**NPTEL**
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# Discrete Mathematics
## Mathematical Induction and pigeonhole principle

## MI - Inequality 2 solution

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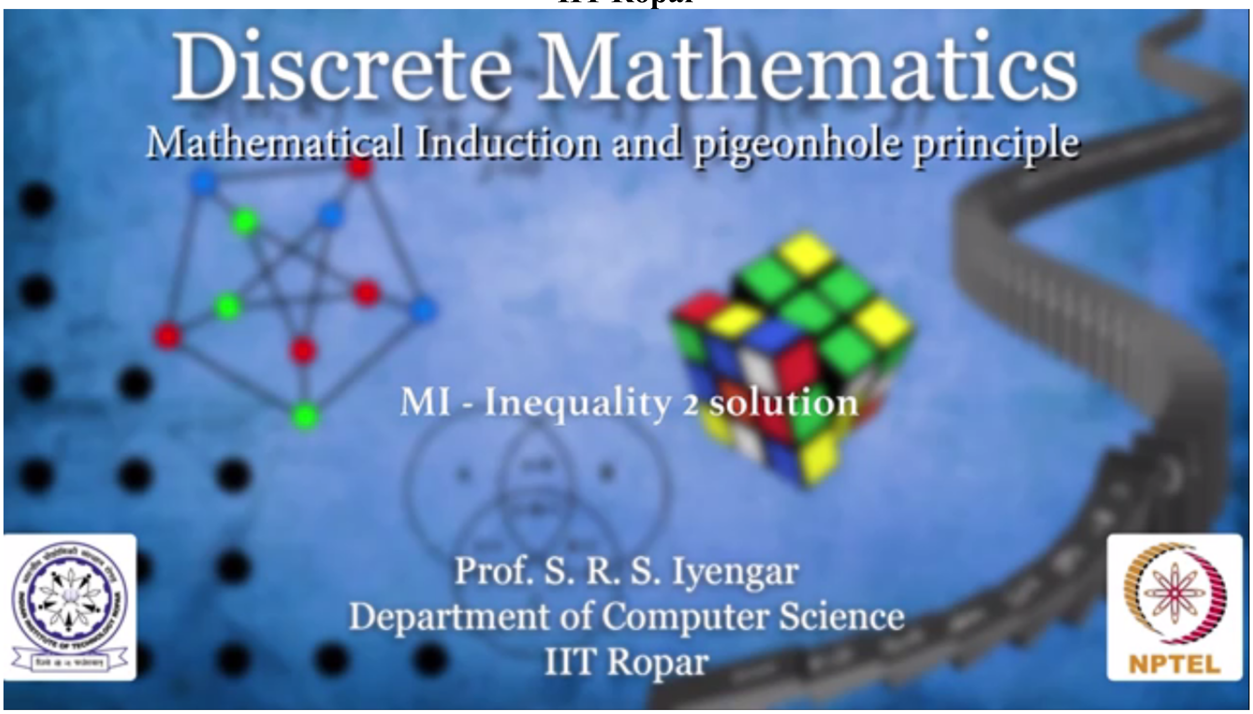
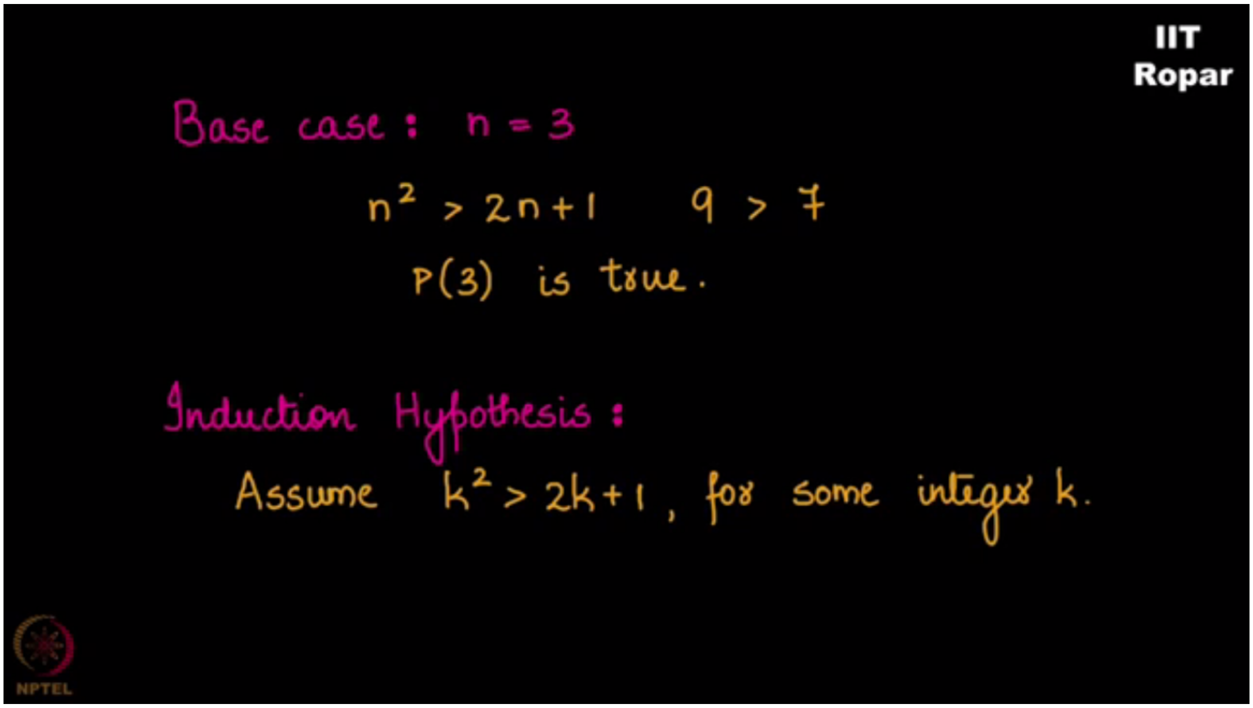
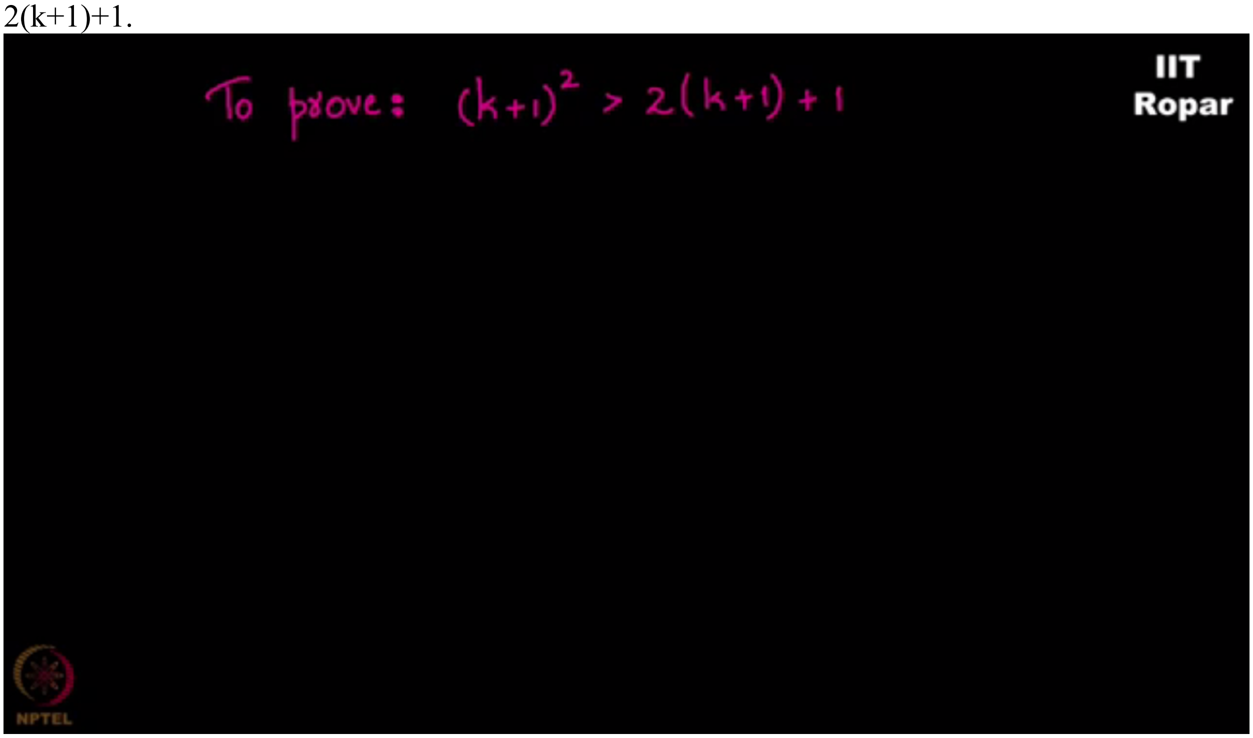



So I stated the problem in the earlier video. Let me prove it using induction. What is the base case? The base case starts with n = 3 and hence we have $n^2$ greater than 2n+1 where n=3 will be 9 greater than 7 and hence the basis step is true.



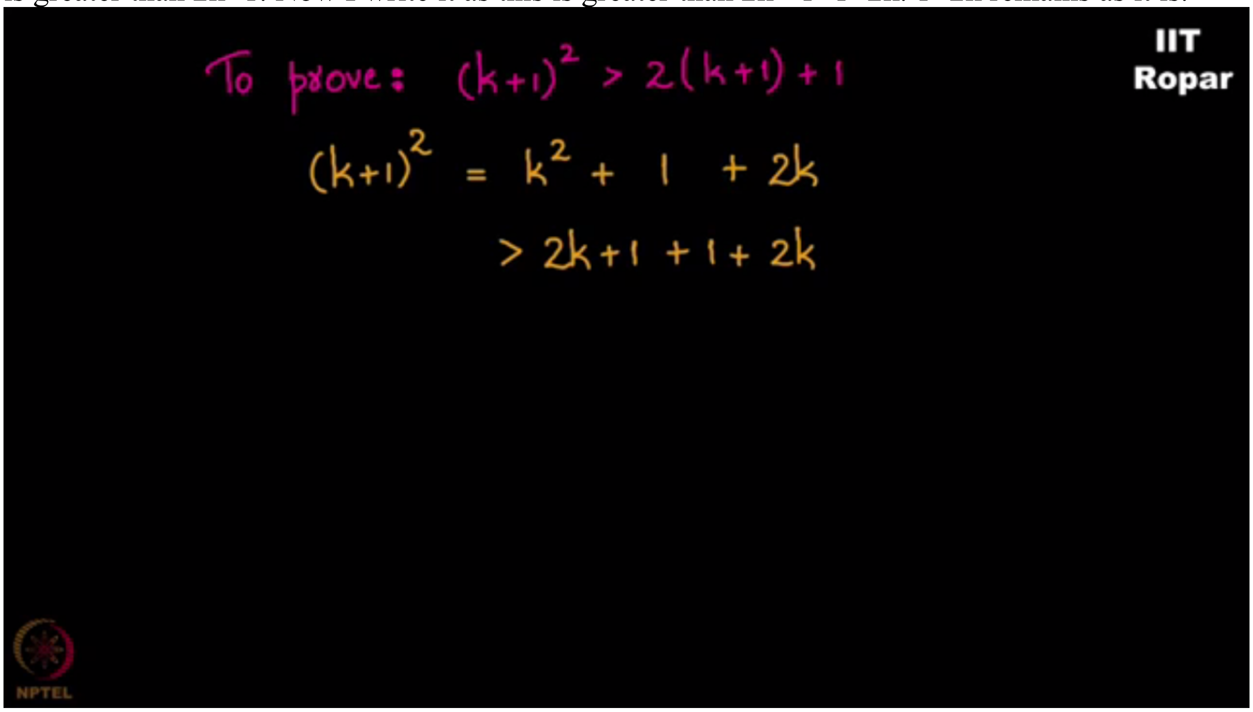

Let me move on to Induction Hypothesis. What do I assume? I assume that $k^2$ greater than 2k+1 for some integer k, positive integer k should be. So this is the Induction Hypothesis.

Now if this is true that if $k^2$ is greater than 2k+1, then I should prove that $(k+1)^2$ is greater than 2(k+1)+1.

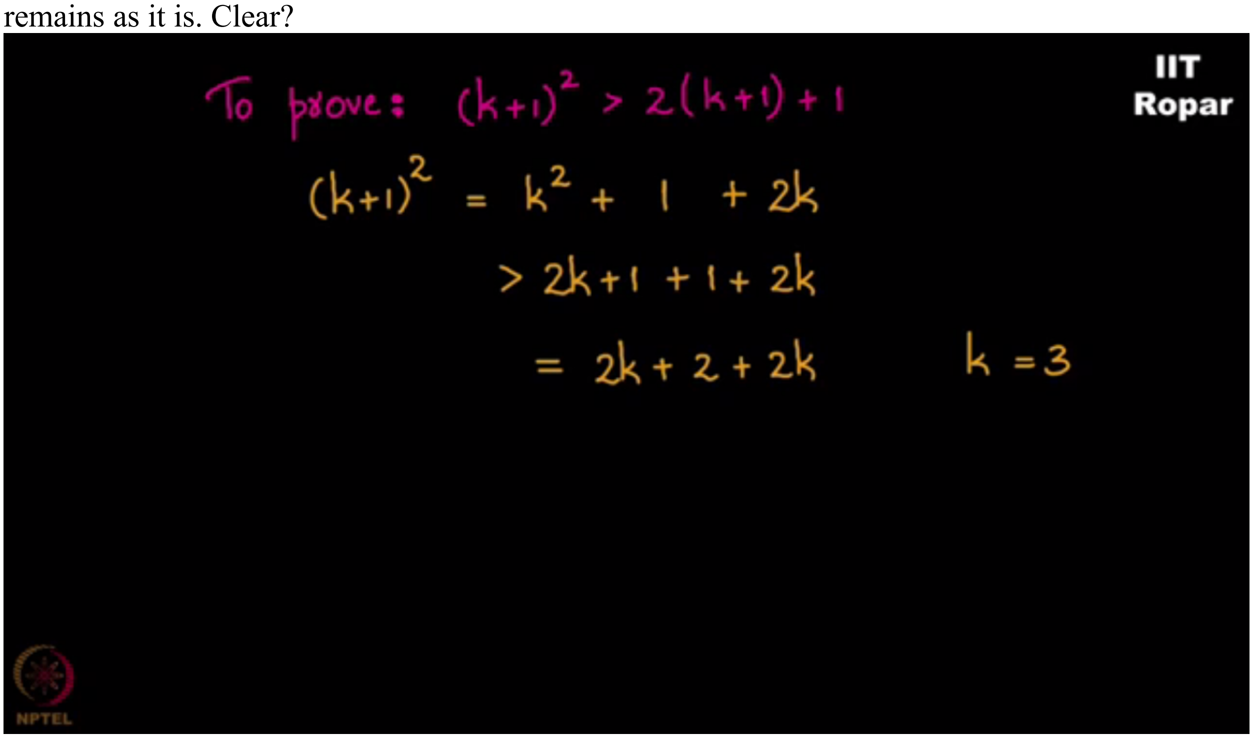



So let me take $(k+1)^2$. $(k+1)^2$ using the identity $(a+b)^2$ goes like this: $k^2+2k+1$, right? Or I can also just rearrange them and write it as $k^2+1+2k$.

Now observe something very carefully. What can I write $k^2$ to be as? From the hypothesis, it is $k^2$ is greater than 2k+1. Now I write it as this is greater than 2k+ 1+1+2k. 1+2k remains as it is.



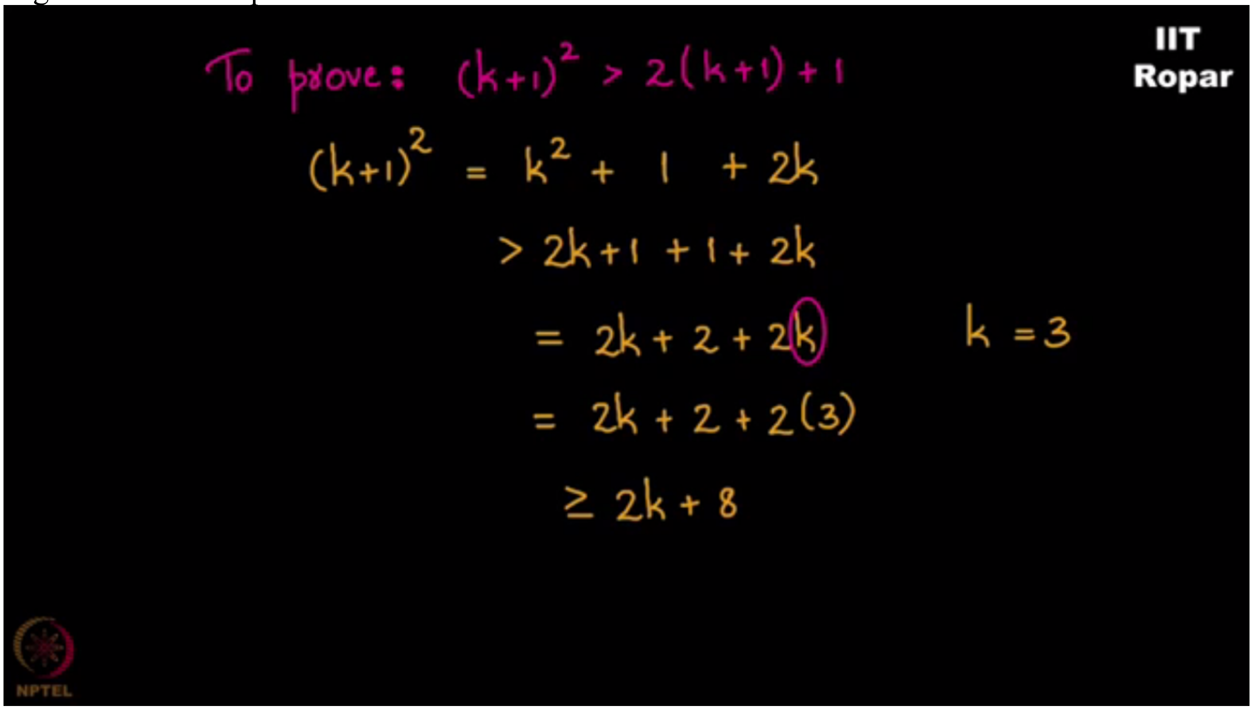

Now this expression is equal to, if you remember, I had explained when we write an equal to sign and when we write greater than sign. I am not going to touch greater than here. It is not affecting the inequality. I am just telling 2k+1+1+2k, this is equal to 2k+2+2k, but the inequality remains as it is. Clear?



Now where did we start from? We started from k greater than or equal to 3. Right? That's how we had started as the basis step if you remember.

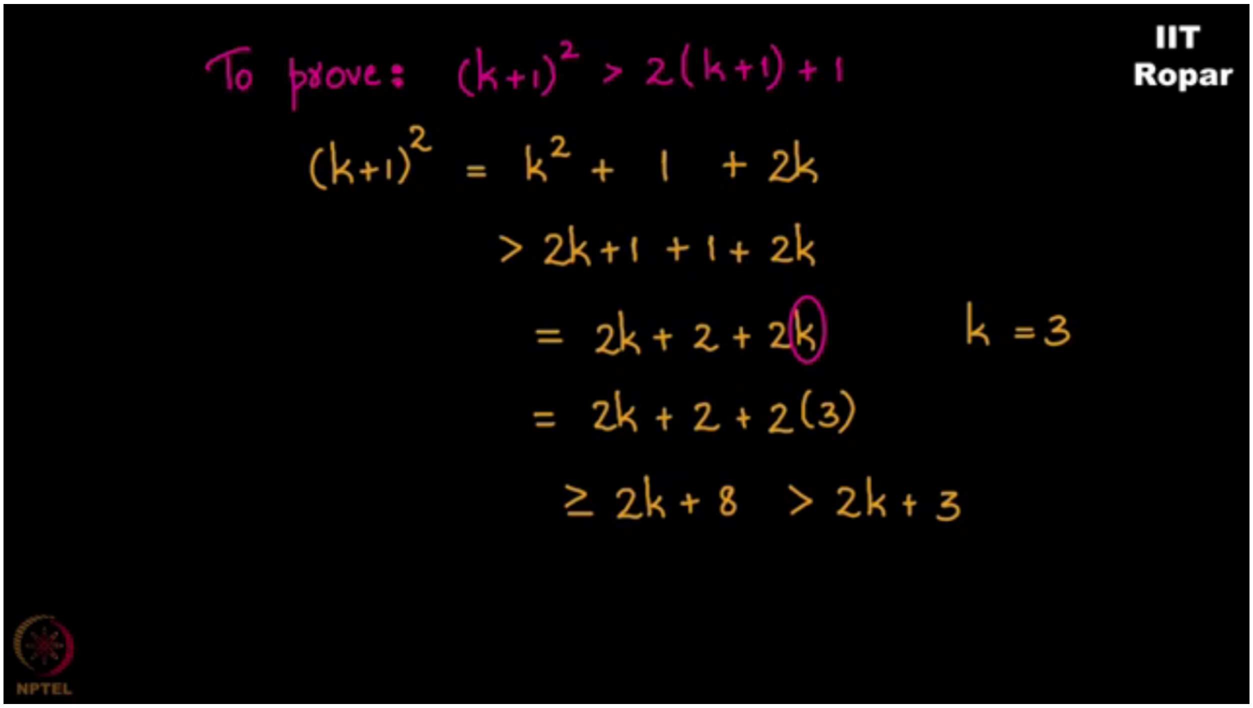
Now let me just substitute 3 here. Why am I doing this? You will get to know later. I'm going to substitute 3 for this k only. Again why? You will understand in the next few steps.

Now 2k+2+2(3), 2 into 3 is 6 plus 2 is 8. Now k can take any value greater than 3 and hence this is greater than or equal to 2k+8.

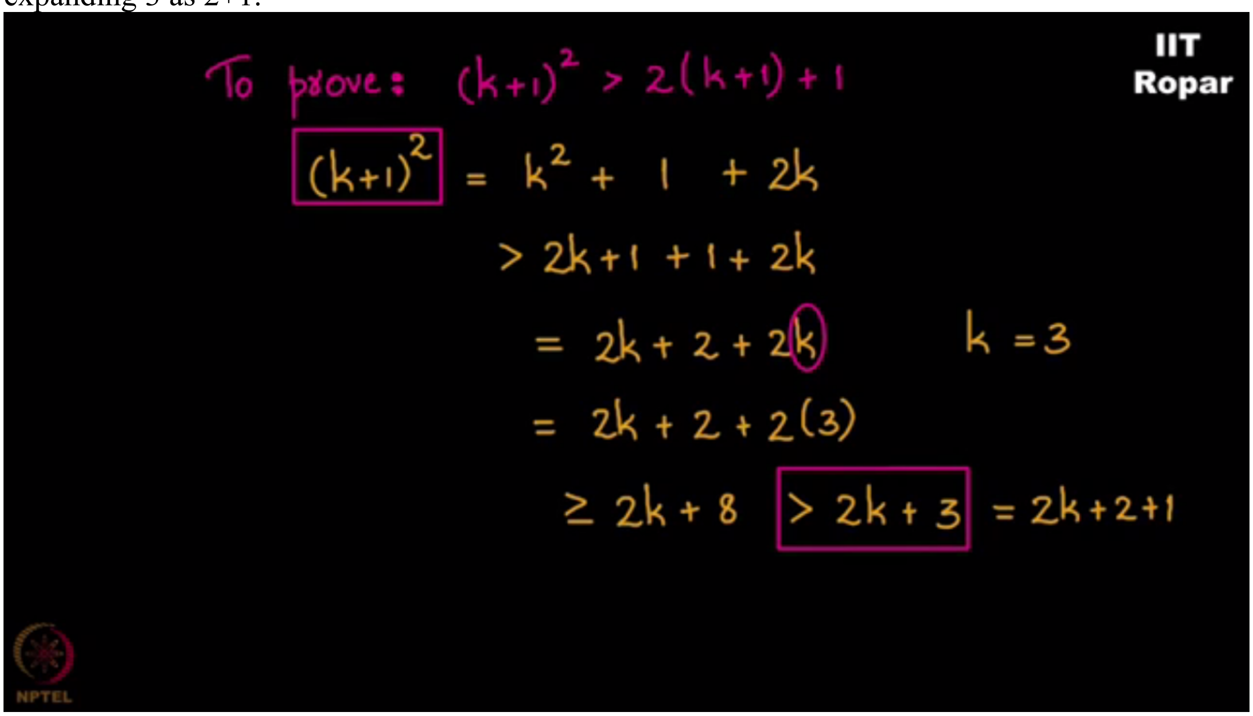



This is the most subtle part of this theorem. You might need to take some time for understanding this portion. This is 2k+8. Now, and this is greater than, strictly greater than 2k+3.

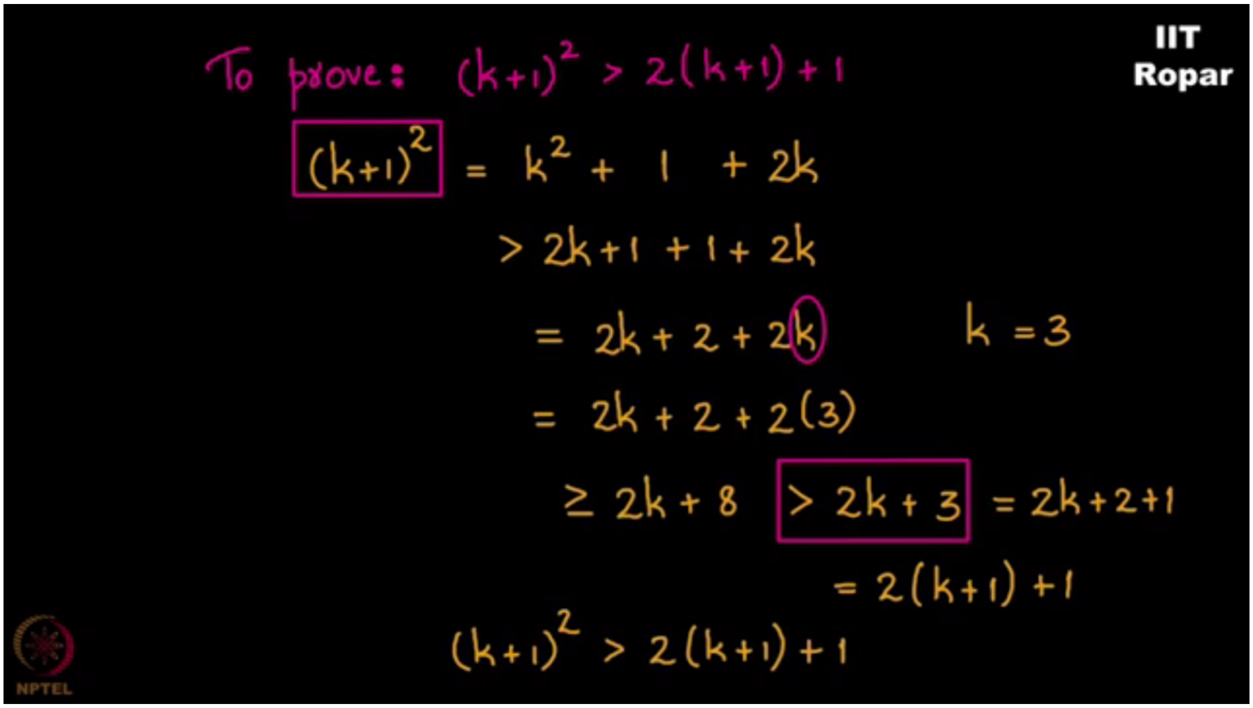
Now you can ask me why did you write 3? You can also write 4 or 5. Yes, but we are concerned only about 3 here. Just observe the proof. So 2k+8, this is greater than 2k+3.




Now what is the inequality? What is the status of the inequality? $k^2+1+2k$ is greater than 2k+3. We have – we are right now in this position. Now this can be written as 2k+2+1. I am just expanding 3 as 2+1.




Now I'm going to take out 2 common here and this step is equal to 2(k+1), (k+1) in bracket plus 1. So now what is the status of inequality? It is $k^2+1+2k$ is strictly greater than 2(k+1)+1. So $(k+1)^2$ is greater than 2(k+1)+1. We have reached the end of the proof.



Take some time to understand each line in the proof. You might need to read between the lines and understand the proof.