Discrete Mathematics

Functions

Inverse functions

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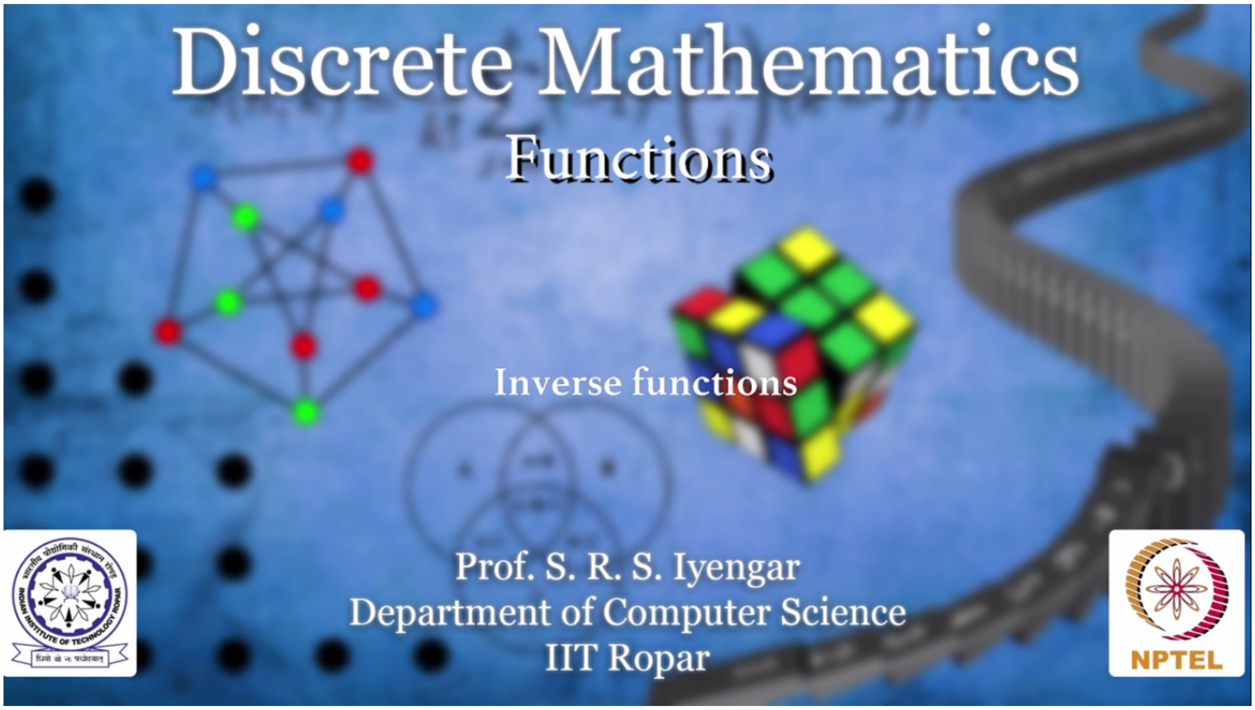


So we have been seeing functions let me talk about one last concept that I expect you all to know really well and that is called invertibility of functions. Inverse of a function.

So do you remember what is a set? A set S is a collection of well defined objects. Now that sounds very complicated. What do you mean by collection of well defined objects? It is just a convention. So let's say a set S comprises of 1, 2, 3, and 4. You see can I write 4 once more here? No. That's not accepted as a convention. That is not the definition of a set. In a set elements do not repeat.

Now you can say why. Why is a set defined that way? Now these are all conventions. These are all accepted norms. These are all some sort of a uniformity in the community of math people that set should be defined this way only for convenince sake.

There is a situation where we may have to include repeated elements. They call it a multi-set. Again why a multi-set? That's a norm. that's a convention. That's some sort of a uniformity that we maintain.

Similarly a function f as you know is defined from x to y we always ensure that one element from the domain never goes to many elements to the co-domain. We always assume that a function is always about element here going to an element here.

Two things we ensure. First thing is that an element doesn't go to two different elements. So when you say f of x what is f of x, you should not have two different values, y and y dash. Correct. It should be only one element. That's the first condition.

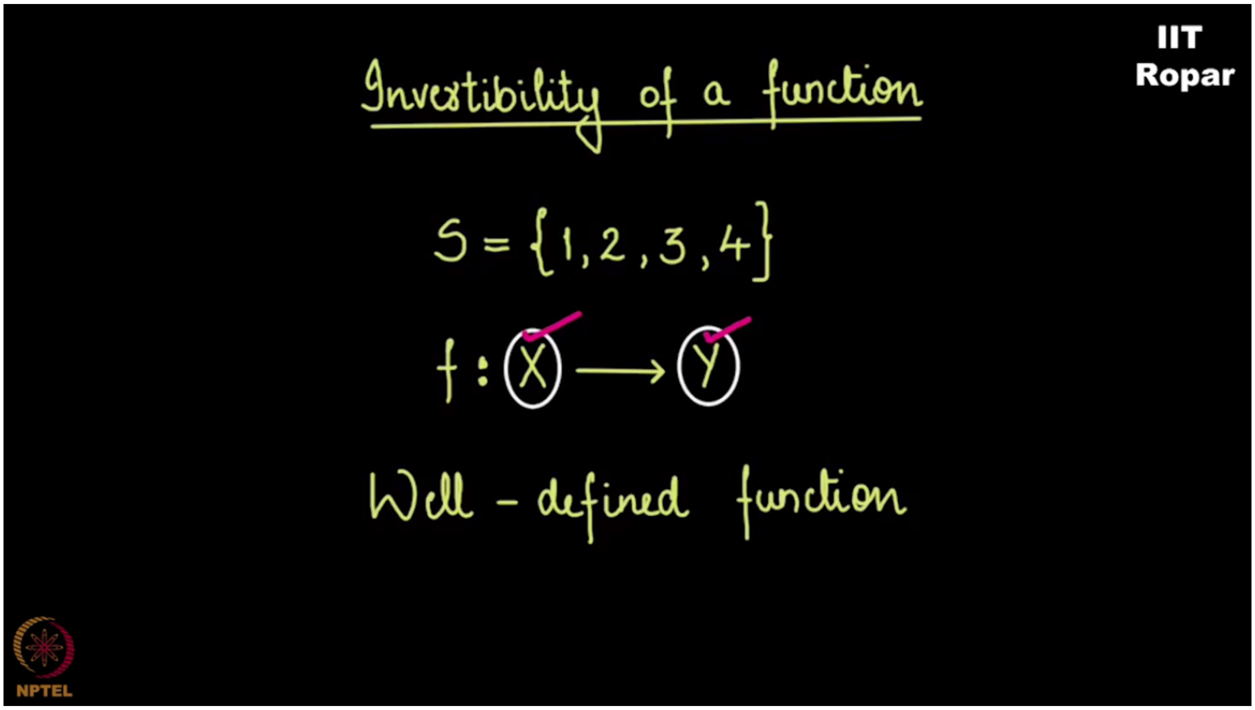
Second condition is every single element should go somewhere. Nothing should be left out here. Although as you know things can be left out this side but it should not be left out this side.

The idea here again is it's a convention that we have been using. Many function satisfy this criteria so we make this as the criteria for a function. It has to have a unique image and every element in the domain should have some image. It should go to some element.

Such a function there is some name to it. In fact it's slightly confusing. They say such a function is well defined function. Now you can ask me you tell what's the function and you say what is well defined function. It doesn't sound good.

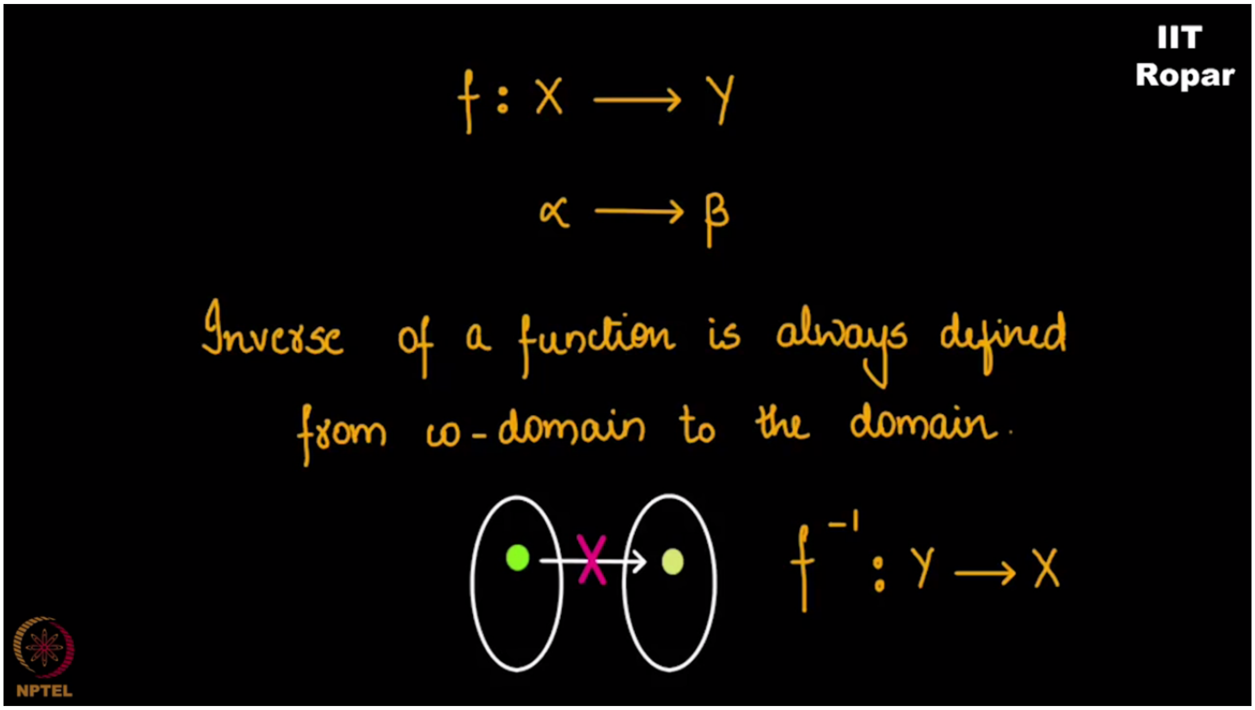
So this is like a difference between a good person and a very good person and a very very very good person. It should clearly state who is a good person, who is a very good person and things like that.

A function is always something that satisfies this condition one and two. Some people call this well defined function. So don't get scared whenever you see this term well defined function. By that they only mean these two properties they ensure is satisfied.



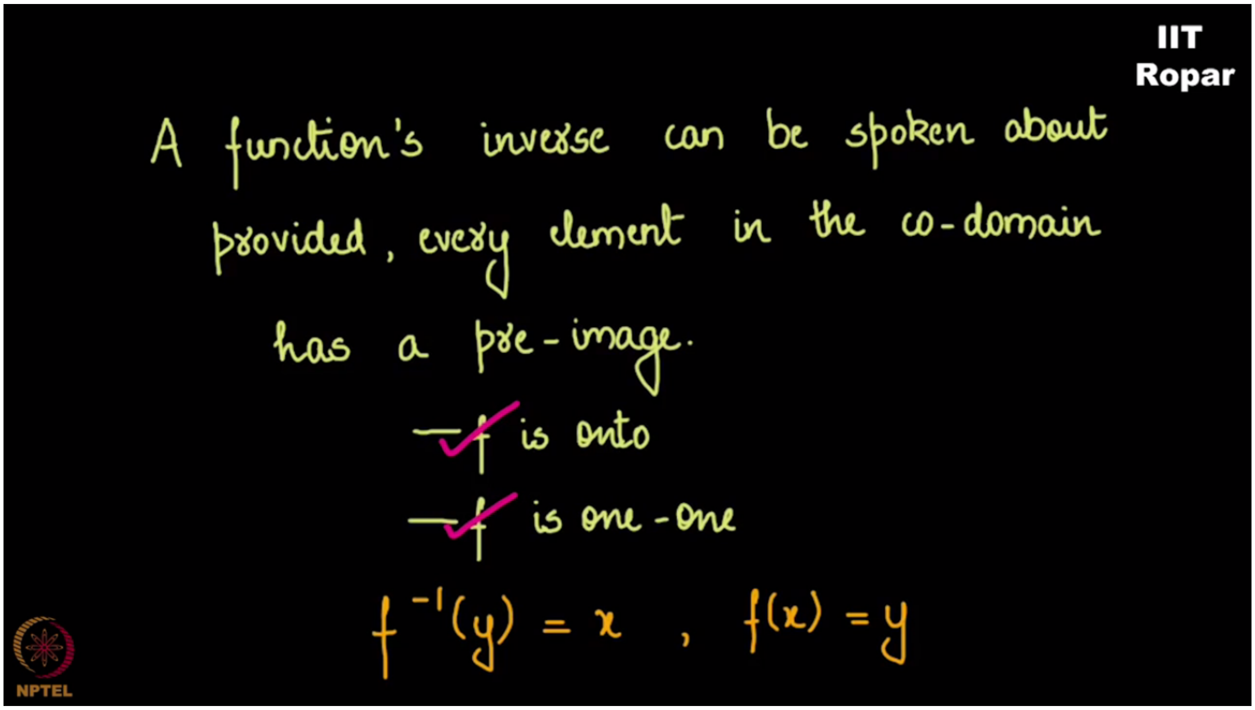
Okay. So now whenever you define a function f from x to y, you can always talk about the inverse of a function which is if this element is going to this element here, inverse of this is basically this element coming back to this element. By that I mean inverse of a function is always defined from the co-domain to the domain.

Now there is a problem when I define it like this. What if there is an element here which is not an image of any element here. What do you mean by f inverse of that element? F inverse is a function I said from y to x. This may not satisfy the property that I told you of well defined which means it must have an image. It may not have an image and another thing is if the function is like this inverse of this element will have multiple elements as its image.



Now that again contradicts our condition for a function. So a function's inverse can be spoken about provided every element in the co-domain has a pre-image which means f is onto. That's a condition. And also no element should have two pre-images which means f is one-one.

Think about it patiently. All I am trying to say is a function f we can talk about its inverse provided it is both a one-one function and an onto function. And once a function is one-one and onto its inverse is only obvious. What is an inverse of an element? Let's say f inverse of y it is that element whose f of x is y as simple as this.



Let us now see some examples of inverse of a function.