NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Problems on Permutations

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So far we have seen what is meant by n factorial, what is meant by permutations. We have seen a video on it. We have seen that the formula goes like this nPr means picking r items out of n items with the order respected and this is equal to n factorial by n minus r factorial. So this is the formula of permutations.

Now let us solve some problems.

The first one. How many three-letter words with or without meaning can be formed out of the letters of LOGARITHMS if repetition is not allowed? So the word given to us this LOGARITHMS. You see LOGARITHMS has 10 different letters; L-O-G-A-R-I-T-H-M-S. So we have to form three-letter words which can or not have meaning and how many ways can we do this? This is the question.

So let me enumerate some of the three-letter words. LOG, OGA, ARI, ART, ARH, and so on. So in how many ways can we do this? So we have 10 letters and we have to form three letter words. So it is the problem of picking 3 letters out of 10 letters. So it is 10P3 which is equal to as the formula says 10 factorial by 10 minus 3 factorial which is same as 10 factorial by 7 factorial and after computation this gives 720 as the answer.

So in 720 ways we can form three-letter words which need not necessarily have meaning out of this word LOGARITHMS.

The next question. In how many ways can the letters of the word LEADER be arranged? So the word here is LEADER. It has six letters but please note there are two E's here unlike in the last question where all the letters were different. So we have LE this is the first one, ADER and this E is the second part. And the question is in how many ways can the letters of this word be arranged.

So we have six letters and two repetitions of E here. So the number of ways to arrange our six factorial by 2 factorial. The six factorial is 6 into 5 into 4 into 3 into 2 into 1 divided by 2 factorial and this is 6 into 5 into 4 into 3 which comes up to 360. Now why did we divide by 2 factorial here? The world LEADER has two E's. I will mention them as E1 and E2. When we arrange the letters of the world LEADER in different ways in case I consider E1 and E2 as two different letters, if we treat them as distinct letters then there are six factorial permutations but since the letters are the same if we don't treat them as distinct we are over counting and hence we divide by two factorial. So the number of ways to arrange the letters of this word are 360.

The next question. A company has 10 members on its board of directors. In how many ways can it elect a president, a vice president, a secretary and a treasurer. So this company has 10 members in its board of directors and four members are to be picked. So it's same as asking the question picking four out of 10 and this goes by 10P4 which is 10 factorial by 10 minus 4 factorial which is 10 factorial by 6 factorial and this is the same as 10 into 9 into 8 into 7 and this comes up to 5040.

The next question. In how many ways can the word HOLIDAY be arranged such that the letter I will always come to the left of the letter L? So the word HOLIDAY has all distinct letters and the number of letters are 7 and therefore there are 7 factorial ways to arrange the letters. Now there is a constraint here; the number of ways such that I comes to the left of the letter L; we have to compute how many times this happens. So you see that either I should be to the left of the letter L or to the right of the letter L. So in exactly half of the arrangements it will happen. So in 7 factorial by 2 and these many number of arrangements letter I will always come to the left of the letter L. So 7 factorial by 2 which comes up to be 7 into 6 into 5 into 4 into 3. So these many number of ways the letter I will come to the left of the letter L.

Let us move on to the next question. Find the number of permutations of the letters of the word CLIMATE such that the vowels always occur in odd places. So the word here is CLIMATE and you see it has no repetitions but the condition given is the vowels should always occur in odd places. Let us understand this. So we see that the word climate has four consonants; C, L, M, and P and three vowels I, A and E. It has the total number of letters are seven and we have seven slots like this and the words can occur here in these seven places. So you see they are odd and even slots like this. So there are four odd slots and three even slots. We have three bubbles and they should occur in the odd places. So in how many ways can we arrange this? So total number of odd places are four and we have three vowels. So the number of phase three vowels can occur in four odd places. The same as for 4P3 which is nothing but four factorial by three factorial this is nothing but four factorial by four minus three factorial and this is four factorial by one which is nothing but 24. So in 24 ways the three vowels can occur in four odd slots.

You see there are only three vowels here and four consonants. So we still have four places remaining after the three vowels take three places. So the four consonants can take rest of the four places in how many ways? In 4P4 ways which again comes up to be four factorial by 0 factorial which is nothing but 1 and we have 24 here. So in 24 ways the four consonants can take four slots. So the total number of permutations which are possible is 24 into 24 and this goes by the rule of product because we even want that the vowels should take the odd places and the consonants taking the rest of the places. So both these events should occur and we have 24 possibilities for each of them and hence the total number of permutations are 24 into 24 which comes up to be 576.

In how many ways can MATHEMATICS be arranged so that the vowels always come together? So the condition here is the vowels must come together. So the world is MATHEMATICS and the vowels here are A, E, A, I and the consonants are M, T, H, M, T, C, S. Yes. So I'll do one thing. I'll consider these vowels as one unit and these consonants there are eight letters here we see that M appears twice and T also appears twice. Let us first compute in how many ways we can arrange these consonants. So there are eight letters and we have two letters which repeat; M and T. So the number of ways of arranging M, T, H M, T, C, S is equal to 8 factorial because there are eight letters by 2 factorial into 2 factorial. This is for M and this is for T. So this goes for the consonants. Now we saw that the vowels were A, E, A, I. In how many ways can we arrange this? This is 4 factorial because there are 4 vowels and they say that A is repeating twice and hence we divide by 2 factorial which is the number of ways of arranging these vowels.

So we have computed differently for consonants and for vowels. So the total number of ways in which this entire world MATHEMATICS can be arranged so that the vowels can always come together are 8 factorial by 2 factorial into 2 factorial into 4 factorial by 2 factorial. Again by the rule of product. You see I treated this A, E, A, I as one unit because they have to always come together and so these many number of ways the word MATHEMATICS can be arranged so that the vowels come together.

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