Discrete Mathematics

Functions

Examples of Onto Function

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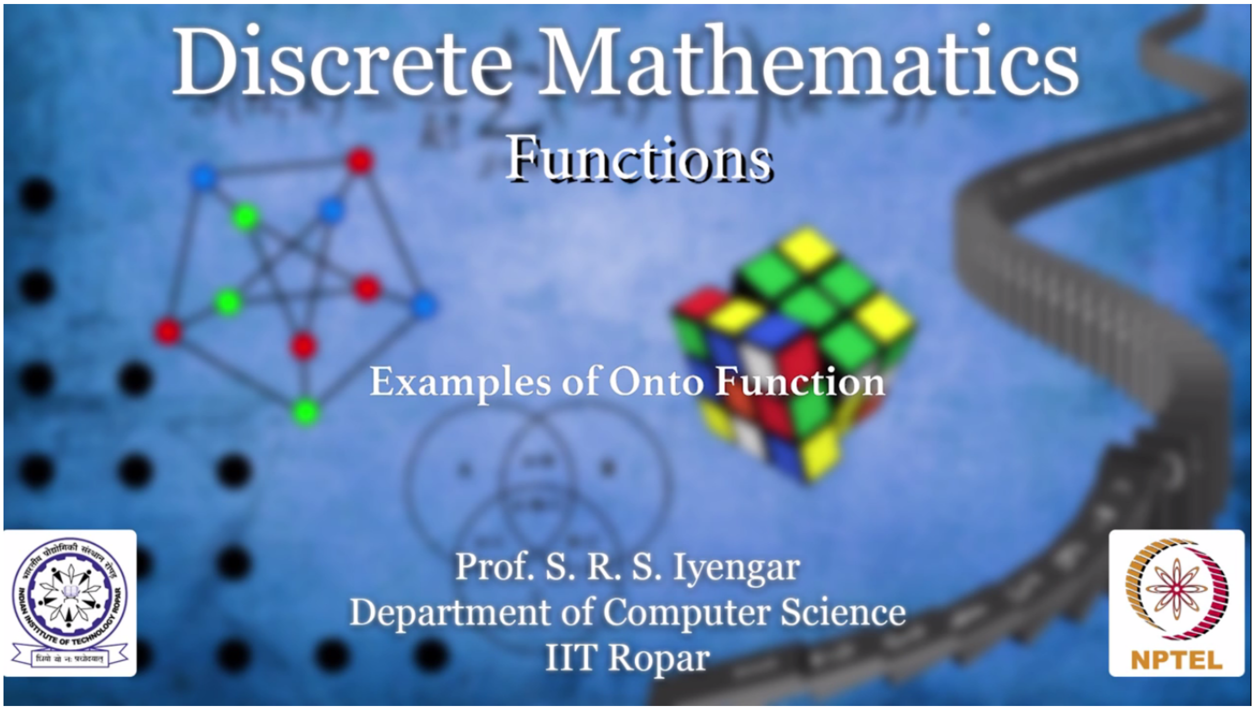


We are now going to see some examples of onto functions. Consider this function f from the set of all natural numbers to the set of all natural numbers defined as f of x is equal to x.

This function where x goes to x is called as the identify function. So let me start enumerating f of 1 is 1. f of 2 is 2. f of 3 is 3. f of 4 is 4 and so on. Each number is mapped to itself.

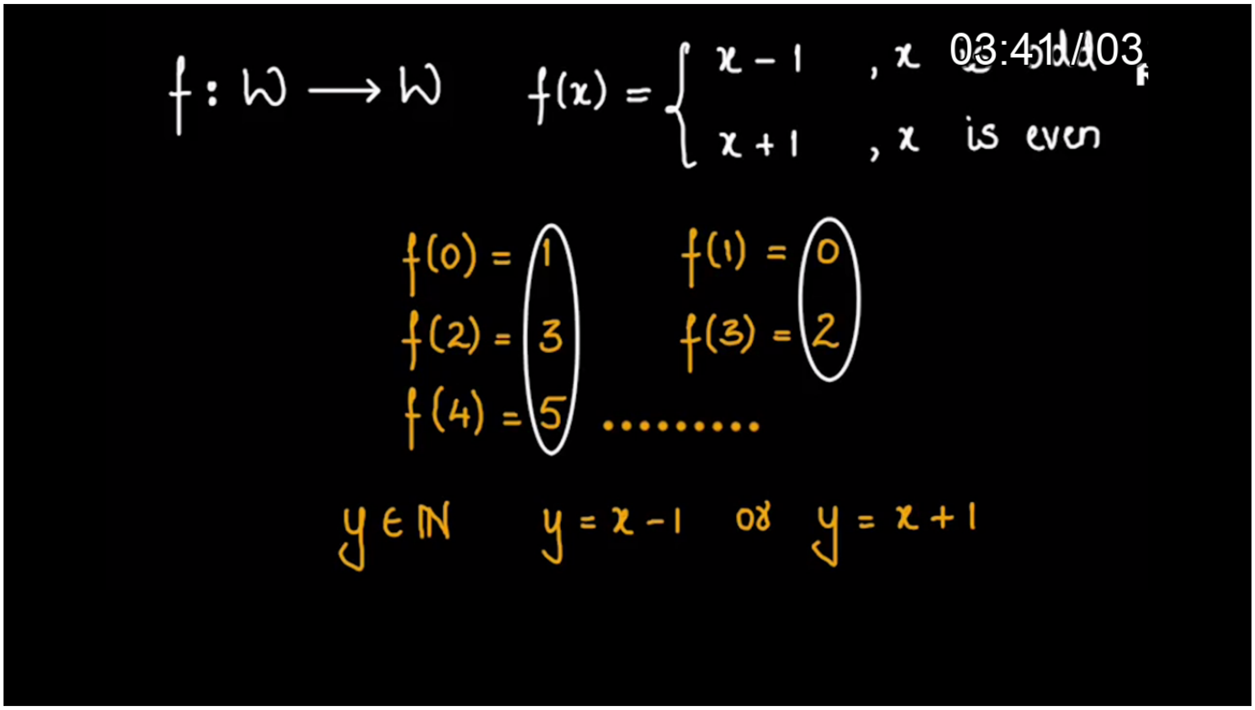
So I have to find out if this function is onto or not. Consider any element Y belonging to the co-domain. Co-domain is the set of all natural numbers. What will be its pre-image? We see that the pre-image is the number itself. Y will be mapped to y itself. And hence every element in the co-domain has the same element as its pre-image in the domain and hence f is onto.

Consider this function f from the set of all integers to the set of all integers defined as f of x is equal to mod x. the modulus function. How is it defined? F of -1 will be 1 that is mod -1 is 1. f of 1 is 1. f of -2 is 2 because mod of -2 is 2. f of 2 is 2. f of -3 is 3 and so on.

You see -1 and 1 are both mapped to the same element 1. And the same goes with -2 and 2 both of them are mapped to 2. Let us observe what is the range here. The range is 1, 2, 3 and so on. So we see that range is the set of all natural numbers but what are we given as co-domain. Integers are co-domain. Do you see that range is not equal to co-domain? And hence f is not onto because we know that if f is onto co-domain has to be equal to range.

Consider this function f. f is defined from the set of all whole numbers to the set of all whole numbers defined as f of x is equal to x-1 if x is odd, and x+1 if x is even. Do you see that f of x is taking two value here and it depends on what x is. If x is odd it takes the value x-1 and if x is even it takes the value x+1.

So we have a domain as whole numbers so 0 is mapped to 1 because 0 is even and hence 0+1 is 1. f of 1 is mapped to 2, 0. f of 3 is mapped to 3. f of 3 is 2, f of 4 is 5 and so on.



Do you see a pattern here? All odd numbers are mapped to even numbers. And all even numbers are mapped to odd numbers. Now let us consider an element y belonging to the core domain. So either it is x-1 or x+1, correct depending on what x is. So the pre-image of y is always either y+1 or y-1 and hence f is onto.