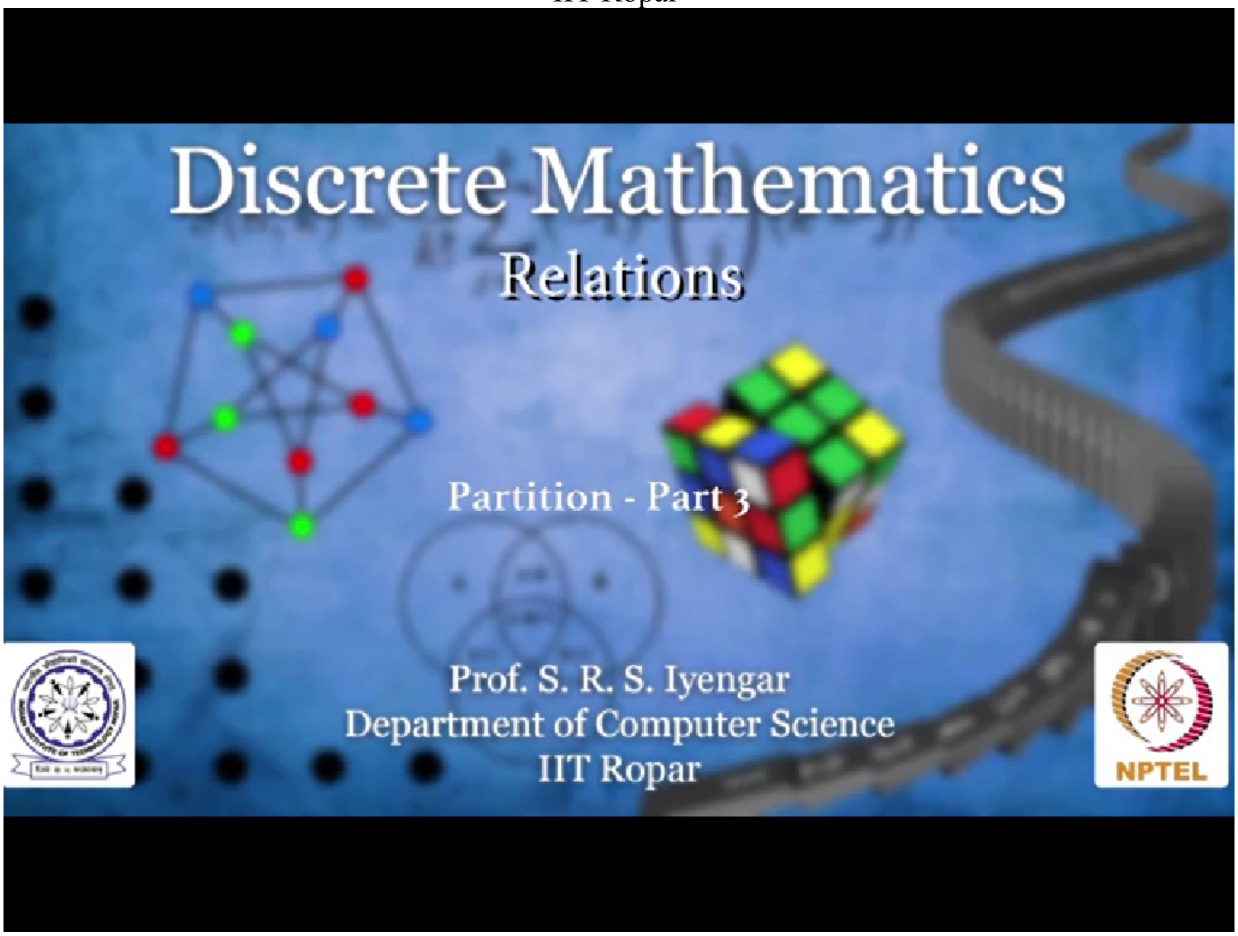
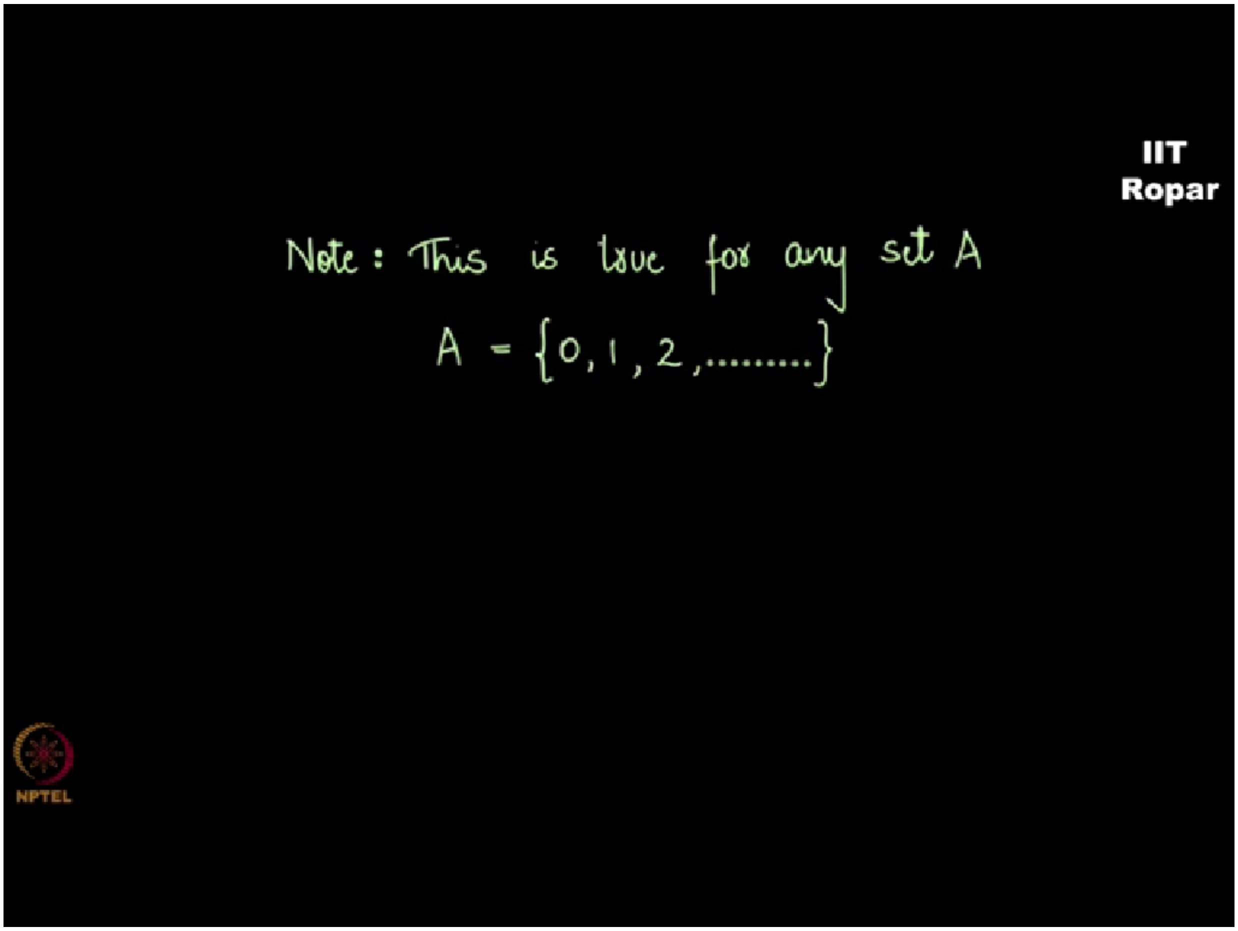
NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Relations
Partition - Part 3
Prof. S. R. S. Iyengar
Department of Computer Science
IIT Ropar

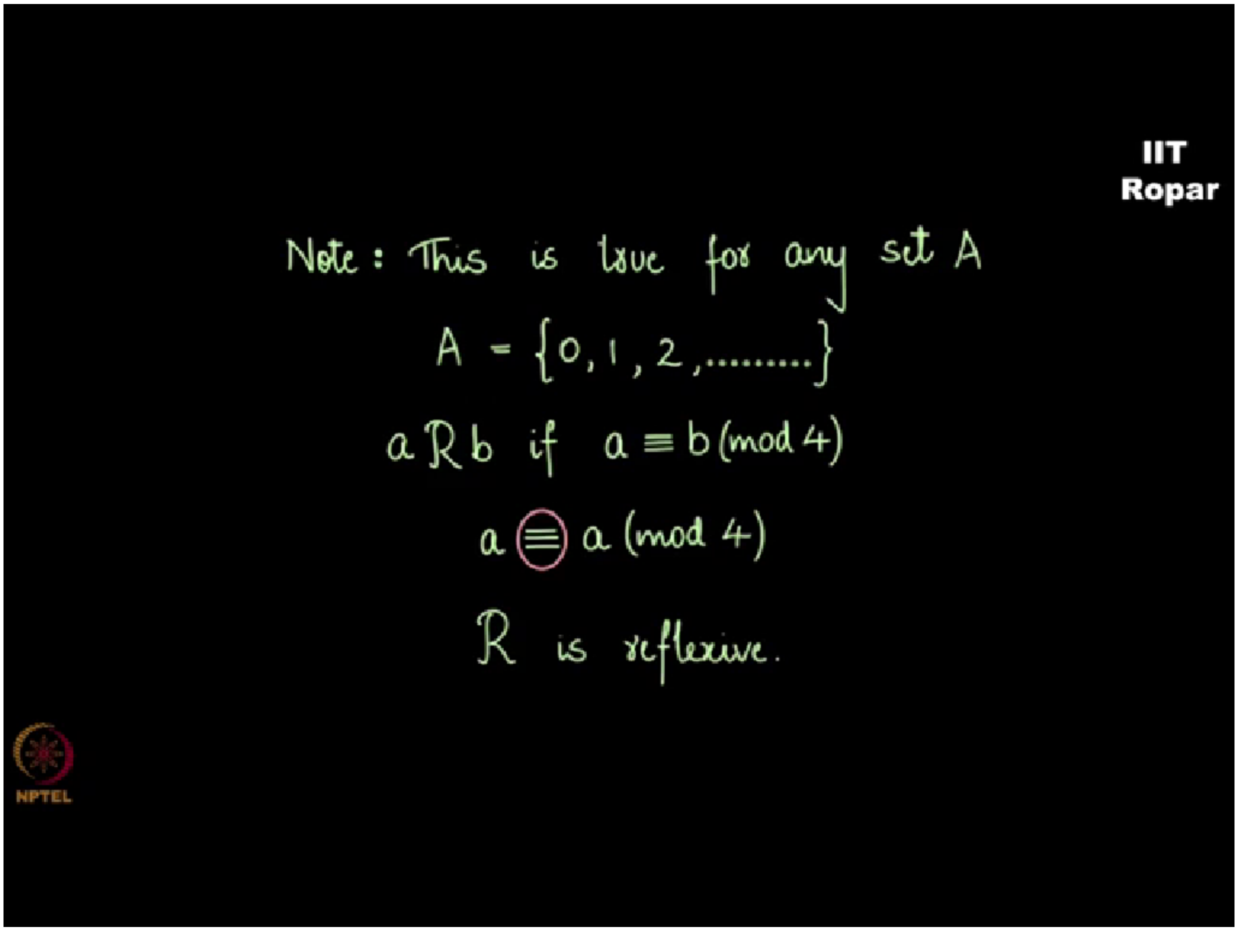


We must note something. This is not just true of finite set A. this is true in general for any set A. I used a finite set with n elements in the previous case, in the previous illustration just to make you all understand. Okay. The set A can actually be infinite.

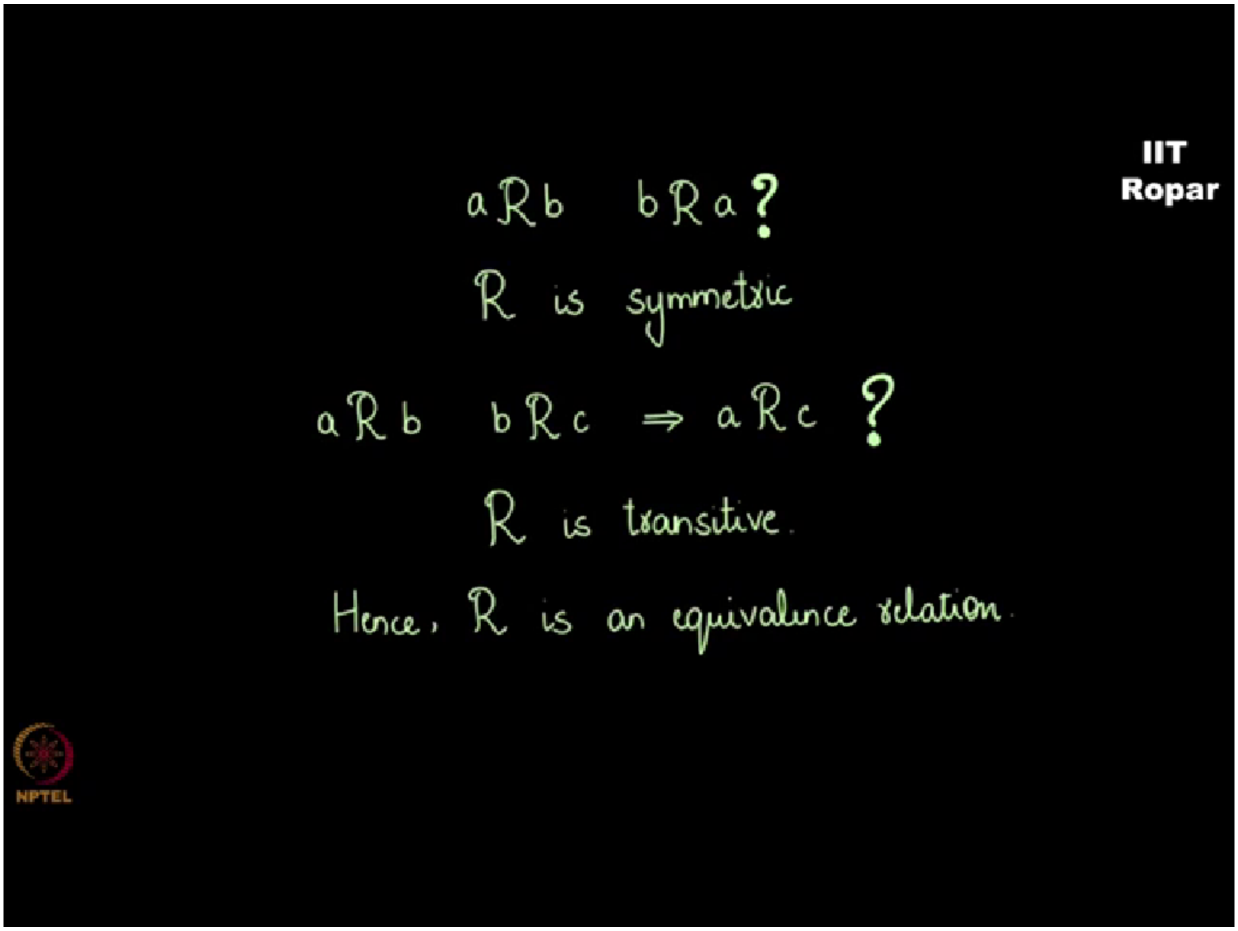


Let’s see a quick example. Let us assume the set A is set of all integers, let’s say 0, 1, 2, set of all nonnegative integers up to, let’s say, infinity. 0, 1, 2, 3, 4, you take everything. All right. And what you do is you say a is related to b if a is congruent to b (modulo 4). By that we mean a and b leave the same remainder when divided by 4.

What just happens here? Let's observe. A is obviously equivalent -- a is equivalent to a (mod 4). Okay. A is a (mod 4) because a and a obviously leave the same remainder when divided by 4, right? That is what we mean by a three lines like this and b (mod 4) means a and b leave the same remainder when divided by 4.



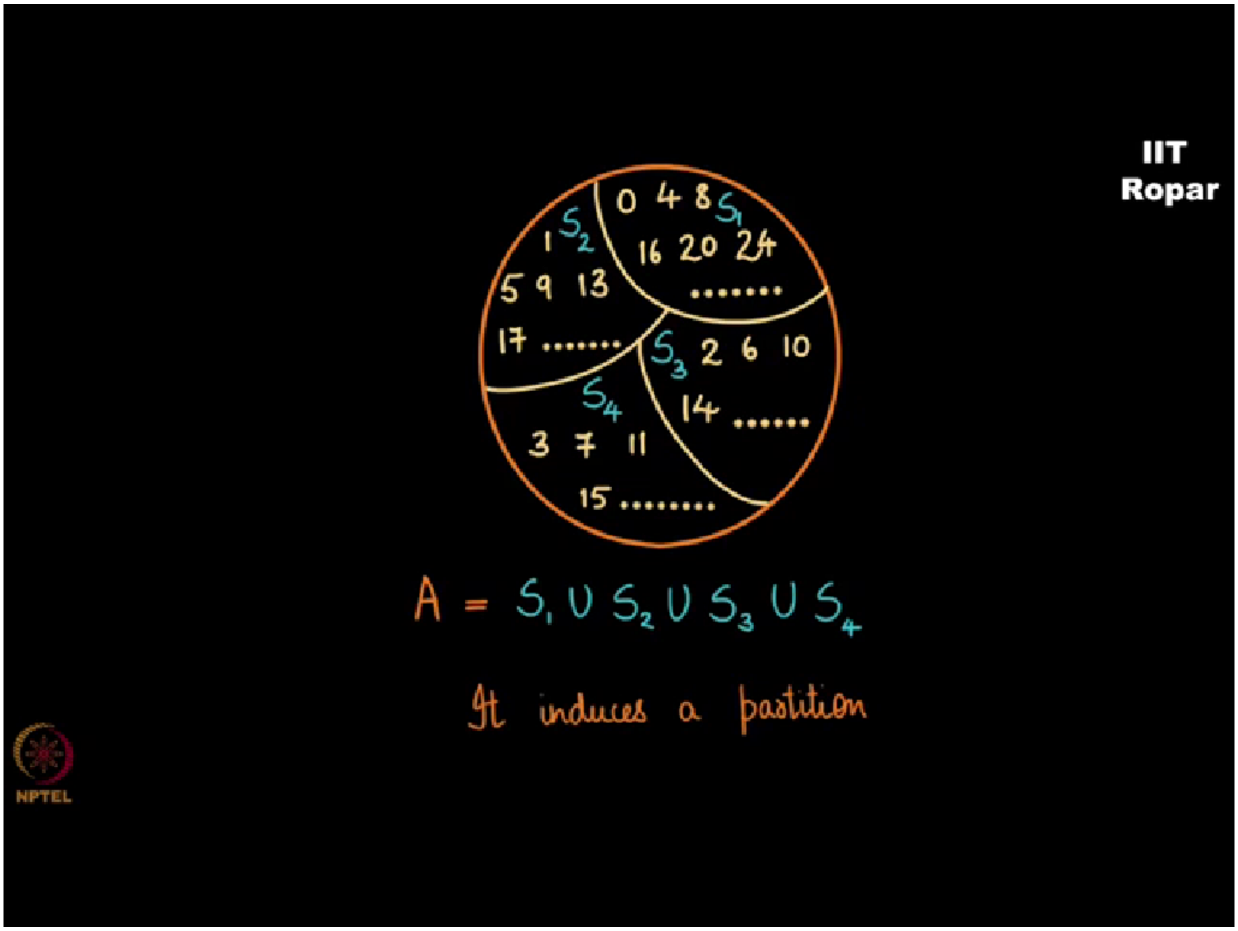
So it is obviously reflexive because a is related to a for all elements a in A, capital A. Okay? And if a is related to b, what can you say about b being related to a? Isn't it obvious? A is related to b means a and b leave the same remainder when divided by 4. If that be the case, don't you think b and a should also leave the same remainder when divided by 4, like what are we even discussing here, right? So symmetric property follows very easily, right?



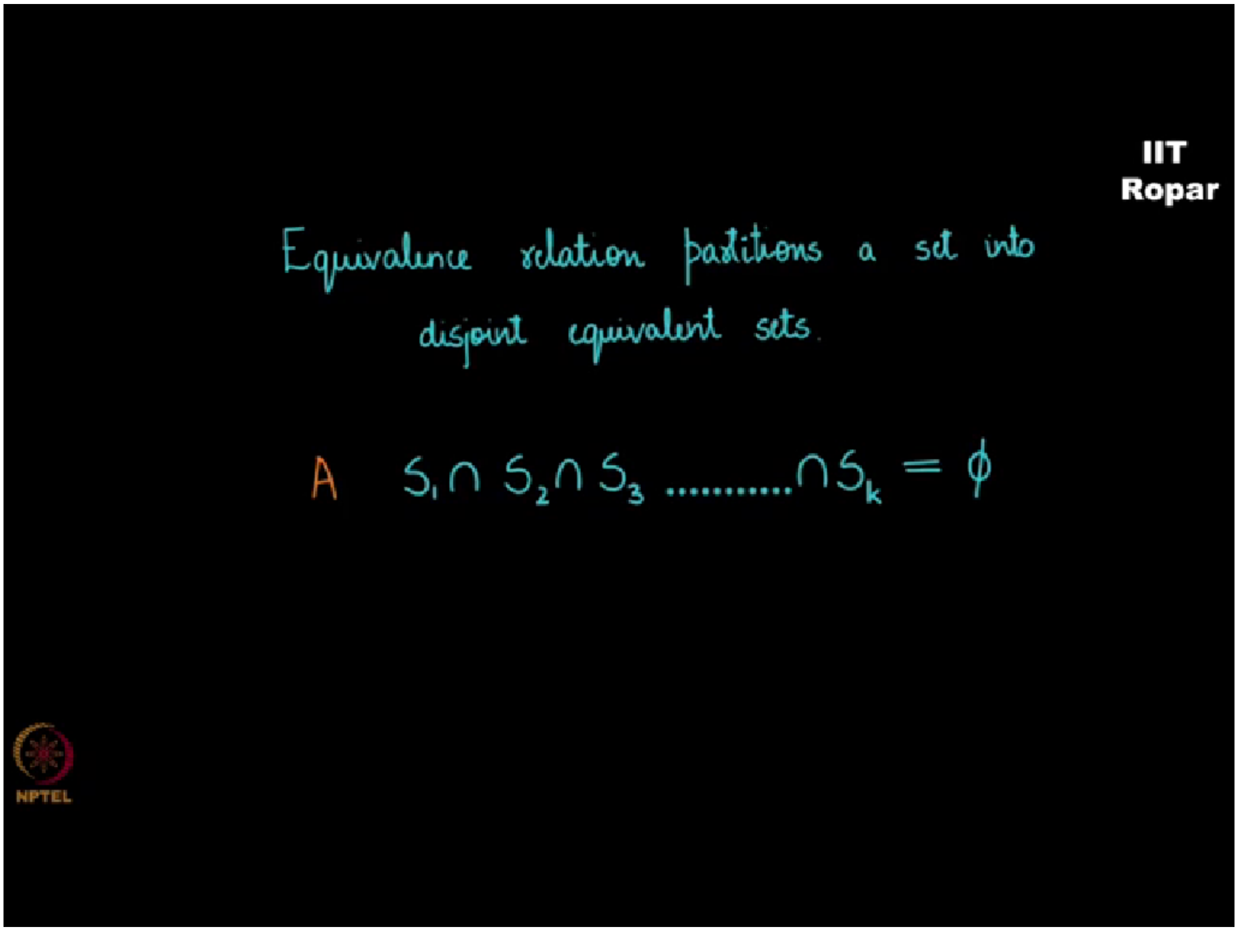
Now transitivity, what is that? A is related to b. B is related to c. It should imply that a is related to c. Is this true? When a and b leave the remainder, same remainder when divided by 4 and b and c leave the same remainder when it's divided by 4, it is only obvious that a and c also leaves the same reminder when divided by 4. Let me not explain. You see why this is true, right? Okay.

So transitivity also follows. So this relation R defined by a is related to b if a and b leave the same remainder when divided by 4 happens to be a reflexive relation, symmetric relation and a transitive relation and hence it becomes an equivalence relation, but will it induce the partition on a? Yes, it does. Very easy to observe.

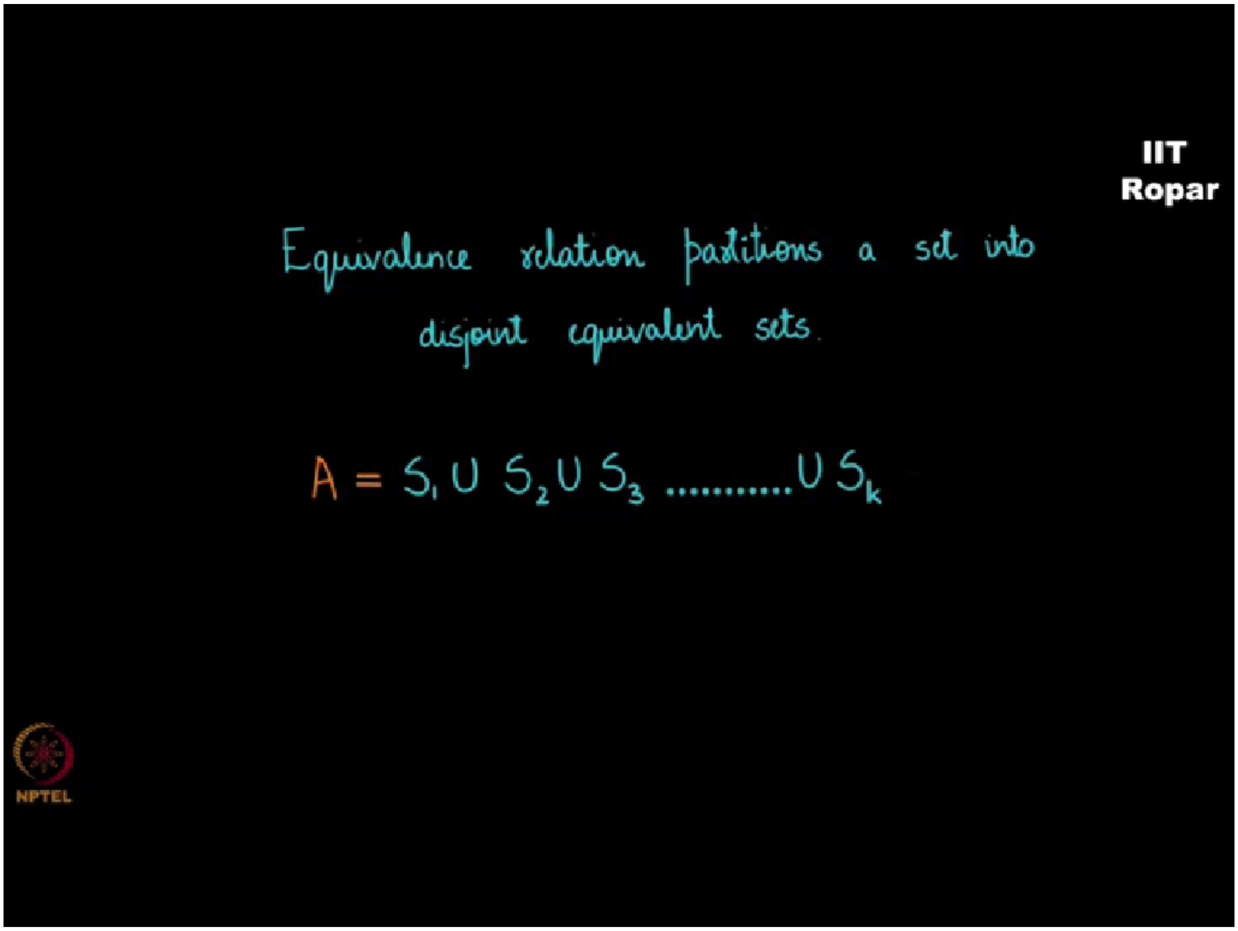
0, 1, 2, 3, let us write this on a -- inside a big circle. You see the whole of this 0, 4, 8, 12, 16, 20, 24 and so on, all multiples of 4 as you can see belongs to one cluster. Let's call it S_1. Okay. 1, 5, 9, 13, 17 and so on belongs to another cluster, let's call it S_2. And the next one would be 2 and then 6, and then 10, and then 14, all of these leave 2 as remainder when divided by 4. They all will form a partition S_3. And your S_4 will be 3, 7, 11, 15 and so on. This will be your S_4 and ta da, as you can see your entire set A, which was all nonnegative integers can now be written as S_1 union S_2 union S_3 union S_4. So a set of all these integers, a relation like that of this where a is related to b if a and b leave the same reminder when divided by 4 or for that matter any number, 4 is just for example, you can even replace 4 by 5, then you will get S_1, S_2, S_3, S_4, and S_5. Observe, right? Okay. So it induced a partition here is all I am saying, right? Okay.



And point to note is it’s an infinite set, which means it doesn’t hold good just for although the proof was for finite sets, it is easy to observe that it is true for infinite sets as well.



As and always, why are we studying this? Many aspects of computing has something to do with these kind of a relation called the equivalence relation. It is important for us to have an understanding that equivalence relation partitions a set into disjoint equivalent sets. I mean, given a set A with n elements, you will get S1, S2, S3 up to S_k is all I mean such that these things are all mutually disjoint. No two sets have a element in common, an element in common and union of these sets will give you back A.



Please remember this. If you don't remember the proof, it's okay. You can always get it by a little bit of thought, but you must remember that equivalence relation induces a partition of a given set.

IIT Madras Production

Founded by

Department of Higher Education

Ministry of Human Resource Development

Government of India

www.nptel.ac.in

Copyright Reserved