

**Probability & Computing**  
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**Lecture - 08**  
**Tutorial 2**

(Refer Slide Time: 00:16)

The image shows a slide with handwritten notes on a yellow background. At the top right is the NPTEL logo. The notes are written in black ink and include the following text:

- Ex 1.5 Mitzenmacher 1<sup>st</sup> ed Pg 17
- 10 dice  $\Pr(\text{sum divisible by } 6)$
- A diagram showing a horizontal bar representing the sum of 10 dice, labeled 'S' below it. The right end of the bar is labeled 'X' with an upward arrow pointing to it.
- The equation  $\Pr(S+X \mid 6) = 1/6$
- An arrow points from the equation to the text  $S=37$ .
- Below that,  $S+X=42$  is written.
- An arrow points from  $S+X=42$  to the number '5'.

In the bottom right corner, there is a small video inset showing a man with glasses and a beard, wearing a light-colored shirt, looking towards the camera.

Let us look at exercise 1.5 in the Textbook [Mitzenmacher](#) and Upfal first edition it is on page 17 ok. What is the question? You have 10 dice; you roll these 10 dice and sum of all the numbers that come up on these 10 rolls. Now the question is what is the probability that the sum is divisible by 6? Ok. It might be tricky to think about it as is.

But let us try to apply the principle of deferred decisions ah. So, now what we are going to do is we are going to focus our efforts on the tenth time ok. So, we are going to focus our efforts on the tenth time and assume that these 9 dice have already been rolled ok. And let us say the these nine dice the there some happens to be  $S$  and the tenth dice which is what we are focusing on comes up with a value  $X$ . So, this question becomes what is the probability that  $S$  plus  $X$  is divisible by 6? Ok.

Well this is this makes it a lot easier to think about because whatever be the value of  $S$  there is exactly one value of  $X$  that when added to  $S$  will make  $S$  plus  $X$  divisible by 6 and that is not being difficult to see. Because let us take an example.

Let us say  $S$  is 30  $S$  is equal to 37 ah. So,  $X$  can take the values from 1 to 6. So, the only value of  $X$  for which  $S$  plus  $X$  is divisible by 6 is when  $X$  is 5 in which case  $X$   $S$  plus  $X$  will be 42 and that is the only value for which this divisibility happens.

And of course, what is it what is the probability that  $x$  would take the value 5 that is 1 over 6. So, this probability is simply 1 over 6. So, one thing we need to be careful about is when we formally write this up we need to apply the law of total probability and a lot of the probability has to be applied over all possible values of  $S$ .

Because  $S$  can take a range of values for every possible value of  $S$  probability of  $S$  plus  $X$  being divisible by 6 is  $\frac{1}{6}$ . So, we need to carefully apply the law of total probability to actually arrive at this result and that is what is expected of you in this exercise.