Probability & Computing
Prof. Jhon Augustine
Dept. Of Computer Science and Engineering
Indian Institute of Technology, Madras

Module – 01
Introduction to Probability
Lecture - 06
Segment 2: How to Understand the World? Play with it!

We are now on a segment 6 of module 1 and the title of this segment is how to understand the world play with it and this actually makes a lot of sense if you look at a small child how does it understand the world around it, it plays little gaze it and then it builds an understanding of the world how the world works, and that is how the child develops.

And what we are going to see is somewhat similar to that in this ah, but in a much more simplified sense, let us say we want to try and understand and build a model a probabilistic model of some system. And ah, but we do not know what the right model is. So, what we do is we play with it, we repeat some experiment to try to understand what the model should be.

And over the course of time, we get a good sense of what the model should be and that is the goal.

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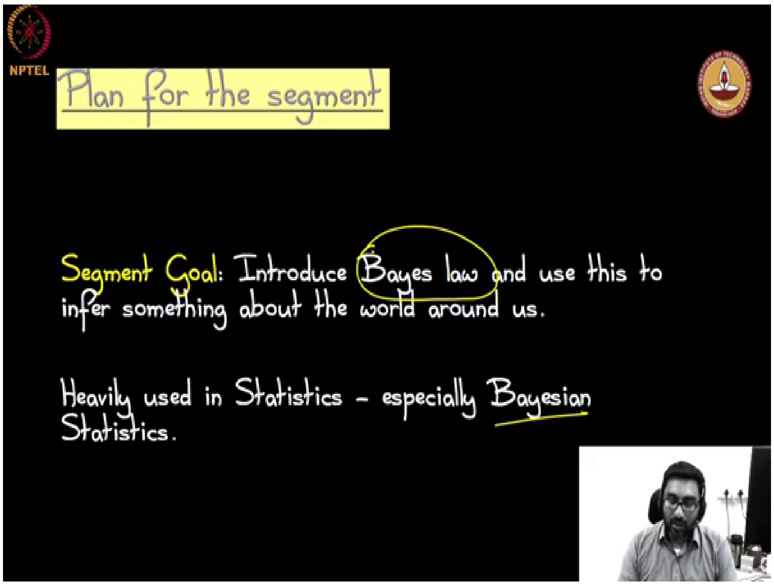

um And we want we want to look at this in this segment is Bayes law which provides the basis for this sort of playing with the world in order to get the best get to build the right model ok. And this is this finds a lot of application especially in Bayesian statistics.

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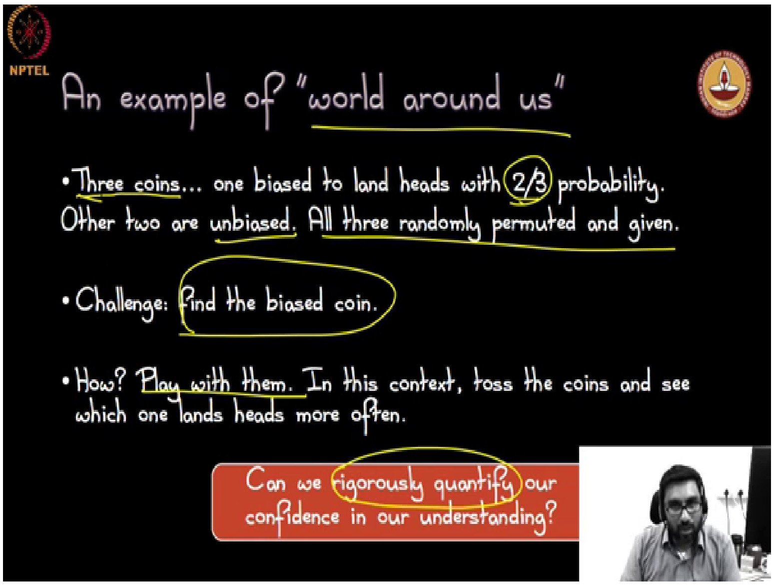

And so, that is what further ado let us look at a simple example where this sort of context plays out. So, here is what we mean by the world around us, we have three coins, one of them is biased it is bias. So, that it will land heads with probability two thirds, the other two are unbiased, but the problem is we do not know which one is biased in which one is unbiased.

So, we are just going to assume that they are all randomly permitted and So, this is the world that we are given and, but we would like to build an improved understanding of this world, build the right model we want to we will find the biased coin we would not know what the biased coin is. And so, what would be a natural thing to do well let us try to play with the world try to play with these coins toss them around, to see which one is likely to be the biased coin. So, this is what we mean by playing with our context ah.

So, let us toy that you know. So, we can do that, but and we can get a sense of what which one is biased, but we want to have a rigorous quantification of our understanding, and that is where our attempt at modelling the situation comes in ok.

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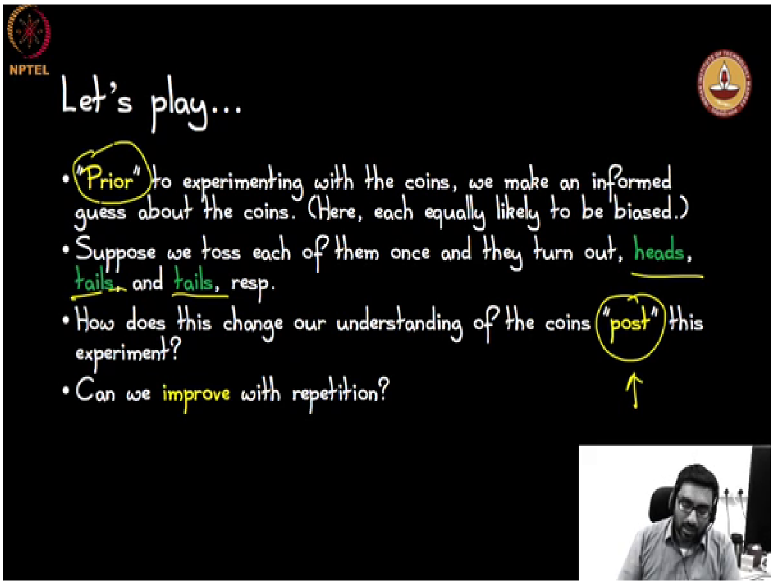

So, what we do is before we start experimenting with these coins, we start with some basic understanding a reasonable understanding of which coin is biased and which one is not biased and. So, this is before we start any experimentation. So, its often called the prior ok. And let us say we toss each of these coins once and ask ourselves you know what is the outcome well say the outcome is, heads tails and tails the three coins.

Now, having seen this outcome we would like to make some inference about which coin is biased in which coin is unbiased. It will not be a deterministic inference, but it will be a probabilistic inference and this because this is an inference that we make after the experiment has been conducted, its called the posterior understanding of the posterior model.

So, before the experiment we had a prior model and after the experiment we get a posterior model, and then we this, but this is only based on one experiment. So, maybe we could try to improve upon this posterior understanding and how do we do that? We just simply repeat the experiment a few times.

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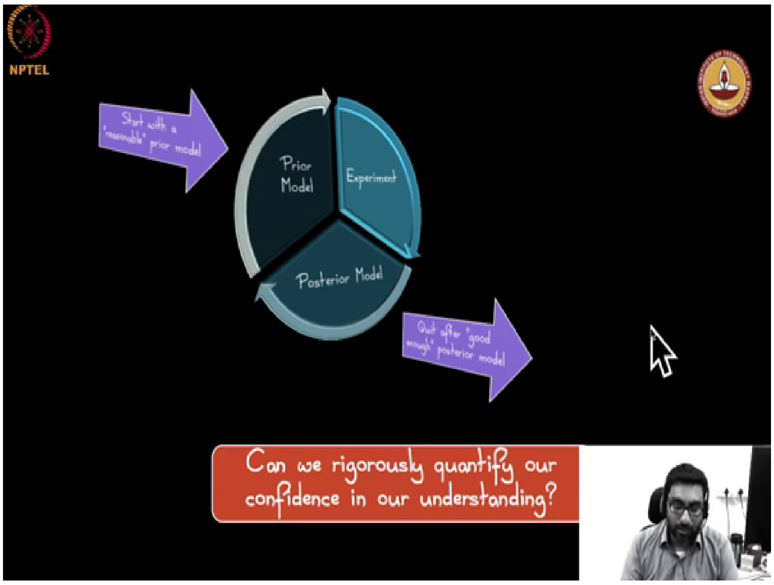

So, here is a pictorial representation of how we can do that we start with a reasonable prior model, we conduct an experiment and based on the outcome of the experiment, we develop a posterior model. But now this posterior model is an improvement over the initial prior model.

So, this posterior we therefore, convert that into our prior. So, and conduct the experiment again and when we conduct the experiment again, we get an even more improved posterior model and with that as our prior we repeat the experiment and so on and so forth. And at some point we are going to reach a stable situation where there is no not much more improvement and so, that kind of indicates that we have reached a good posterior model and we break out of this loop.

So, this is this is very typical Bayesian inference in this context. So,. So, the question is can we rigorously quantify our confidence in our understanding. So, we have run done these experiment a few times, we what is how good is our understanding these are some of the questions that we would like to answer.

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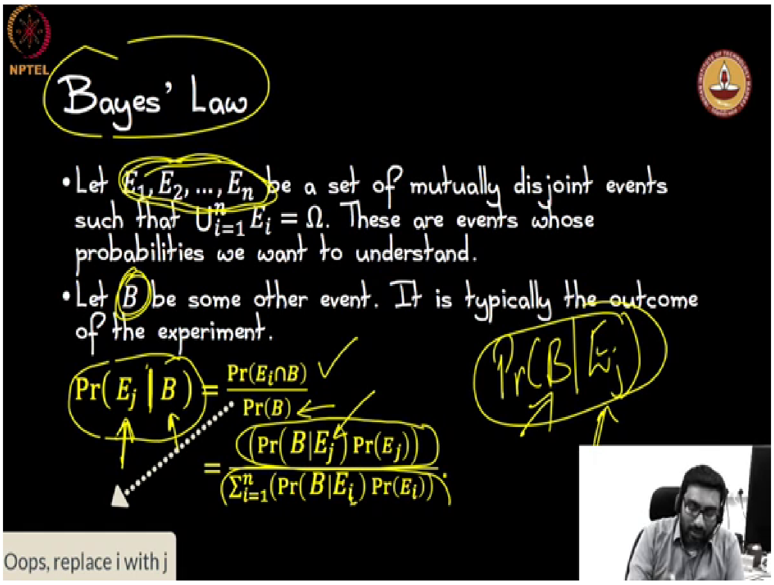

And the key underlying principle that will help us approach this these this question is Bayes law.

And So, to understand Bayes law let us consider some events E 1 through E n the probabilities of these events is what we are interested in. This is this is the model that we want to build and these are we are going to assume that these are disjoint events, and b is some other event this is this is usually the outcome of some experiment and based on this outcome, we want to update the probabilities of these events. So, we basically want to get these probabilities of these events E js and this given that after the experiment we got this B as our outcome.

So, this is this is what we want and this is a conditional probability. So, we can apply the formula that is very straightforward. And Bayes law is very simple you just look at the denominator that is just the probability of the event B. And now you expand that out using the law of total probability and similarly the numerator you apply the formula for conditional probability.

But important thing is in the right hand side you are going to condition on the E E j values on the probabilities on the on the E j events So, what you notice here is on the left hand side we want the probability of the E events Ej conditioned on the outcome of the experiment, but on the right hand side what we are going to do is switch the conditionalities.

So, conditioning on the E j s we want to plug in the probabilities of get the probabilities of the of the event B. And why is this important and useful well because the E j is on the right hand side corresponds to our prior model.

So, based on our prior assumption, we can actually compute things of this nature. Given that we have a prior understanding of E j, we will be able to compute the probability of B. So, this is something we will typically be able to handle and that is the type of probabilities we have on the right hand side.

And because on the now we are in the right hand side, we have probabilities condition the conditional probabilities of this type we can actually compute them and when we can compute them what Bayes laws law gives us is a way to compute the left hand side where the conditionality is reversed and it in the focus now is on getting the probabilities of the E js which is the posterior model ok.

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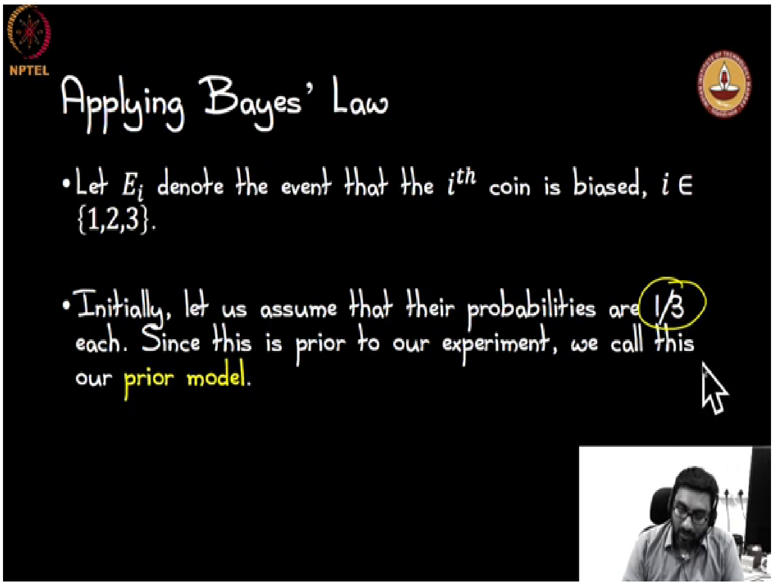

Let us actually work out an example and things would become a lot more clear ok. So, let us go back to the coin tossing example we have three coins we do not know which one is biased one of them is and so, let us let e E i denote the event that the ith coin is biased ok. So, E 1 means the first coin is biased E 2 means E a second coin is biased and E 3 means the third coin is biased and of course, these are all disjoint events.

And we want to understand and build the model of which coin is the biased coin ok. Initially we do not know anything about these coins. So, the reasonable prior to begin with is that with probability one third, each one of them is the biased coin that is so, that is pretty much all we can do because we do not know any more information about the coins ok.

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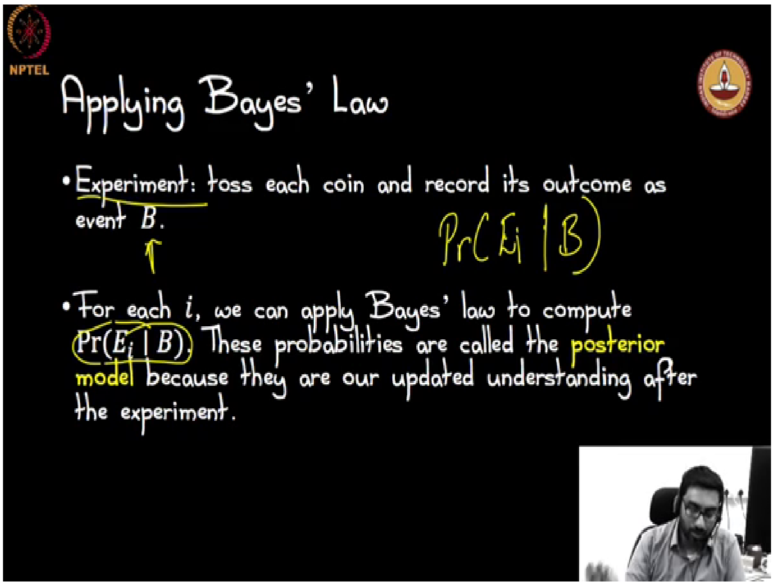

So, this is our prior model now let us try to apply Bayes law for that we will need to run some experiment and using the and we need to be able to use the outcome of that experiment, that event B in order to make some improvements to our model.

So, basically compute the posterior model. So, now let us say we toss each coin and B is the outcome of each of those three coins. Now for now based on B we would like to understand what is the probability of E i given that B came out as the outcome um.

So, this is what we want to compute these are the posterior probabilities.

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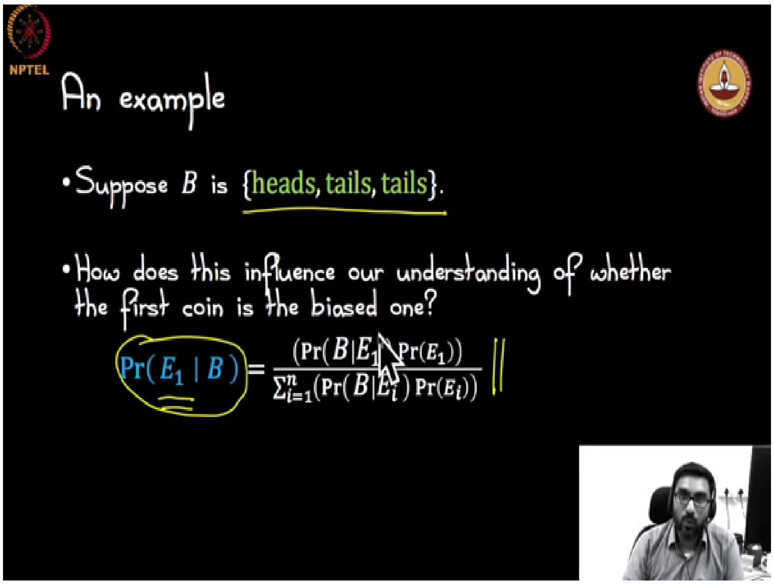

And let us look at an example to see how this can be worked out. So, let us assume that this is our outcome B. The first coin came out heads the second and the third coins both came out tails ok. Given that this is our outcome how does this influence our understanding of whether the first point is the biased one. So, that is the event E 1, E 1remember refers to the event that the first coin is biased we want to understand this probability ok.

Now, all we have to do is apply the Bayes law formula and notice that on the right hand side we have the conditionality you switched we are now conditioning on E 1 ok.

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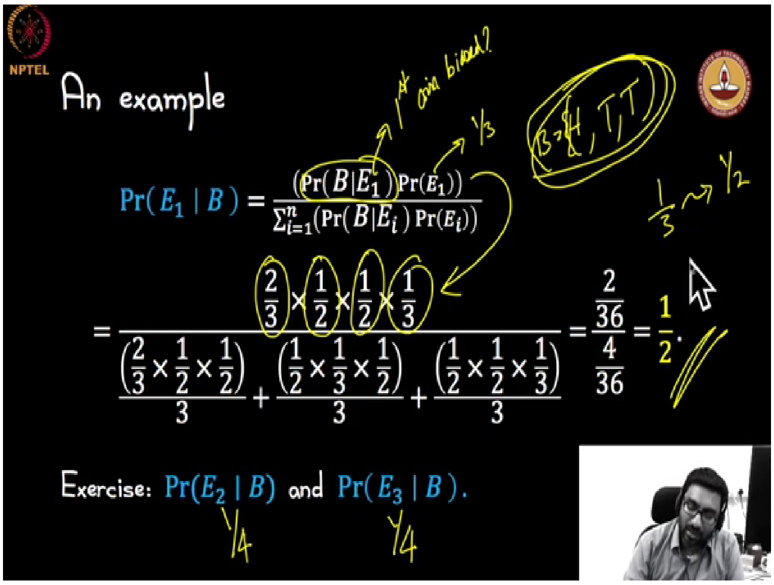

That is nice because for each of these we already So, we know this one this is a third that is a prior model.

Ah we also know given that each coin is equally likely to be biased what is the probability that you will you will see the outcome B how what is that well this is this one third refers to this probability of E 1. Remember B is heads tails ok. So, what is the outcome of what is the property of getting a heads given that E 1 the given E 1 which is which means that the first coin is biased.

So, given first coin is biased what is the probability that that point will come out heads well we know that its two thirds ok. And given that the first coin is biased what is the probability that the second coin will come out tails well if the first coin is biased then the second and the third coins are unbiased. So, they will come out tails with probability half each ok.

So, and which means that the outcome B is the product of two third times one half times one half because these three coins are tossed independently which. So, now, you see that the numerator values we are able to get all the numerator values and in similar fashion you can fill out all the denominator values as well.

And if you work it out it comes out to a half. So, if we see the outcome heads tails, then our posterior understanding of the first coin changes you from one third which was a prior probability that the first coin is biased, noting that that was the only coin that showed up heads we have been able to update our understanding to say that look looks, its more likely that the first coin is the biased one we cannot save deterministically, we are still not far from being sure about it, but certainly our belief that the first coin is the biased coin has been bumped up and its gone to about a half.

And if you work out the other two it will you will work will be able to work it out that these are both a quarter these are the probabilities that e two and e three are the biased once given this particular outcome. And this makes sense if you both of them showed up as tails they are not likely to be the biased coin.

So, this one experiment changed our understanding a little bit and the this still is not you know we still do not have complete understanding, will have to perform this experiment again. So, this one half one forth one forth will become our new prior model, and we will have to repeat to further refine our understanding of the words and that is what is shown over here we started off with one third, one third, one third, we ran an experiment.

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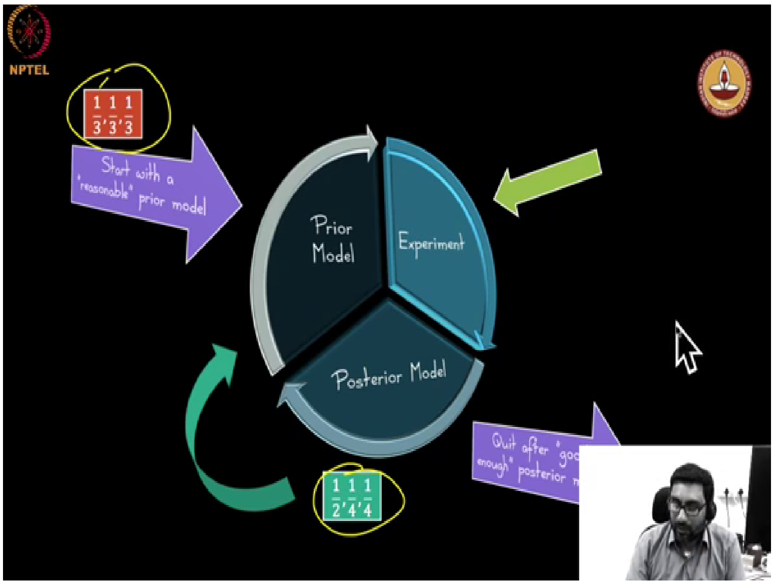

We updated our probabilities and use this updated probabilities to run the experiment again, and then further we will keep refining getting a will get a better posterior model, which is the individual probabilities of E 1, E 2 and E 3 and that will become our prior model run the experiment again, we will go through this loop a few times at some point it will stabilize and we will know that this is the right answer at that point in time where done.

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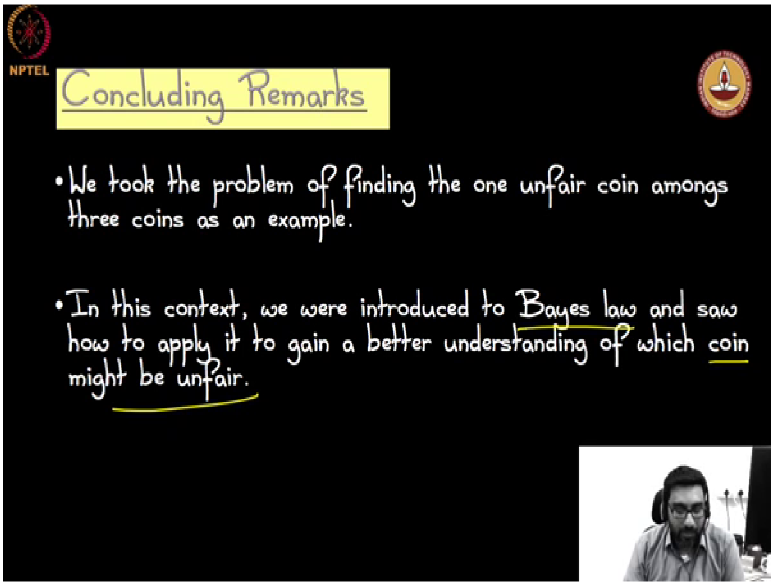

So, this is the general framework of applying Bayes law to understand the world around us and in this segment what we have seen is we have shown how to apply Bayes law you know in a fairly simple setting wherein we try to figure out which of the coin might be the unfair coin ok.

So, with that we can conclude our understanding of Bayes law of course, this is just a mere introduction, there is a lot more to this.

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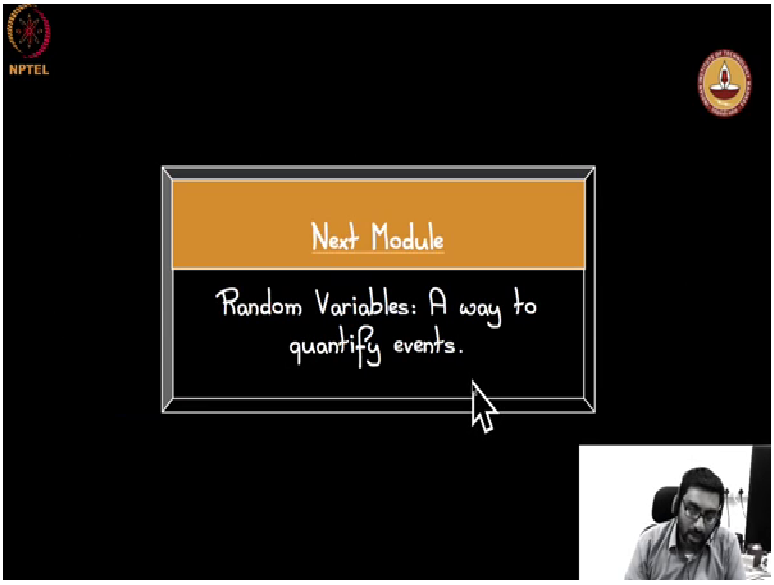

Ah And I urge you to Google it up and explore some more about it ok. And this with this we come to the end of the first module, in the in so, far we have only worked with probabilities and events and an important notion forum especially in the algorithm design context is random variables, which we will be studying in more detail in detail in the upcoming second module.

Thank you.