Discrete Mathematics

Functions

Examples of Bijection

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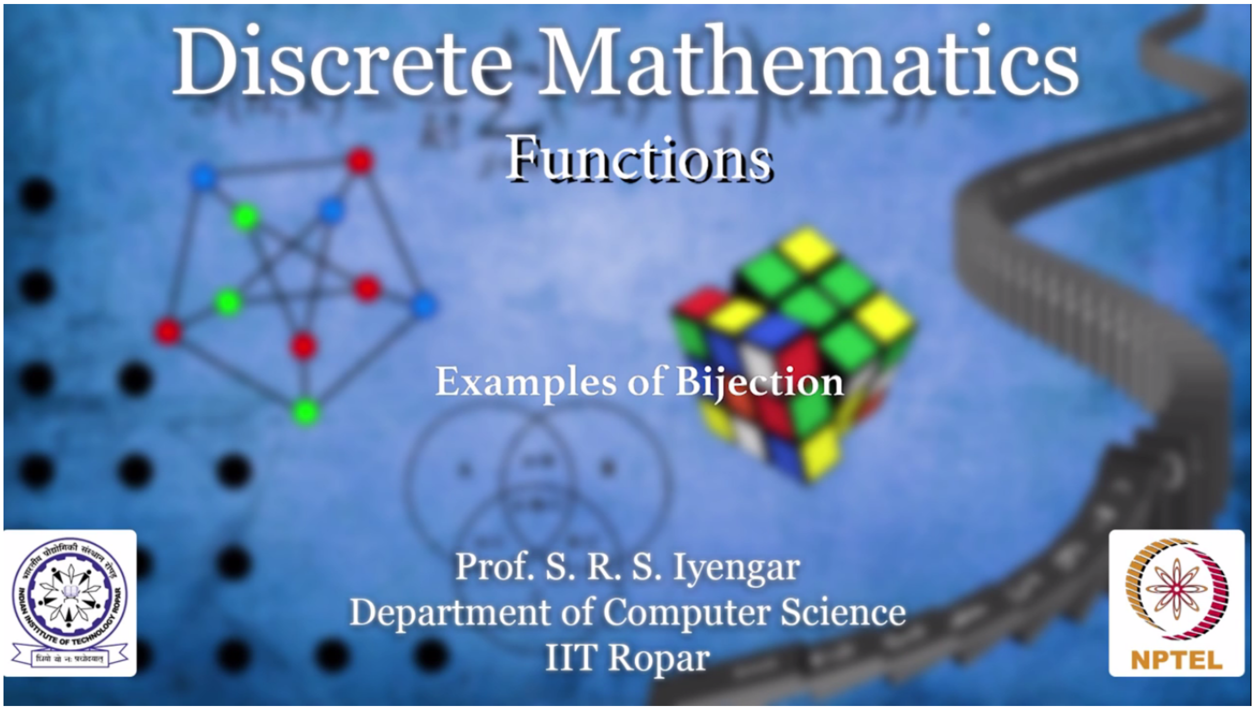


So we are proceeding with a few examples of bijection. Please note how do we say a function is bijection? If it is both one-one and onto. And to say that a function is not a bijection it is enough to show that either it is not one-one or not onto. If one of them is not satisfied then it is definitely not a bijection.

So let me quickly recollect one easy bijection that we have been seeing from week one that is the mapping from the set of all three digit binary numbers to the subsets of the set 1, 2, 3 is a bijection. I am not going to discuss the proof here as we have been telling it and you all must be very clear with it.

So this one bijection that we all know. Consider this function f from the set of all months, January, February, March, so on upto December to the set 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So I have the set of my months as the domain and numbers from 1 to 12 as the co-domain.

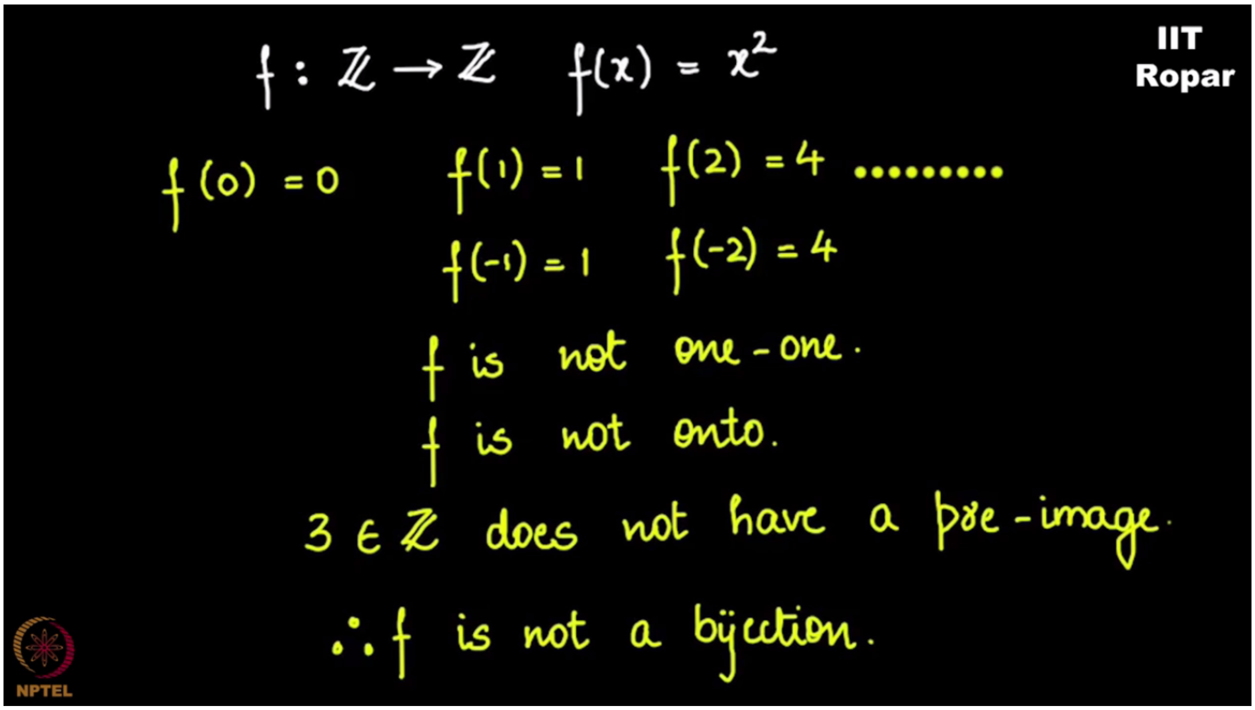
I define the function as every month going to the number in its order, that is January will go to 1, being the first month. February will go to 2, being the second month. March will go to 3. April will go to 4 and so on.

So if I write my months as M1, M2, M3, upto M12 then my function is Mi is mapped to I. F of Mi is equal to I where what is my I, I is from 1 to 12.

Now do you see that every month is mapped to a unique number here? And every number from 1 to 12 has some month associated to it. You see if you observe deeply you can see that it is both one-one and onto and hence this function is a bijection.

Take two minutes to think and then proceed ahead. Consider this function f from the set of all integers to the set of all integers where f of fx is equal to x square. So f of 0 is 0, f of 1 is 1, f of 2 is 4 and so on. But f of -1 is also 1, f of -2 is also 4. Do you see that f is not one-one. F is also not onto. Why? Because you see there is this element 3 in the co-domain is not having a pre-image. Root 3 square is 3 but does root 3 belong to our domain? No. Hence the function is not one-one and not onto. Obviously f is not a bijection. But it suffices to show that either it is not one-one or onto and hence is not a bijection as I mentioned earlier.

Consider this last example. The function f from the set of all rational numbers to the set of all rational number defined as f of p/q is equal to p. Seems interesting. F of ½ goes to 1. f of 1/3 goes to 1. Why? Because p/q is mapped to p. So what is p/q? 1/3, ½ and what is it mapped to? Both are mapped to 1. For that matter ¼ is mapped to 1. 1/5 is mapped to 1 and so on. You must quickly conclude that f is not one-one and hence we need not proceed further. F is definitely not a bijection.



These were a few examples. Of course, there will be several many. We will leave it to you as an exercise to explore further.