NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Relations
Cartesian Product
With
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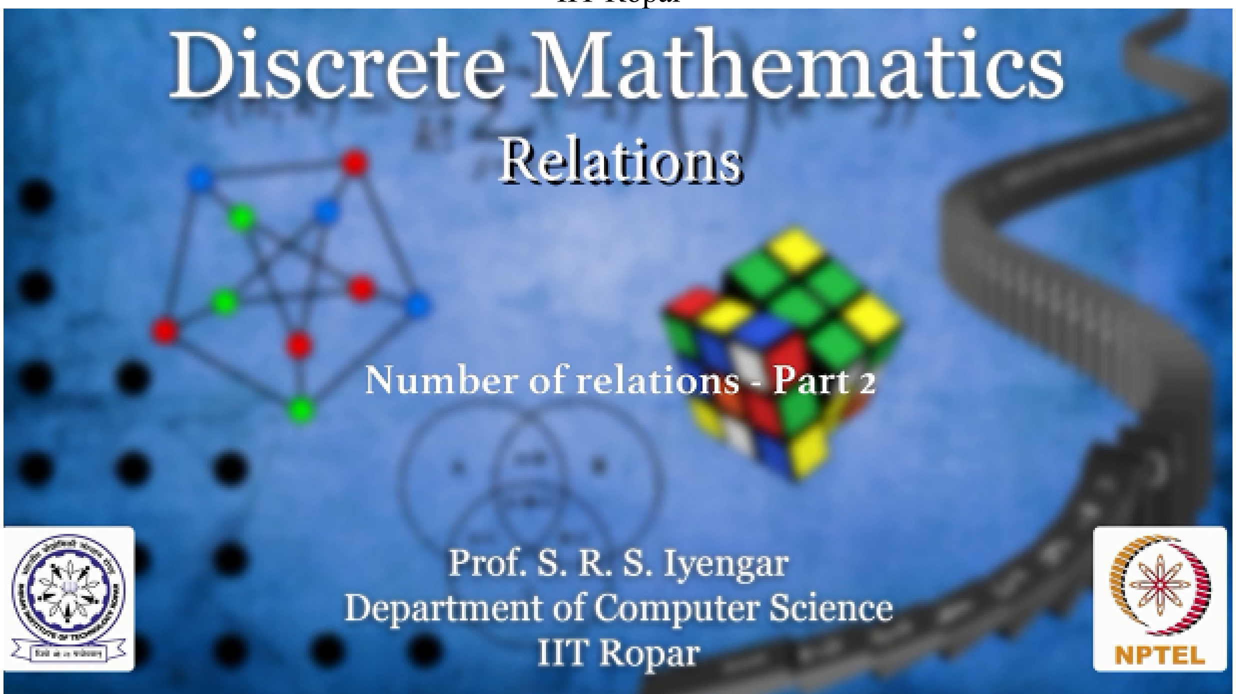

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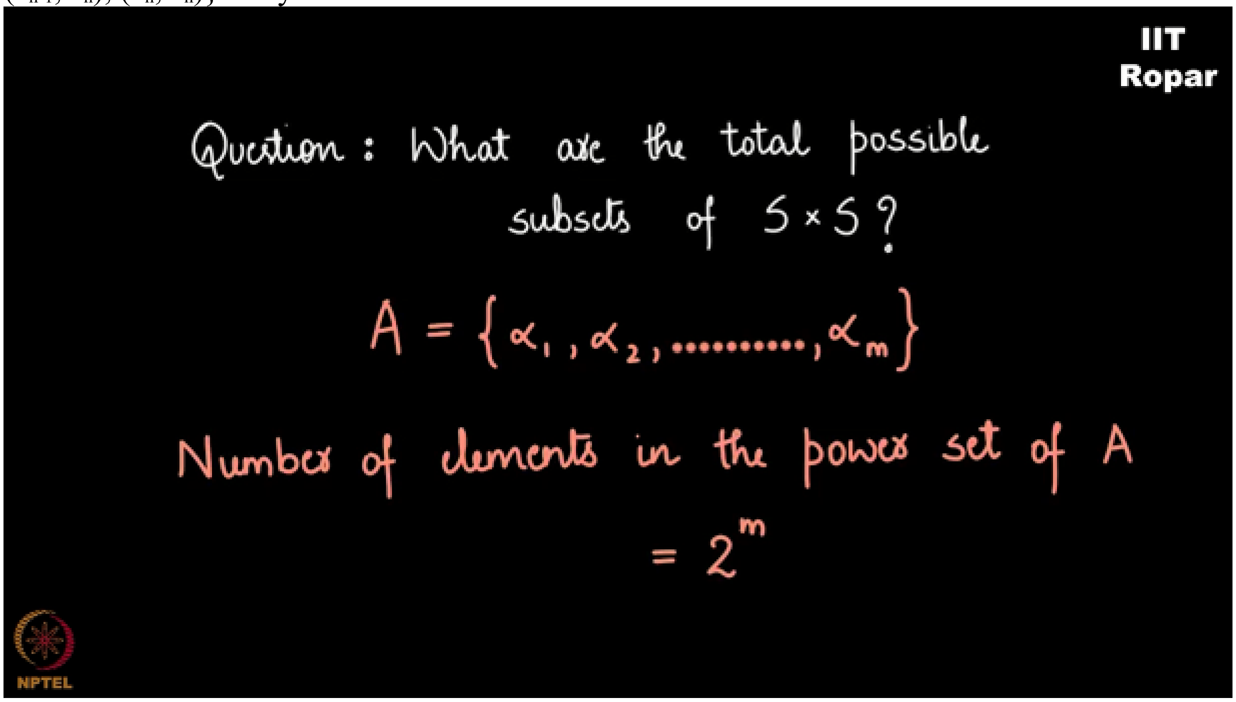
$$S = \{a_1, \ldots\ldots, a_n\}$$

$$S \times S = \{(a_1, a_1), (a_1, a_2), \ldots\ldots, (a_{n-1}, a_n), (a_n, a_n)\}$$

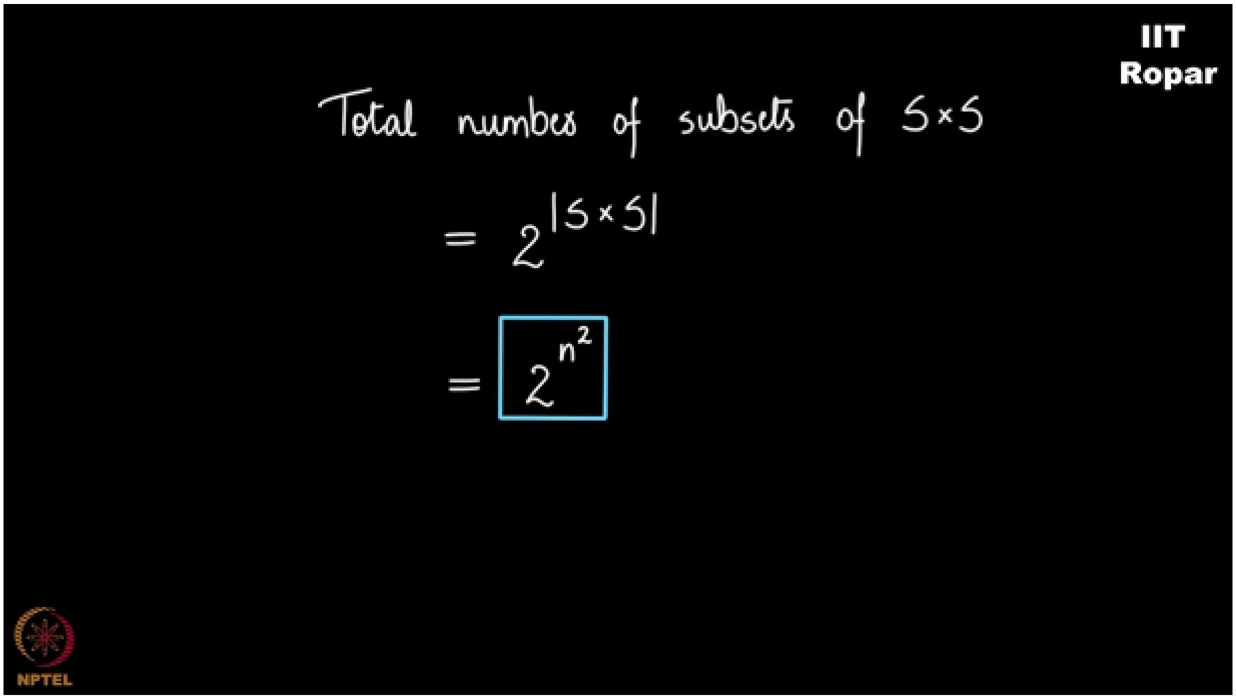
Total number of elements in $S \times S = n^2$

So, formally speaking, if we had the set S with not just five elements but let's say, in general, n elements a1 to a n, S x S will have the following elements. From $\{(a_1,\ a_1)\ (a_1, a_2),\ \ldots\ldots\ldots\ldots\ (a_{n-1}, a_n), (a_n, a_n)\}$. So you'll have total number of elements in S x S will be n^2 elements.



Our question is, what is the total possible subsets of this S x S. So before that, we all know that in general when you take a set A, comprising of some, let's say, n elements, alpha 1 alpha 2 up to -- let's not say n, let's say m elements. We know the total number of subsets of this. It's called the power set, if you remember. Total possible subsets is -- you basically write all possible sets here, all possible subsets of A here. The number of elements in this power set will be 2^m, correct.

The result is straightforward. You can map this to all possible binary numbers of length m. There are 2^m of them. Every binary number represents whether you pick an element from the set A or not, right. We have discussed this before. It's a straightforward observation.



Now the total number of subsets of S x S will simply be, 2 to the power of the number of elements in S x S, which is 2^{n2}. Why? Because every possible subset of S x S is a valid relation, and hence, there are 2^{n2} o valid relations, and that is the number of total number of relations on S.

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