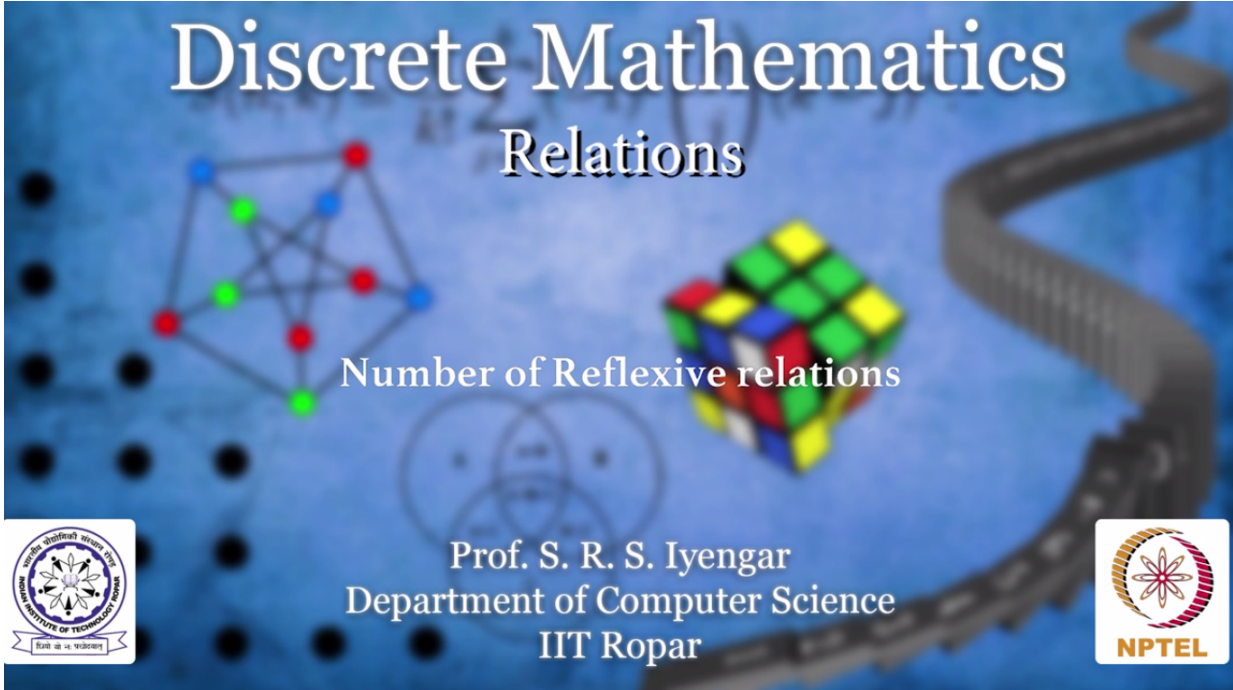




NPTEL
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Discrete Mathematics
Logic
Number of Reflexive relations



Discrete Mathematics
Relations

Number of Reflexive relations

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So we are going to ask the same question again and again. As you know discrete math is all about counting in fact more than 50% of the subject is all about counting. So let us again ask a counting question. We defined what is a reflexive relation right. So let me consider a set S as usual which comprises of n elements a_1, a_2 up to a_n and now I ask the question what are the total number of reflexive relations on S . What do I mean by that?

Reflexive relation

$$S = \{a_1, a_2, \dots, a_n\}$$

What are the total number of reflexive relations on S ?



Remember in the previous lecture we saw we asked the question what are the total number of possible relations in general. Now I am asking the question what are the total possible reflexive relations on a set with n elements. So let me think. How will a relation R look like? It is definitely a subset of $S \times S$ but not all subsets of $S \times S$ will be a valid reflexive relation. It is that subset which has all the elements $a_1, a_1, a_2, a_2, \dots, a_n, a_n$. These n elements are definitely going to be there but the rest of the elements, the rest of them, rest of them may or may not be there. Take a pause and then think about what I'm saying. These n elements must not should be there.

$$R \subseteq S \times S$$

Not all subsets are valid reflexive relations.

$$\{(a_1, a_1), (a_2, a_2), \dots, (a_n, a_n)\}$$

n elements

THINK!



But the rest of them may or may not be there and all possible ways in which you can put the rest of them, in quotes, “rest of them” in all possible ways in which you can include this is what makes all possible relations that are reflexive. So what is that? That is going to be straightforward. In $S \times S$ definitely you need to include a_1, a_1, a_2, a_2 up to a_n, a_n . The rest of the elements which are n^2 in number I need to exclude these entities, these n elements which is $n^2 - n$ and all possible ways in which I can choose the rest of them that I stated here which is going to be as and always 2 to the power of $n^2 - n$.

$$\text{Total number of reflexive relations} \\ = 2^{n^2 - n}$$

So the total number of reflexive relations happen to be 2 to the n square minus n. Again why? That's because I need to put these n elements for sure and the rest of them I have 2 to the rest of them choices. The rest of them happens to be n square minus n and hence the answer for my question happens to be 2 to the n square minus n.

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