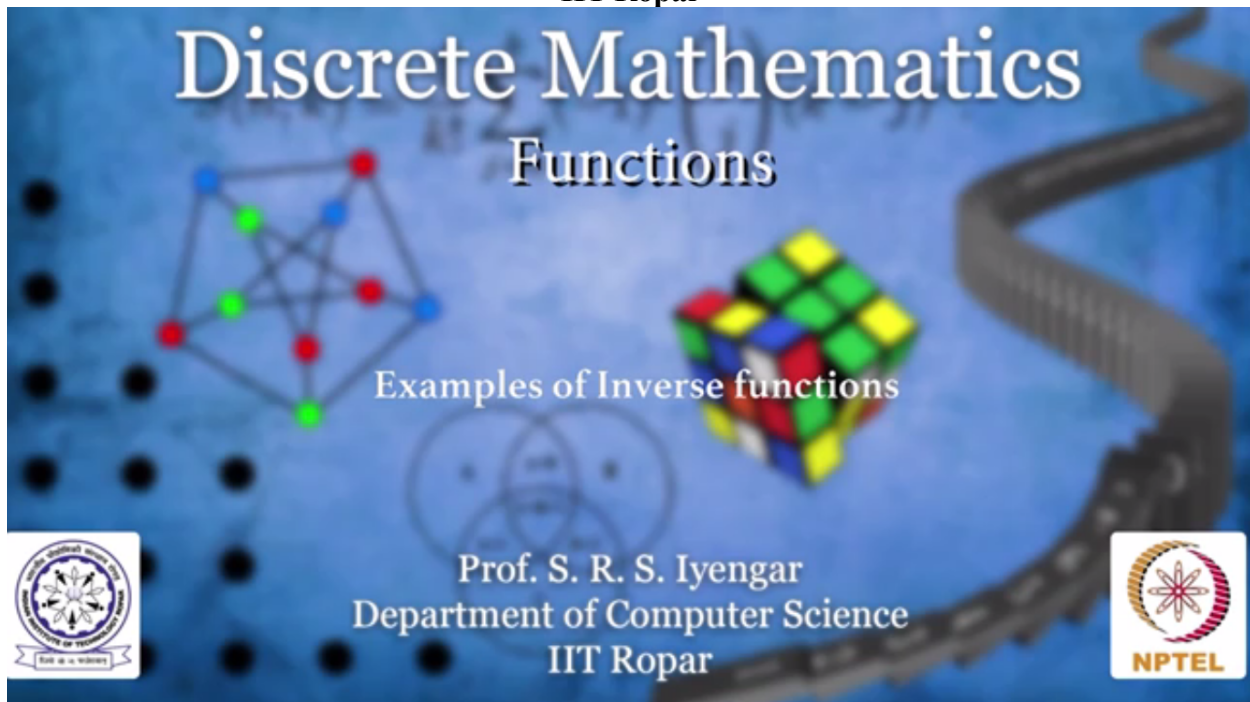


**NPTEL
NPTEL ONLINE COURSE**

**Discrete Mathematics
Functions**

Examples of Inverse functions

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Let us now see some examples of inverse functions. Consider this function f from integers to integers defined as $f(x) = 3x + 2$. Now $f(x)$ I will take it to be an element in the co-domain rather than the range. I will write it as y . So y is what? It is equal to $3x + 2$.

Let me compute x from this equation. $y - 2$ is equal to $3x$ and hence x is $(y-2)$ by 3. So what is x ? x is the inverse of y . Rather x was mapped to y and hence f inverse y happens to be $(y-2)$ by 3.

$$1. f: \mathbb{Z} \rightarrow \mathbb{Z} \quad f(x) = 3x + 2$$

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$$y = 3x + 2$$

$$y - 2 = 3x$$

$$\therefore x = \frac{y - 2}{3}$$

$$f^{-1}(y) = \boxed{\frac{y - 2}{3}}$$



Pause and think how the steps have proceeded and you will be able to understand it better. So the inverse of every element y belonging to the range of this co-domain is $(y-2)$ by 3.

Consider this function f from set of all real numbers to integers defined as $f(x)$ is equal to greatest integer function that is this square bracket of x . We see that $f(0.32)$ is 0; $f(0.45)$ is 0; $f(1.798)$ is 1; $f(1.45)$ is 1; $f(1.32)$ is 1 and so on.

So do you see that several elements will be mapped to 1?

$$2. f: \mathbb{R} \rightarrow \mathbb{Z} \quad f(x) = [x]$$

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$$f(0.32) = 0$$

$$f(0.45) = 0$$

$$f(1.798) = 1$$

$$f(1.45) = 1$$

$$f(1.32) = 1$$



Now how do we find inverse of 1? Let me tell you the inverse will be unique. One element cannot be having two or three inverses. Right? You must know that by now by the definition of function.

So will it be possible for us to find an inverse here? Definitely not, but let us reason this out. If f is from \mathbb{R} to \mathbb{Z} , f inverse will be from \mathbb{Z} to \mathbb{R} . What is f inverse of 1? It will be associated to several numbers in the real line and hence it will not be a function and therefore we cannot find the inverse of this function. It is not invertible.

$\mathbb{Z} \cdot f: \mathbb{R} \longrightarrow \mathbb{Z} \quad f(x) = [x]$

$f(0.32) = 0$
 $f(0.45) = 0$
 $f(1.798) = 1$
 $f(1.45) = 1$
 $f(1.32) = 1$

$f^{-1}: \mathbb{Z} \longrightarrow \mathbb{R}$ Not a function.
 $\therefore f$ is not invertible

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The last question: if f is a function from \mathbb{R} to \mathbb{R} defined as $f(x) = x/2$, so I am taking the half of that element each time, what is the inverse here? $f(x)$ is y . So what does y equate to? $x/2$ and therefore what is x ? Very simple. It is $2y$. Right? So f inverse y happens to be $2y$.

$$f: \mathbb{R} \longrightarrow \mathbb{R} \quad f(x) = \frac{x}{2}$$

$$f(x) = y = \frac{x}{2}$$

$$\therefore x = 2y$$

$$f^{-1}(y) = 2y$$



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