Discrete Mathematics

Functions

Providing a Function is One-One

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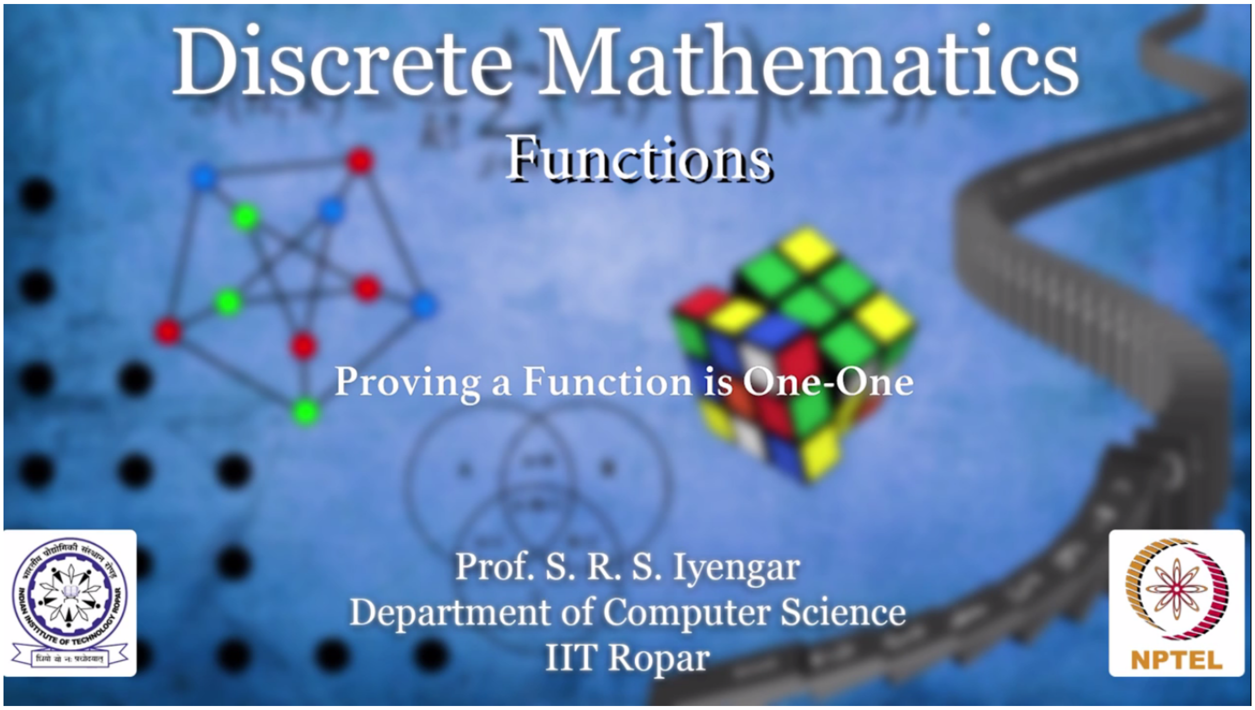


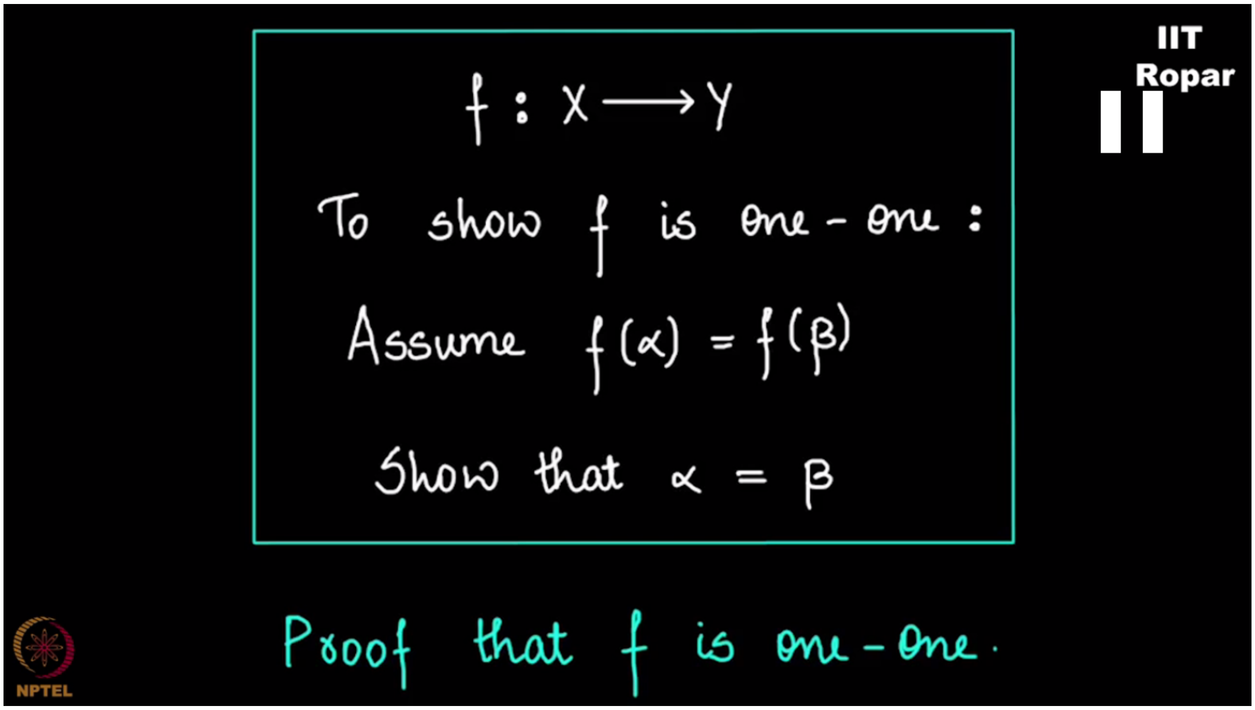
Here is a slightly involved concept. Although straightforward to explain what the question is, you will take some time to even understand what is this question all about. The question goes like this, given a function f how do you show that it is one-one? Why are we even asking this question? Just look at this pictorial representation of a function. You see that as I have told you function is all about someone shooting from the domain on to something in the core domain. You ensure that not two elements of the domain goes to the same element in the co-domain. If this happens we are done. We can call the function to be one-one but then when the function is an infinite function by that I mean it has a domain with infinitely many values you cannot be pictorially trying to – you cannot pictorially try to see are there two elements going to the same element right. It is practically impossible to check this for infinitely many elements.

So there is a proof technique that you can use. What is that? Let me explain that using it using a very simple straightforward example. Consider the function f from positive integers to positive integers defined by f of alpha is equal to 7 alpha. Alpha X and you can use anything. Okay so basically 1 goes to 7, 2 goes to 14 and so on. Is this a one-one function? Yeah it can be. You can dismiss this since a yes it's a one-one function. It's very obvious. No two elements can go to the same element but how do you prove it. Let me prove it very-very patiently. Observe although in this example you feel there is no need to prove but I'm teaching you how to prove that a function is one-one.

When will a function not be one-one? When two elements they go to the same element. Let's assume such a thing is happening here. What do I mean by that? There is a alpha and there is a beta such that f of alpha is going to that element where f of beta is also going okay and they are not the same or rather I'm sorry they're the same. F of alpha is going to an element. F of beta is another element and these two things are the same that is when we have a problem with this function being a one-one function. It will not be a one-one function.

So let me assume such a thing is happening. F of alpha is being equal to f of beta and these two elements alpha and beta are a different elements in the domain of course. Now f of alpha is 7 alpha and f of beta is 7 beta you know 7 times a number is equal to 7 times another number means that alpha should be equal to beta. Why you can cancel of 7 here correct. Alpha will be equal to beta. Now that contradicts the very assumption that alpha was different from beta.

In plain English you know what I just did I should but in case alpha and beta goes to a same element according to the function then alpha and beta will become the same which means you cannot see a structure like this. You cannot have a alpha and beta different things going to the same element when you say the function is one-one. Given that here the function f of alpha is 7 alpha is a one-one function, it doesn't let you have a structure like this. So whenever the structure is not there you say the function is one-one and this is a proof strategy by this I mean whenever you are given a function f from a domain x to co-domain y and if you are asked to show that f is one-one you should simply say assume f of alpha is equal to f of beta and then show that alpha gets forced to become beta which in itself is a proof that f is one-one.



We will see a couple of examples where the function f is non-trivial that it's one-one. It's not very obvious that is one-one but we'll show using this proof technique that it is indeed one-one.