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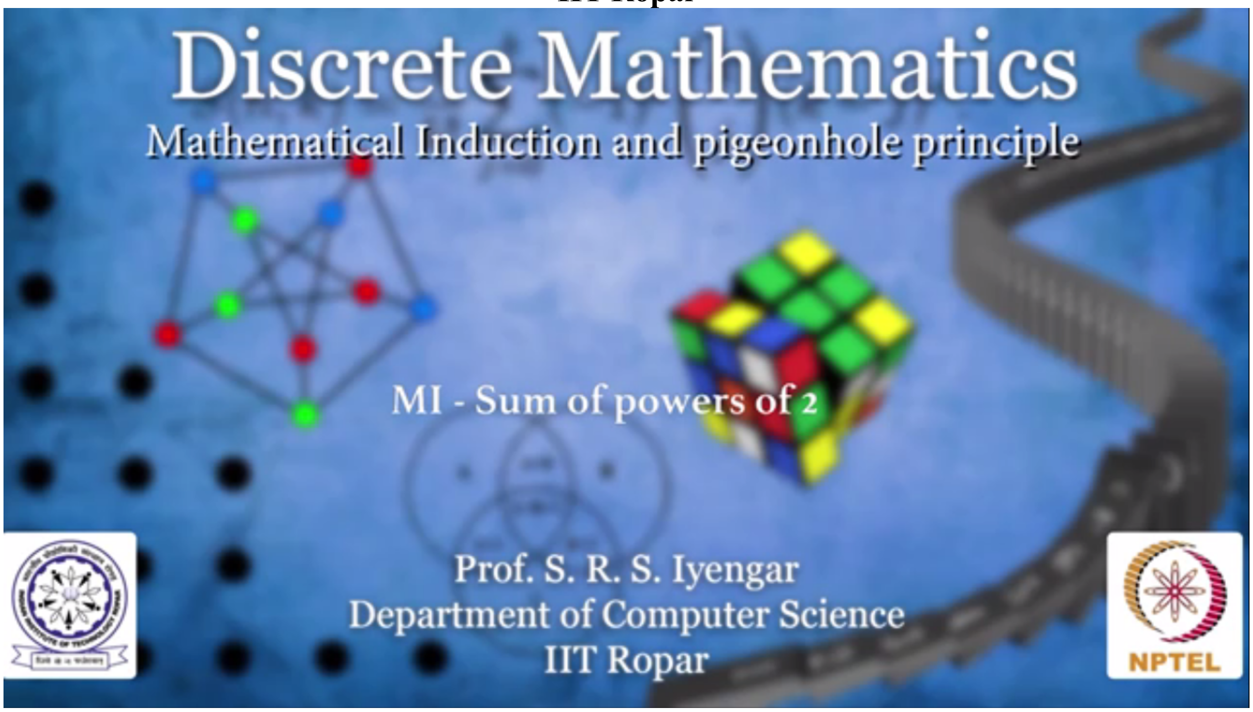
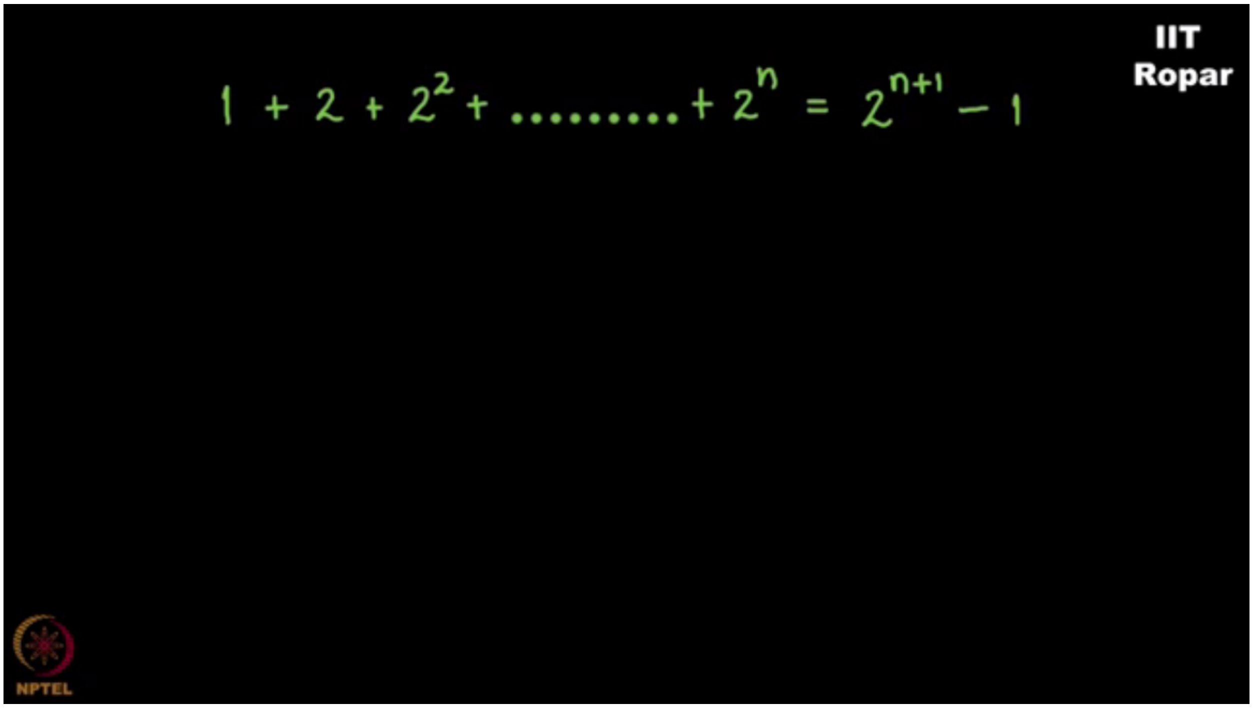
Discrete Mathematics
Mathematical Induction and pigeonhole principle

MI - Sum of powers of 2

Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar

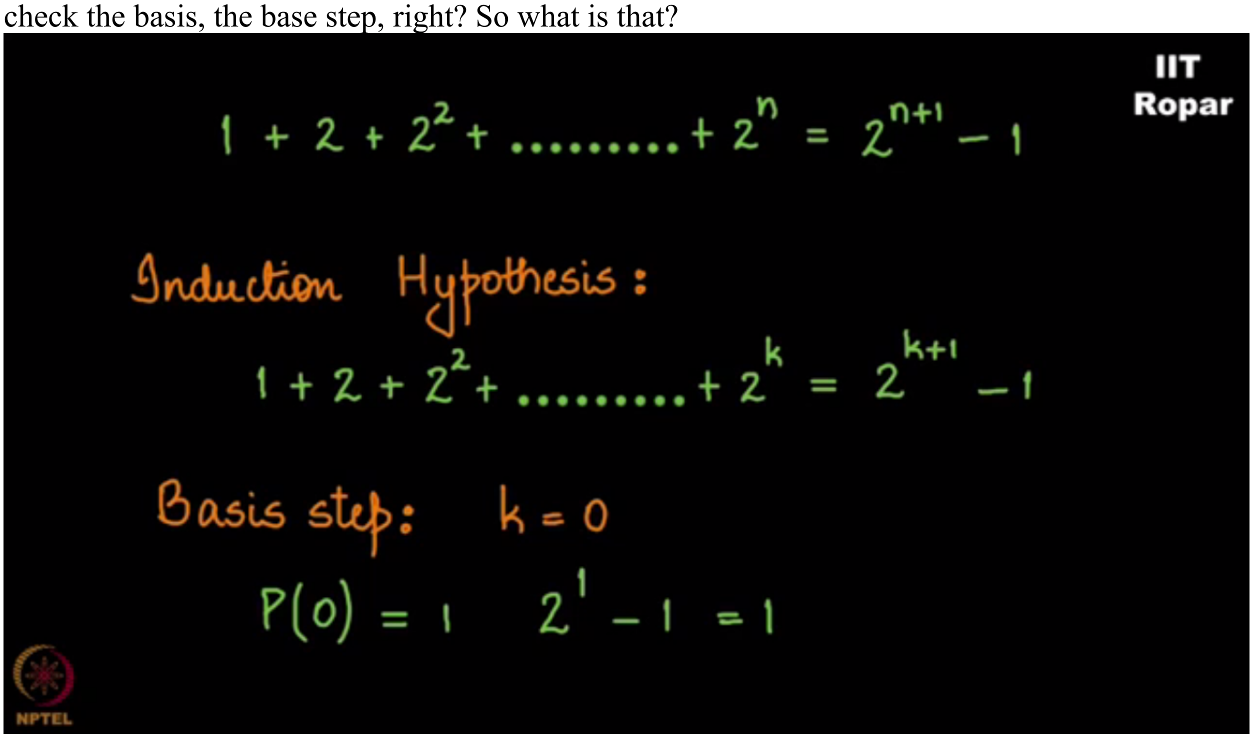


Look at this problem. $1+2+2^2+$ so on up to 2^n is $2^{n+1}-1$. How is this true? Pause the video and try it all by yourself. You needn't necessarily use induction here, but I will give you a proof using induction.



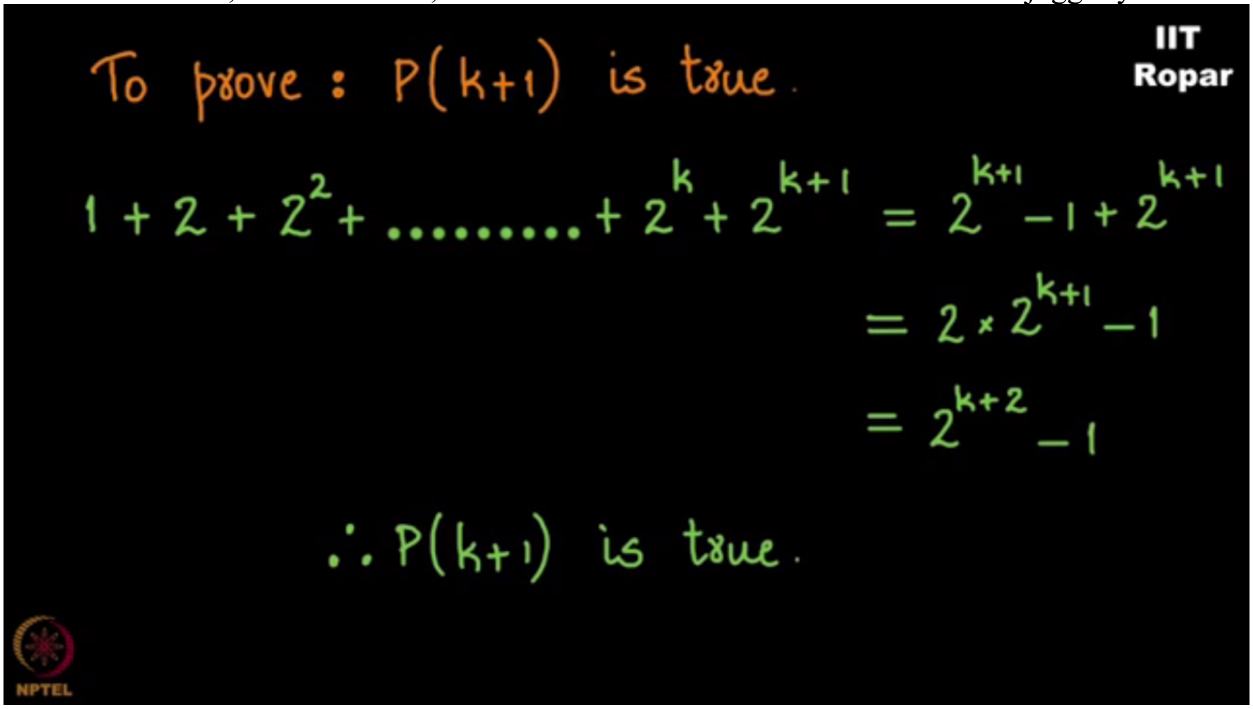
Okay. So now you are back. Probably, you tried proving it without induction. I'm sure some of you succeeded. Some of you didn't. Don't worry. If you understand the binary representation of a number, you will get a hint how to proceed with a solution for this problem.

Anyways, forget all that. Let's now try to see if we can use induction. So let me put, let me consider $1+2+2^2$ up to 2^k equals $2^{k+1}-1$ as my Induction Hypothesis. Right? Firstly, I should check the basis, the base step, right? So what is that?



When k=1, this will be I will check it for let's say 0. P(0) is when it is simply 1, which is equal to 2^{0+1}-1, which is equal to 1. So now the Induction Hypothesis is I'm assuming P(k) to be true, which is $1+2+2^2$ up to 2^k is 2^{k+1}-1.

We need to prove that 2 to the -- for P(k+1) this is true. So what do we do? Simply add one extra term on the left side. That is what you mean by P(k+1), which is $1+2+2^2$ up to 2^k+2^{k+1} which is equal to we know by Induction Hypothesis that this is 2^{k+1}-1 and then I am adding one more term here. Look at this, one more term, which will make it 2^{k+1} and now I do a small jugglery.



I take 2 out here and I basically observe that $2^{k+1}+2^{k+1}$ is two times 2^{k+1}. I write 2 outside. By outside I mean a x a+a is 2a, right? So I get this minus 1, which is same as 2^{k+2}-1 and that is what I wanted, observe, right? P(k+1) is also true whenever you assume that P(k) is true and hence the given formula holds good. We thus have illustrated that with mathematical induction.

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