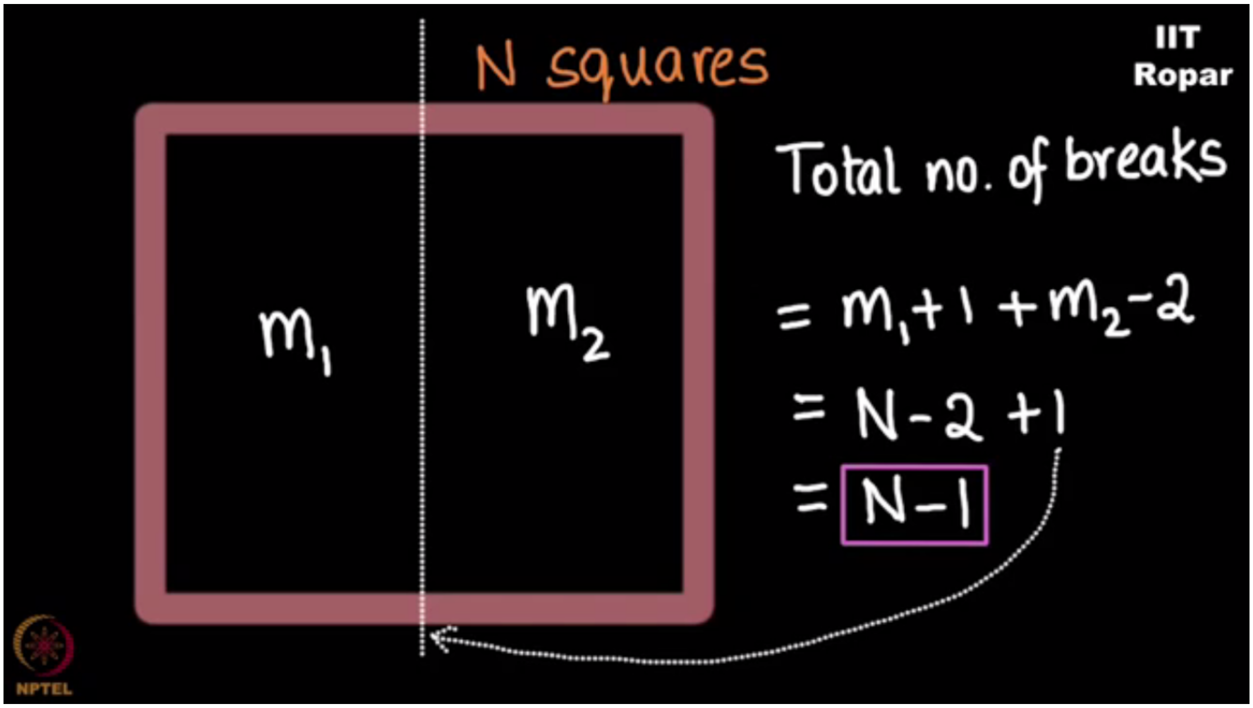


Now I take this 100 chocolate bar and then break it into two pieces along some line. Maybe I will get a 60 and a 40. But I know a chocolate bar less than 100 obeys this rule that in N-1 cuts you will get the chocolate pieces if you know what I mean.

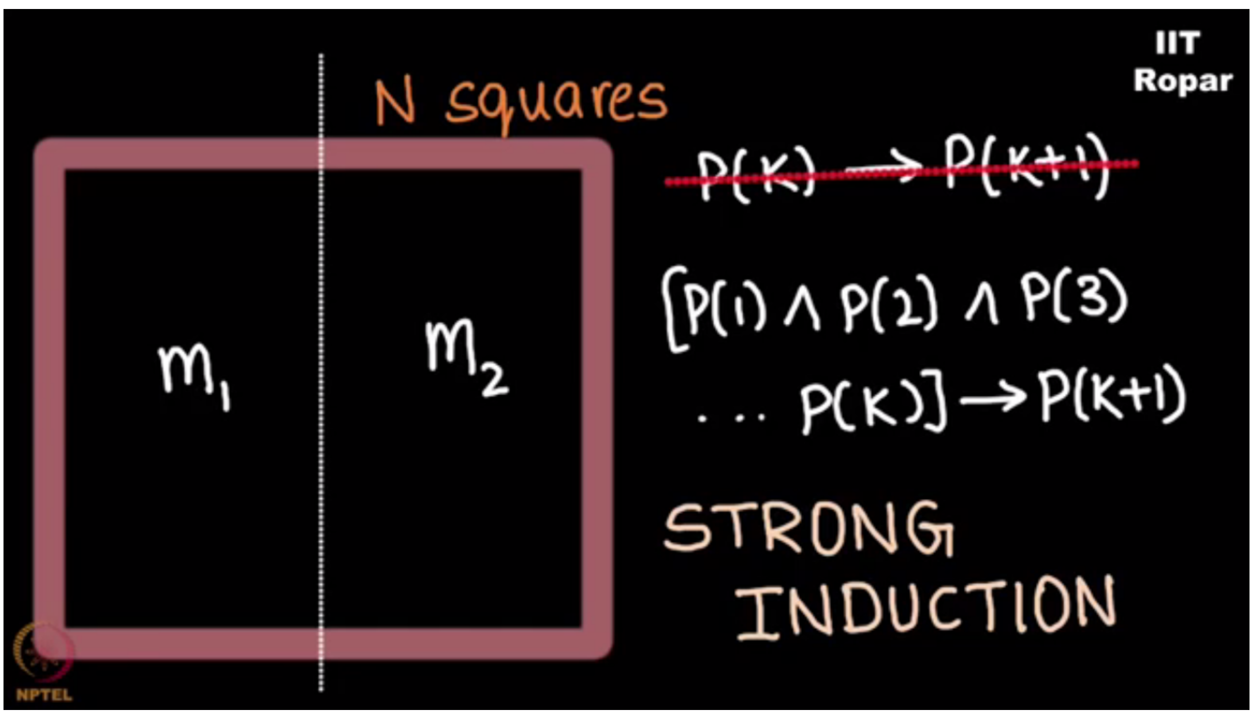


So for 60, it is 59 and for 40, it is 39. 59+39 is 98, but then you made one piece you see already. So you should add 1 to it, which makes it 99.

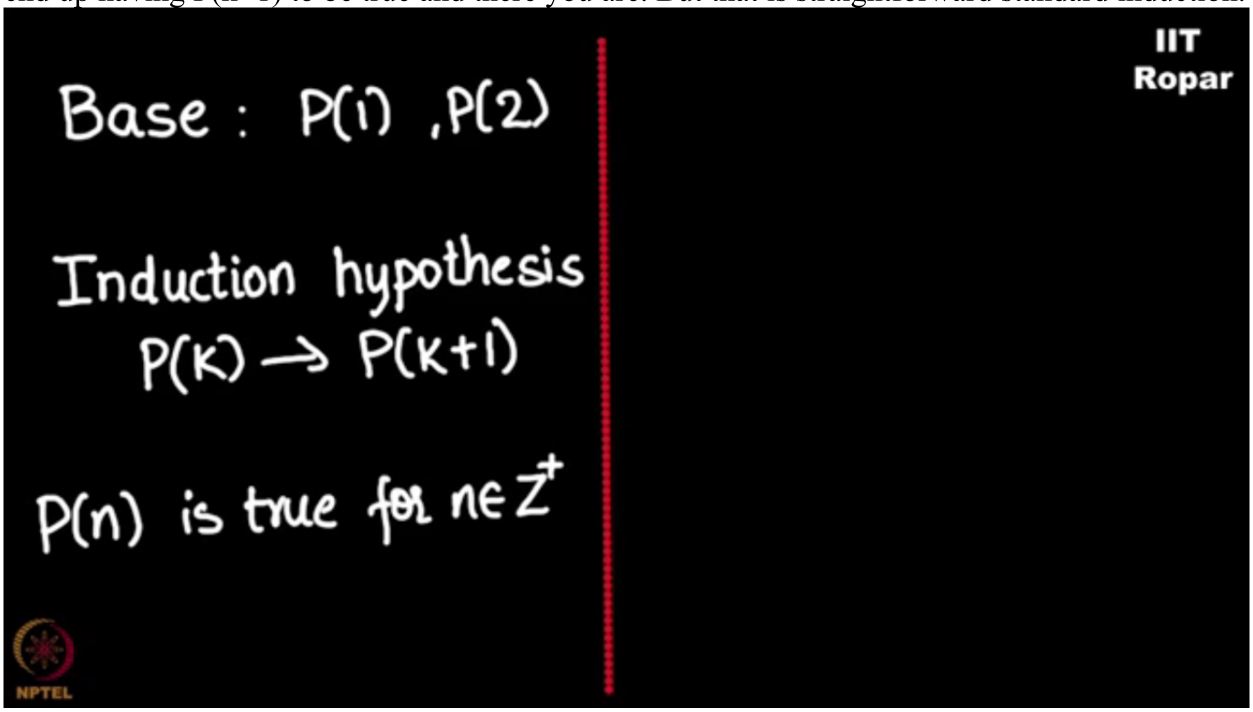
You see what I did. You take any chocolate bar with n squares. Break it. It will become $n=m_1$ pieces on one side plus m_2 on the other side. You break it only once and you get m_1 and m_2 and this m_1 you know can be achieved, can be broken completely with m_1 - 1 breaks and m_2 can be broken completely with m_2 - 1 breaks, which means total number of breaks with which you can completely dismantle this chocolate of size N pieces is m_1 - 1+ m_2 – 1, which is $m_1 + m_2 – 2$. $m_1 + m_2$ is N. It’s N – 2, but then you broke it once to make it m_1, m_2, which means you should add 1 to it and you will N – 1.



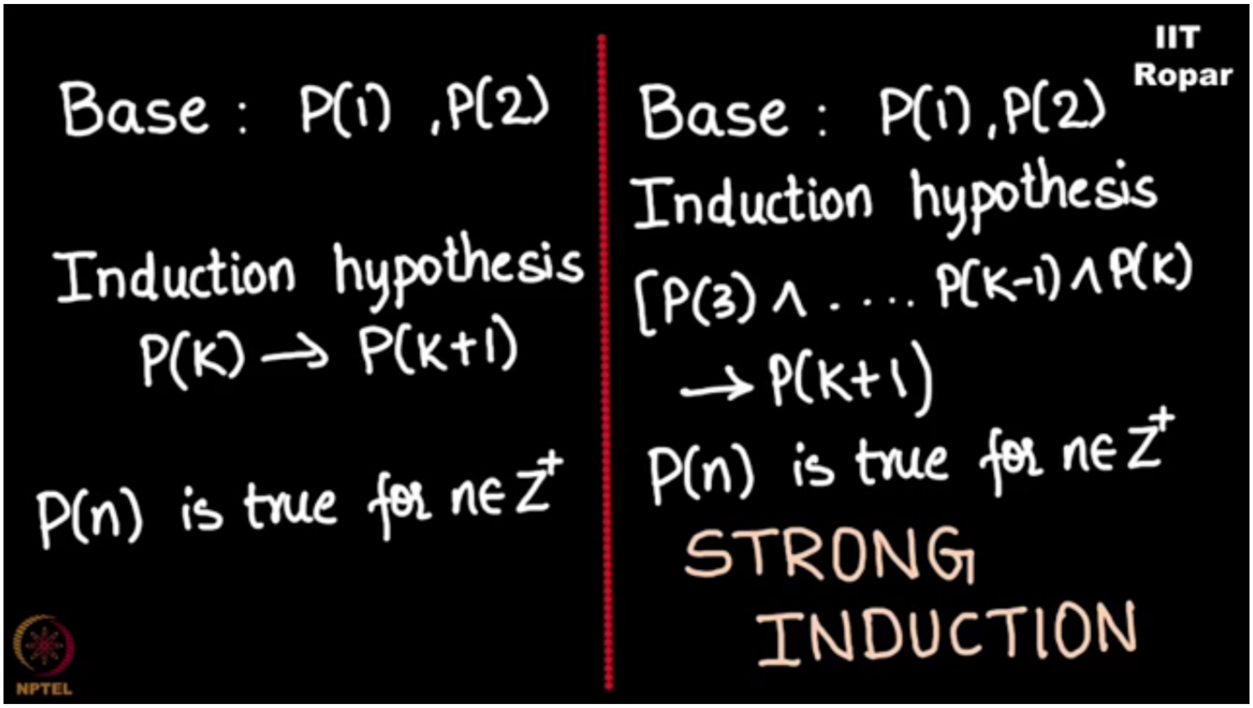
Did you observe something very special about this problem? If you observe deeply, you'll realize that this is not the typical induction that we do. Assume something to be true for P(k) and show that it is true for P(k+1). We didn't do that. We assumed that it is true for everything which is k or less than k. We observed that P(1) is true; P(2) is true; P(3) is true; P(k) is true. Everything is true and hence P(k+1) is true. Such a induction strategy is called as the Strong Induction strategy.



Let me repeat it. Typical induction goes this way. You have a base step. You have a Induction Hypothesis and then you conclude. You show that P(1) is true, P(2) is true and something like that in the first few steps and then you show that whenever you assume P(k) to be true, you will end up having P(k+1) to be true and there you are. But that is straightforward standard induction.



In what I just now explained, all you do is you show that whenever it is true for everything which is for P(k), for P(k-1), for P(k-2) so on for everything less than k, P(i) is true for i -- k or less. If you can show that it’s true for P(k+1), then it is true for all values. Right? This is called the strategy of Strong Induction.



It takes some time for things to sink in. You may want to see more problems where it becomes very familiar to you. So let us look at another problem after which I am sure you will be sure that you understand Strong Induction.

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