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Discrete Mathematics

Let Us Count

Combinations - Part 4

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We are now going to see how nCr is equal to n minus 1 Cr plus n minus 1 $r-1$. Okay. So this just follows from our definition of nCr . What is that? nCr is given by factorial n divided by factorial r times n minus r factorial. Now that becomes your left hand side. Right. Perfect. Now what is the right hand side? n minus 1 Cr which becomes n minus 1 factorial divided by r factorial times n minus 1 minus r factorial plus and this n minus 1 Cr minus 1 becomes n minus 1 factorial divided by r minus 1 factorial into n minus 1 minus r plus 1 factorial. Why is that? I just subtract n minus 1 and r minus 1 rather rather r minus 1 is subtracted from n minus 1. That's why you get n minus 1 minus r minus of 1 and minus minus becomes plus so you have a minus 1 and a plus 1 here which gets canceled which gives you simply n minus r factorial here.

So this is the same old-school method of trying to prove a formula by showing that the left hand side is indeed equal to the right hand side. Let's see how we can proceed. As you can see there is an n factorial in the numerator on the left hand side and n minus 1 factorial on the right hand side. So I will take n minus 1 factorial common on the right hand side and then bring that down to left hand side which makes it n factorial divided by n minus 1 factorial which you all know what is 10 factorial by 9 factorial. It is simply 10 right. So n factorial divided by n minus 1 factorial happens to be n . Correct. So you get an n here and these things numerators become one and one on the right hand side. Okay. And then next.

Do you observe r factorial in the denominator throughout and an r minus one factorial here? So what I do is I will multiply the numerators by the r factorial. So what do I get? r factorial gets cancel here when I put r factorial numerator on the left hand side and then r factorial again on the

right hand side gets canceled but when you put an r factorial here in the second term r factorial divided by r minus 1 factorial gives you r so r remains the numerator right and then the next step, do you see what's happening? I'm multiplying throughout by n minus r factorial, right. And that gives me simply a little observation one more step tells me this simply makes this n equals n minus r the first term becomes n minus r right, rest gets canceled plus r . So n equals n minus r plus r plus r minus r gets canceled. This implies n equals n of course left hand side is equal to right hand side and so you go back and you say that the formula is indeed right.

So how do we show that nCr is equal to $nCn-r$, right, in a very mathematical way. So what is nCr factorial n divided by factorial r into factorial n minus r , correct. But what is n choose n minus r ? It is n factorial divided by n minus r factorial times n minus of n minus r factorial. You see n and minus n gets canceled and this becomes r here and as you can see the left hand side is equal to right hand side. Remember this is how we used to show a lot of things to be true in our school days basic algebra right. So LHS is equal to RHS and hence we show that nCr is indeed equal to $nCn-r$.

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Funded by

Department of Higher Education

Ministry of Human Resource Development

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