Discrete Mathematics

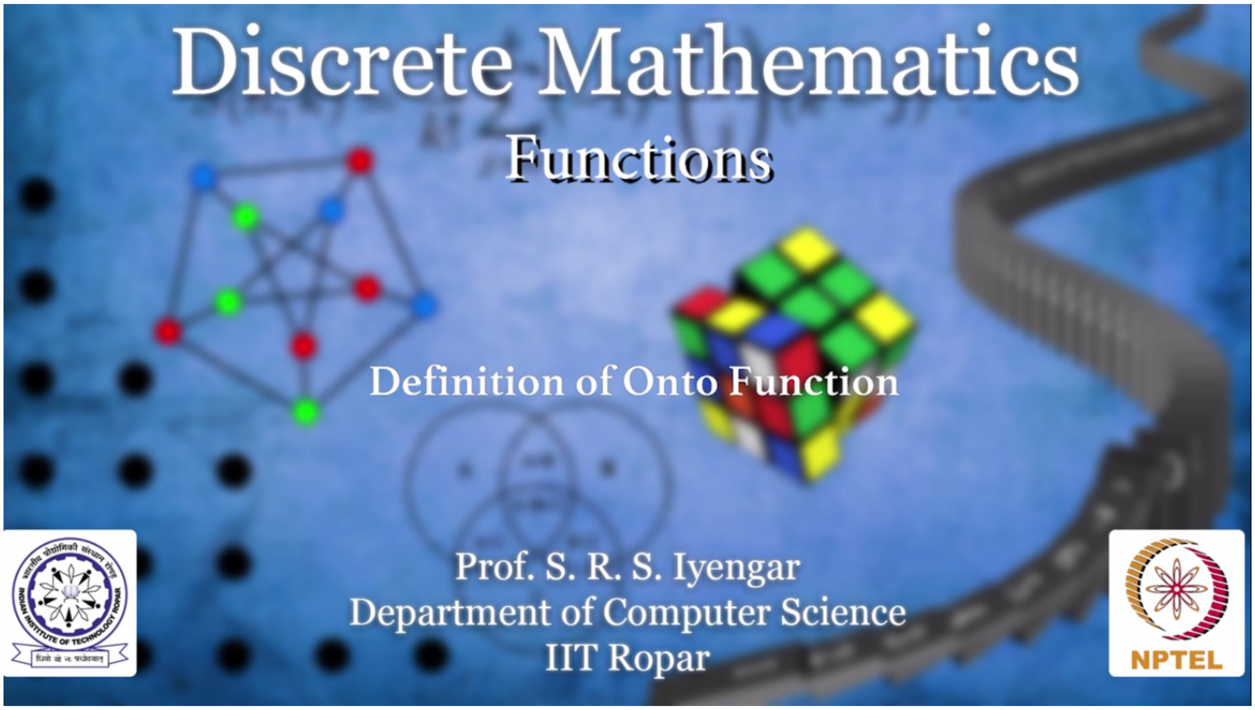
Functions

Cardinality condition in Bijection - Part 2

Prof S.R.S. Iyengar

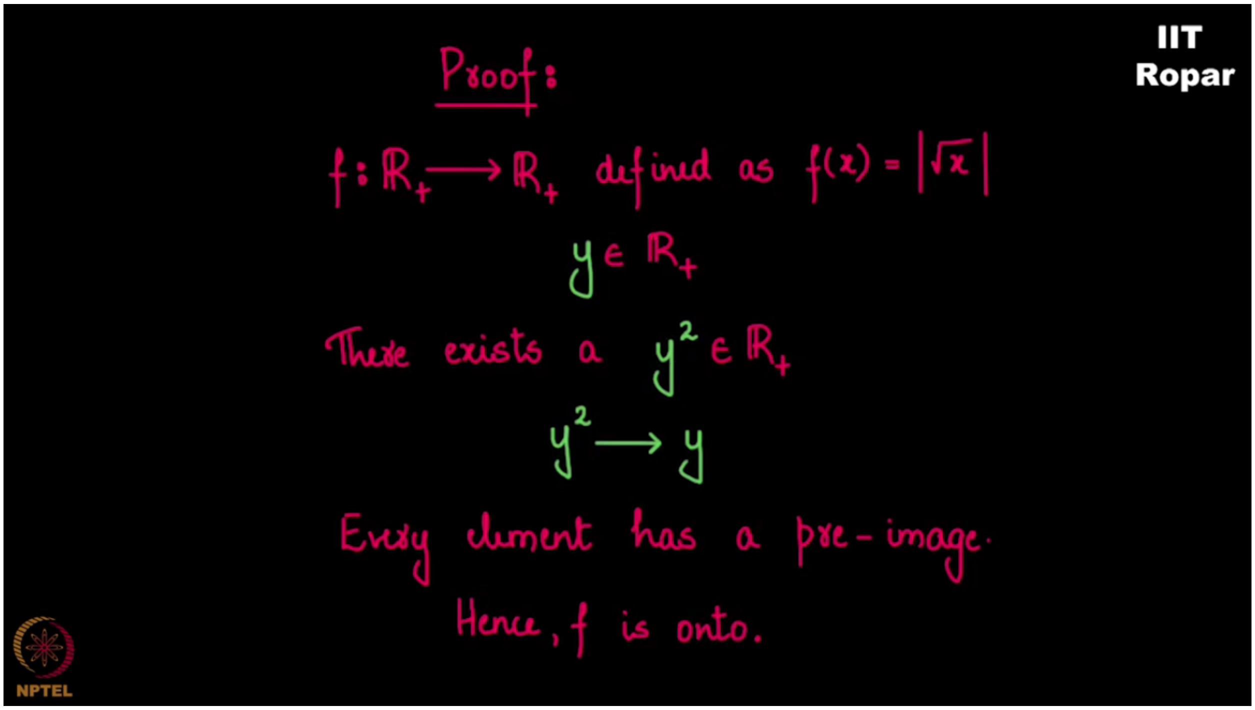
Department of Computer Science

IIT Ropar



Consider this example. A function f from set of all positive real numbers to set of all positive real numbers defined by f of x is equal to square root of x and modulo of it. Why am I taking the modulo of it? If you don't take modulo it gets a little confusing. You see square root of 4 can be both +2 and -2. There is no such function, functions are always defined as things where elements of domain go to a single element of co-domain. Two elements from domain can go to one element of co-domain. But one element of domain cannot go to two elements of co-domain. So if I say f of x equals square root of x there is this confusion that 4 can go to +2 or -2. That is why I am putting a modulo here. And also observe my co-domain is a set of all positive real numbers. There is no room for negative real numbers here. Right.

What is this function? How does it look like? You pick an element of your choice in the co-domain. Let's say 7.6 that belongs to co-domain. It's a positive real number. Is there an element which goes to 7.6? What is that? It is 7.6 square whose square root is 7.6. So you pick any element here. Why? There exist an x such that f of x is equal to y. Now that is a very formal definition of a onto function. So how is an onto function defined? An onto function is defined by the following statement let me make it clear. Let me write it down, for every element y in the co-domain there exists some x in the domain such that f of x is equal to y.



So let's prove that. Do you think this function is onto? Looks like it but let me prove it using this definition I have stated here. How do I prove it? Pick any element y from the co-domain R+. I say there exists an x, what is that x? That x is nothing else but y square. When you pick x as y square, y square indeed is going to y and y square belongs to R+. And it is going to y as per our requirement. So every element has a pre-image. And hence the given function is indeed onto.