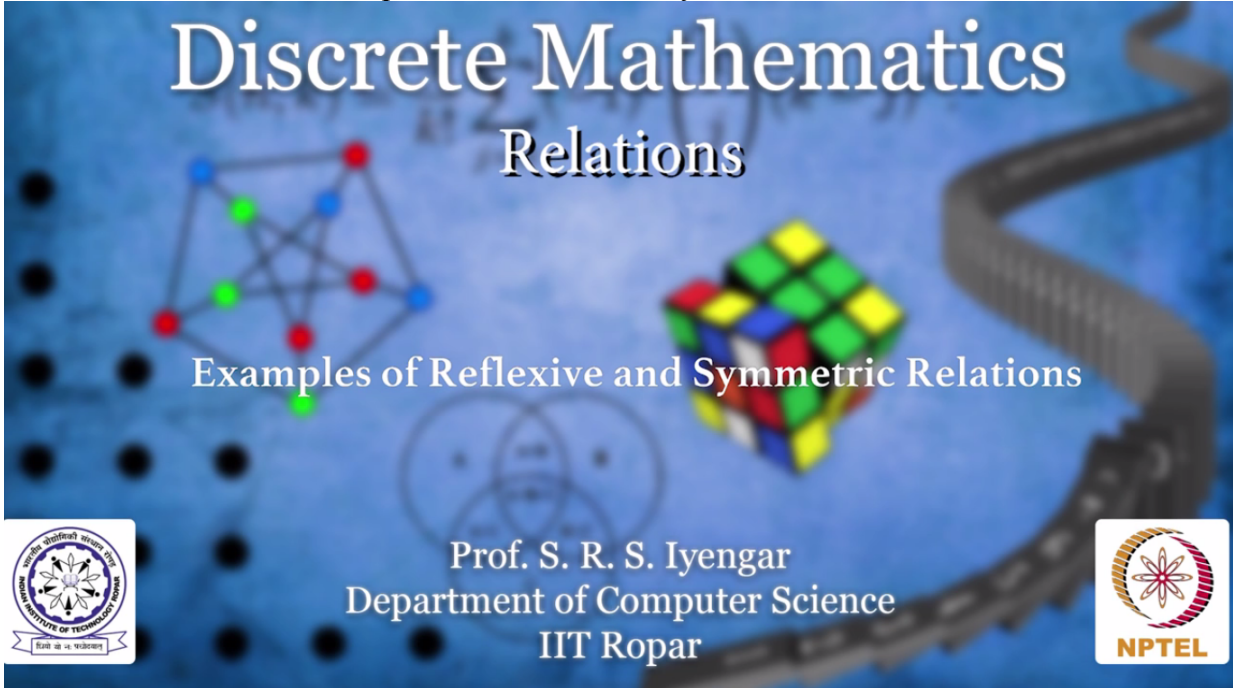


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Discrete Mathematics
Logic



Examples of Reflexive and Symmetric Relations



Discrete Mathematics
Relations

Examples of Reflexive and Symmetric Relations

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Now that we have seen what are reflexive and symmetric relations, let us see these concepts in action by solving a few problems. Consider this set A equal to $1, 2, 3, 4, 5$ and let R be a subset of $A \times A$ and I am going to write R as these elements. Set of all these elements. One. The question is is R a reflexive relation? So A has precisely five elements and in R I see $1,1, 2,2, 3,3, 4,4, 5,5$. Yes R is reflexive because every element of A has an a,a belonging to R and hence R is reflexive.

$$1. A = \{1, 2, 3, 4, 5\} \quad \mathcal{R} \subseteq A \times A$$

$$\mathcal{R} = \{(1, 1), (1, 4), (2, 2), (2, 3), (3, 3), (3, 1), \\ (4, 4), (5, 5), (5, 1)\}$$

Is \mathcal{R} a reflexive relation?

Ans: \mathcal{R} is reflexive

$$(a, a) \in \mathcal{R}$$



Consider this relation as set of all a, b such that a, b are natural numbers and b is equal to a square. So whenever I write a, b , b will be a square. Is this relation reflexive? So let me just try enumerating \mathcal{R} so \mathcal{R} will have the elements $1, 1$, I will start with 1 , the first natural number. So $1, 1$ square which is 1 and then I go to 2 . $2, 2$ square is 4 . $3, 9$. $4, 16$, $5, 25$ and so on.

Now you see $1, 1$ belongs to \mathcal{R} . 1 is 1 square and hence it is there in \mathcal{R} . But $2, 4$ is not equal to $4, 2$ because 2 is not 4 square. And hence but $2, 2$ doesn't belong to \mathcal{R} you must be guessing the reason because 2 is not equal to 2 square and hence the reason is the same for all the other natural numbers such as $3, 3$, $4, 4$, and so on. We don't have these elements in \mathcal{R} and hence this is not a reflexive relation. Consider the relation on the set of integer \mathcal{R} equals x, y such that x square is equal to y square. So \mathcal{R} will be having all those elements whose squares are equal.

$$2. \mathcal{R} = \{(a, b) \mid a, b \in \mathbb{N}, b = a^2\}$$

Is \mathcal{R} reflexive?

$$\mathcal{R} = \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), \dots\}$$

$$(1, 1) \in \mathcal{R}$$

$$(2, 2) \notin \mathcal{R}$$

\mathcal{R} is not reflexive.



So is this relation reflexive? Is this relation symmetric? So \mathcal{R} is going to have all those elements such as $0, 0$, $1, 1$, $1, -1$, $-1, 1$. You see if you are observing you can see that -1 square is equal to 1 square. $2, 2$, $-2, 2$, $2, -2$, $3, 3$, and so on. So these will be the elements of \mathcal{R} .

Now for a relation to be symmetric A, B should be equal to B, A . Is that happening here? $1, -1$ the squares are 1 . And $-1, 1$ again the squares are 1 . So for any element A , $-A$ it is always equal to $-A$, A . A and $-A$ belong to integers and hence this relation is symmetric. Consider this relation on natural numbers. \mathcal{R} equals a, b , where a and b are natural numbers and a into b is 14 . Is this relation symmetric? So what are the elements of this relation? We have to pick only natural numbers and hence \mathcal{R} is going to have $1, 14$, $2, 7$, $7, 2$ and $14, 1$.

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3. Consider a relation natural numbers.

$\mathcal{R} = \{(a,b) \mid a,b \in \mathbb{N}, a \cdot b = 14\}$. Is \mathcal{R} symmetric?

$$\mathcal{R} = \{(\underline{1}, \underline{14}), (\underline{2}, \underline{7}), (\underline{7}, \underline{2}), (\underline{14}, \underline{1})\}$$

Anything else? No. It is precisely these elements because we are concentrating only on natural numbers. 2,7 is there. 7,2 is there. 1,14 and 14,1 all are respectively equal pair wise. And hence this is a symmetric relation.

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