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Discrete Mathematics
Mathematical Induction and pigeonhole principle

MI - Inequality 1 (solution)

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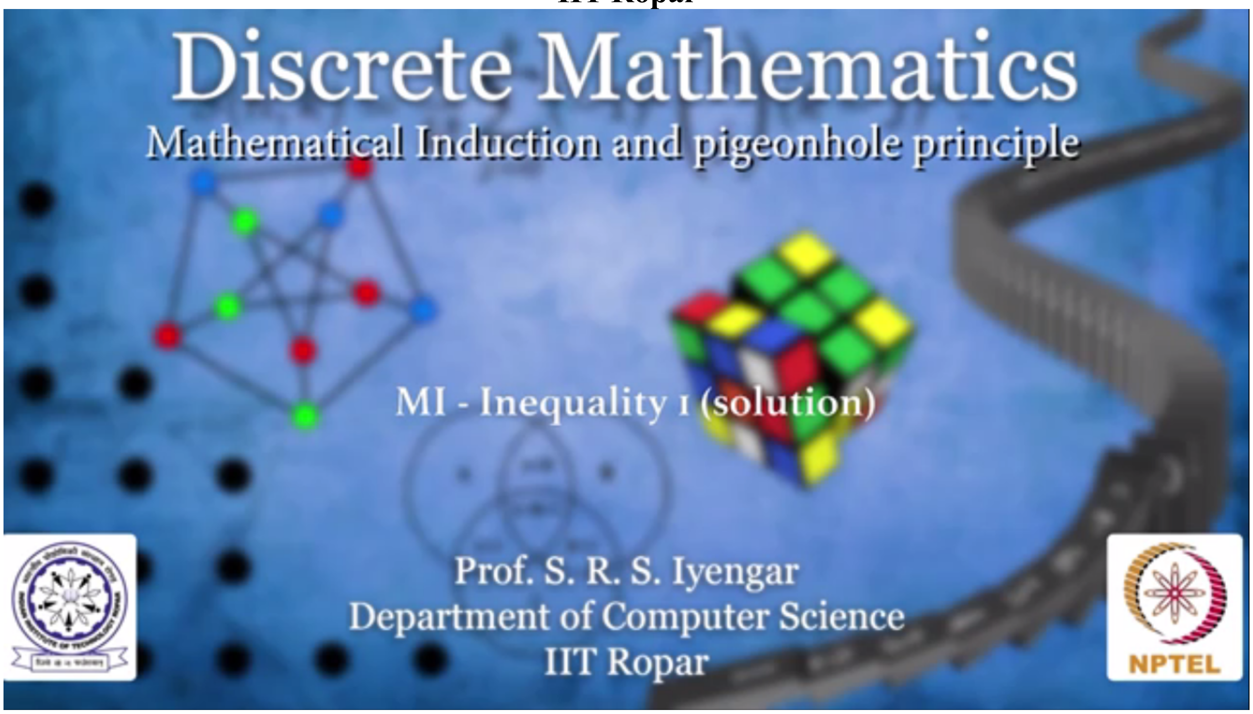
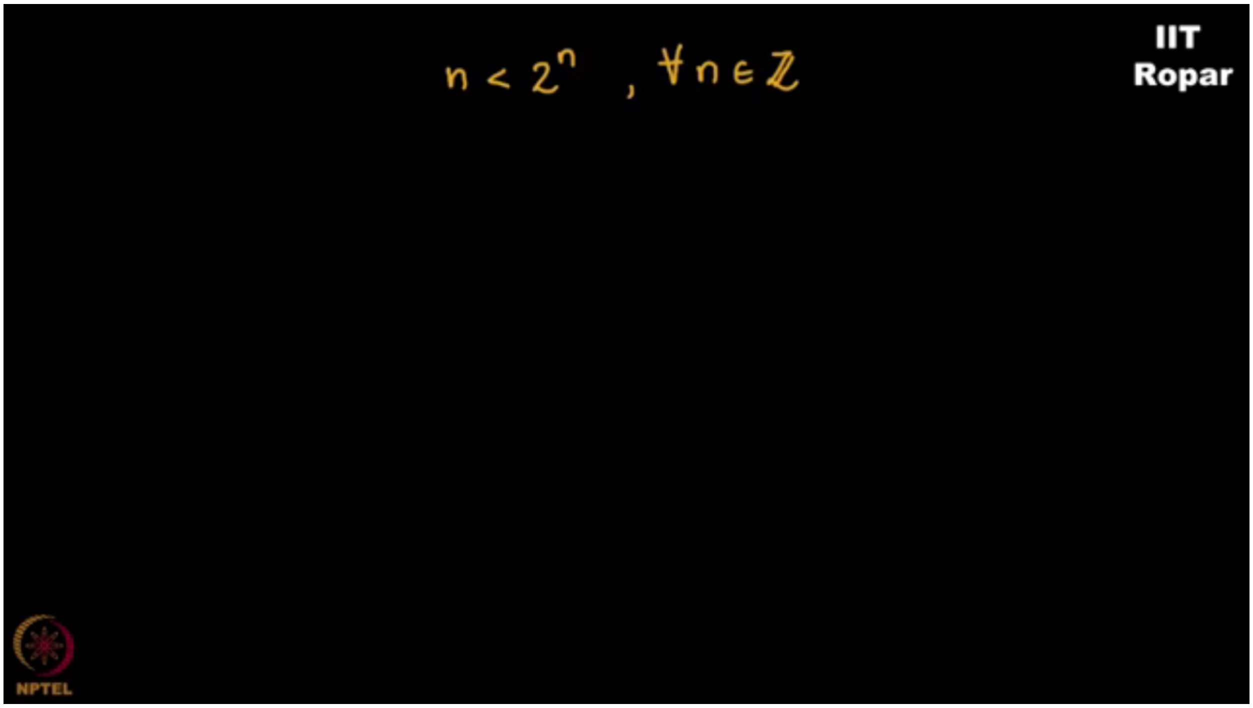
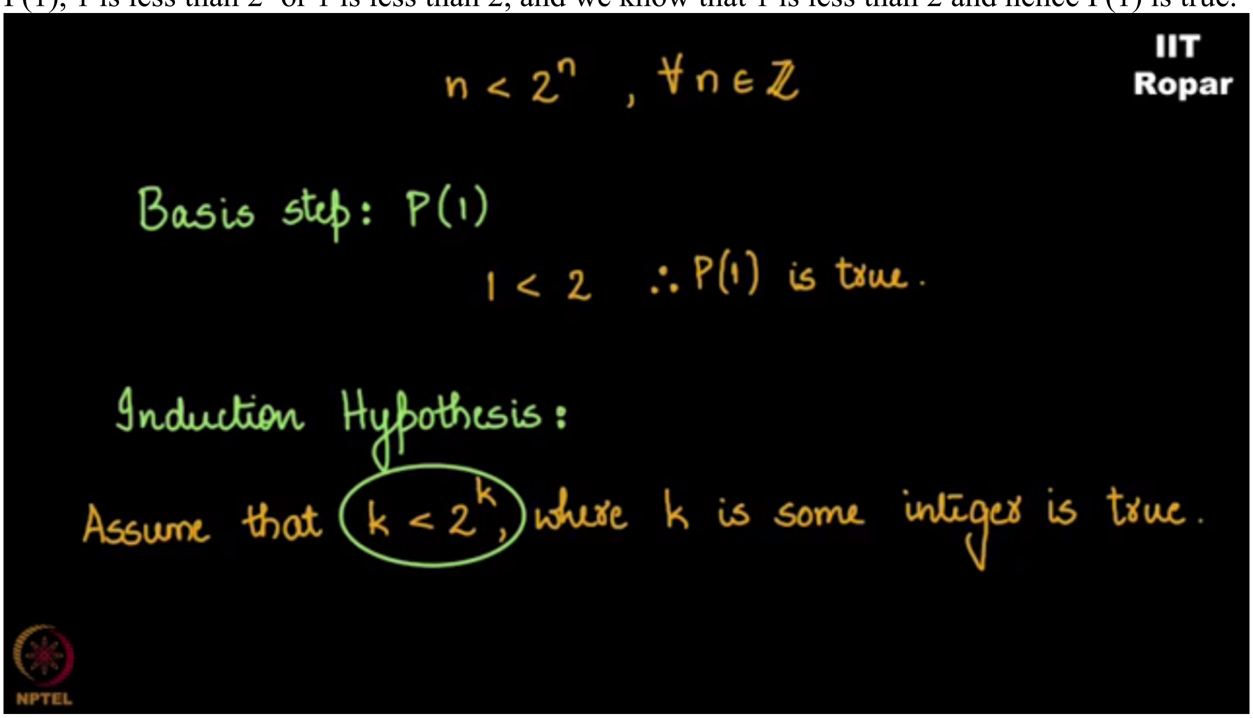


Let us now see the solution of the problem which I stated in the earlier video. What was that? n is less than 2^n for all n belonging to the set of integers. You see it is integers.



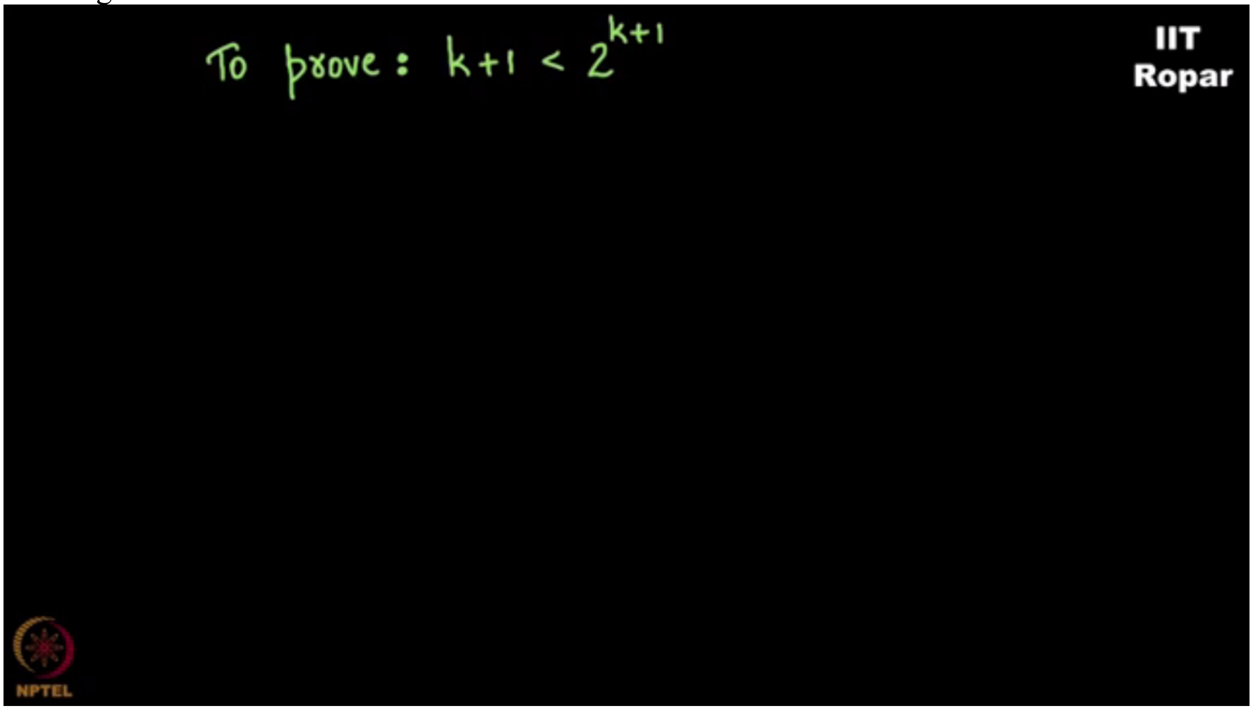
Now as usual, we proceed with the same induction method. What do we do? I consider basic step; then the hypothesis; then I prove it. Okay.

Now let me begin with the basic step or the basis step. You can say anything. For $n = 1$, which is P(1), 1 is less than 2^1 or 1 is less than 2, and we know that 1 is less than 2 and hence P(1) is true.

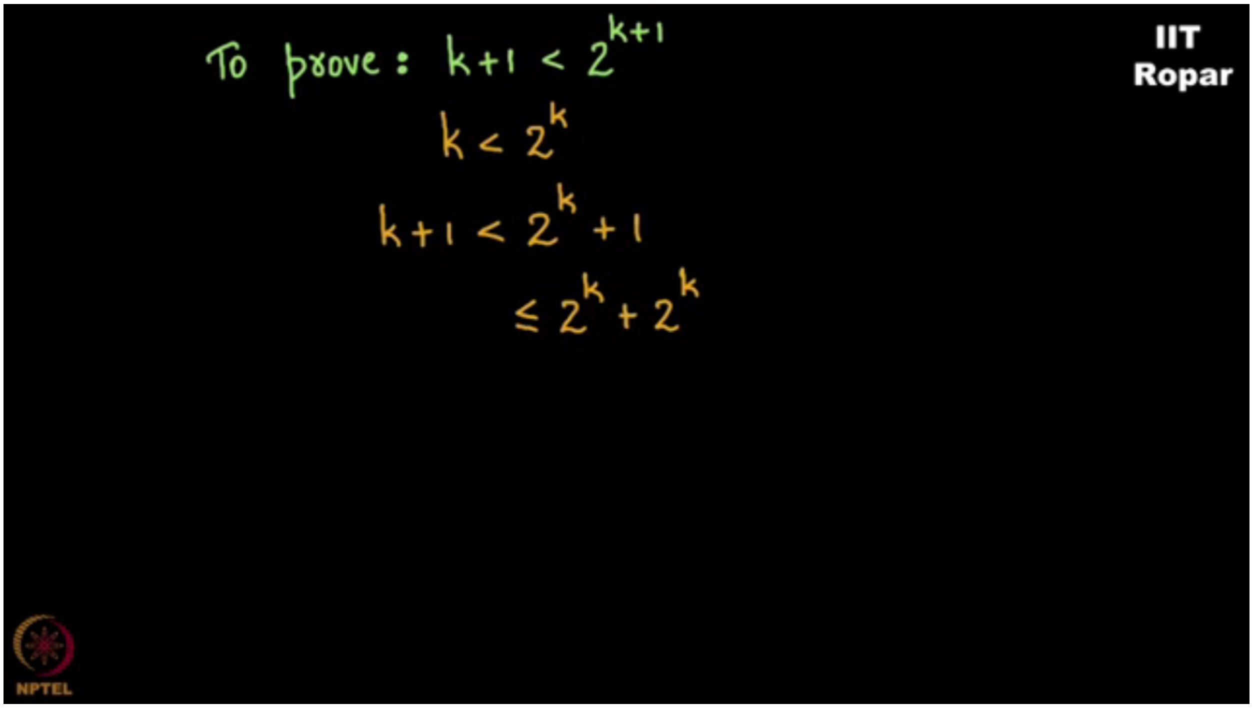


Let us move on to the Induction Hypothesis. What does it say? I will assume that k less than or equal to 2^k where k is some integer is true. I assume that this inequality is true for some integer k.

Now what do I prove? If this happens to be true, that is if k is less than 2^k, what do I have to prove? Then k+1 is less than 2^{k+1}. If this happens, then this inequality holds true. I have to prove this. Right?



Now let me begin. I am assuming that k is less than 2^k. So let me write that down, k is less than 2^k. Now as we have already seen, if we add something on left-hand side, we should compensate by adding the same thing on the right-hand side. Do you remember the proof of 1+2+3 up to n, the sum of n integers? We have done this there. So I add 1 on both the sides. k+1 is less than 2^k+1. Right? This 2^k+1 is less than or equal to 2^k+2^k. Observe, observe.

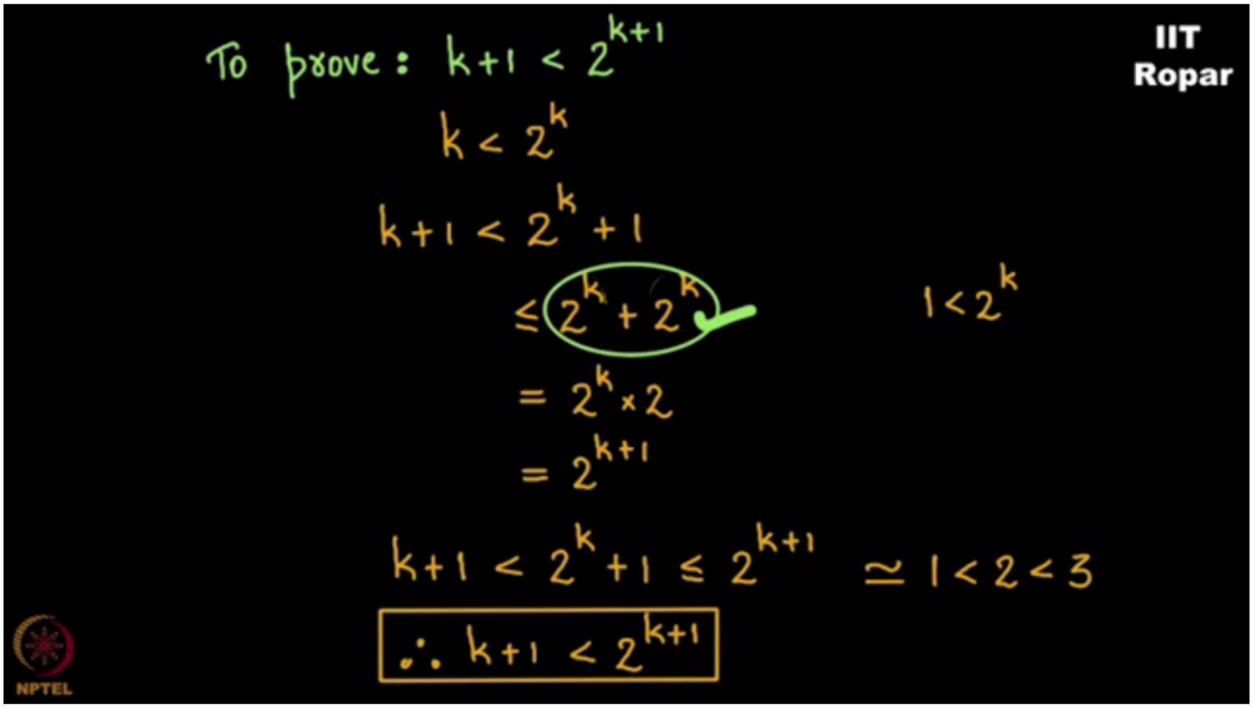


Let me explain. It must strike to you that 1 is definitely less than 2^k. Right? For any k, 1 is less than 2^k and hence I am just inserting that here. 2^{k+1} is less than or equal to $2^k + 2^k$. You see it hasn't affected the inequality at any cost.

Now this $2^k + 2^k$ is equal to 2^k x 2. You see I am not replacing the inequality with = sign. I am just telling that this portion is equal to. Please don't get confused here. Whenever we write this, it always means that the previous step is equal to this step. We are not going to touch the inequality. Clear?

So 2^{k+1} is less than or equal to $2^k + 2^k$ and this portion is equal to 2^k x 2, which is nothing but again I am writing an = sign, 2^{k+1}.

Now I can replace $2^k + 2^k$ by 2^{k+1} and hence the inequality now becomes k+1 is less than 2^k+1, this remains as it is, less you or equal to 2^{k+1}. Is it clear? Here it is 2^k +1 and in the next step it is 2^{k+1}. So the exponent here is k+1.



Now observe something. It is equivalent to telling 1 is less than 2 less than or equal to 3. What does it mean? 1 is less than or equal to 3. This was just an example. What can we conclude here? k+1 is less than 2^{k+1}.

Observe carefully. Pause at various steps and go through the proof again. So, hence, k+1 is less than 2^{k+1} when we assume that k is less than 2^k. Only if we assume this can we say that k+1 is less than 2^{k+1} and hence the proof.

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