

Probability & Computing
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Lecture - 07
Tutorial 1

(Refer Slide Time: 00:12)

The slide content is as follows:

We are playing a tournament, where we stop as soon as one of us wins n games. We are evenly matched, so the probability each of us wins each game is $1/2$, independently of other games. What is the probability that the loser has won k games when the match is over?

Handwritten notes on the slide include:

- A binomial coefficient formula: $\binom{n+k-1}{k} \frac{1}{2^{n+k}}$
- A sequence of $n+k-1$ games, with k wins marked by red checkmarks and the final game marked by a green checkmark.
- The expression $n+k-1$ circled in red.

Let us now look at this very simple problem in this corner in this problem, we were playing a tournament. Where there is a winner and there is a loser who is the winner, we stop as soon as one of us the winner wins n games ah.

So, the two players are evenly matched and so, if you look at any one particular game in isolation, the probability that one of the players wins is exactly a half as independent of all other games. Now the question is that you know what is the probability that the loser has won k out of those, all the games that were played ok.

So, we are now interested in the case where the winner has one n games, and the loser has one k games ok. So, always the winner is going to be the one who wins the very last game. So, if you list down the sequence of games, the very last game would have been won by the winner ah.

Among the rest of the games, some games were won by the winner some games were won by the loser and out of all of these games exactly k of them were won by the loser

that is the event that we are interested in we are interested in the event where the loser wins in exactly some k games in the among the first $n + k - 1$ games ok.

Ah. So, if you if you fix one such configuration, clearly the probability that you get this configuration is going to be nothing, but $1/2$ raised to the power $n + k$. Why is that well because there are a total of $n + k$ games played out of which k where 1 by the by the losing player.

But we want to ensure that those k losses occurred within the first $n + k - 1$ games and we should account for the fact that any such subset of k games could have been won by the losing player.

So, which means this is this is for one particular configuration how many such configurations are there, well there is a total of $n + k - 1$ such games out of which k have to be won by the loser and so, there are $n + k - 1$ choose k such configurations possible and each one of them occur with this probability. So, the final answer is given over here.