Probability & Computing
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Module - 01
Introduction to Probability
Lecture - 01
Segment 1: A box of chocolates

So, we are now going to start with Module 1. I am going to start with the first segment and this segment I am going to title it a box of chocolates I will see what the box of chocolates is about shortly.

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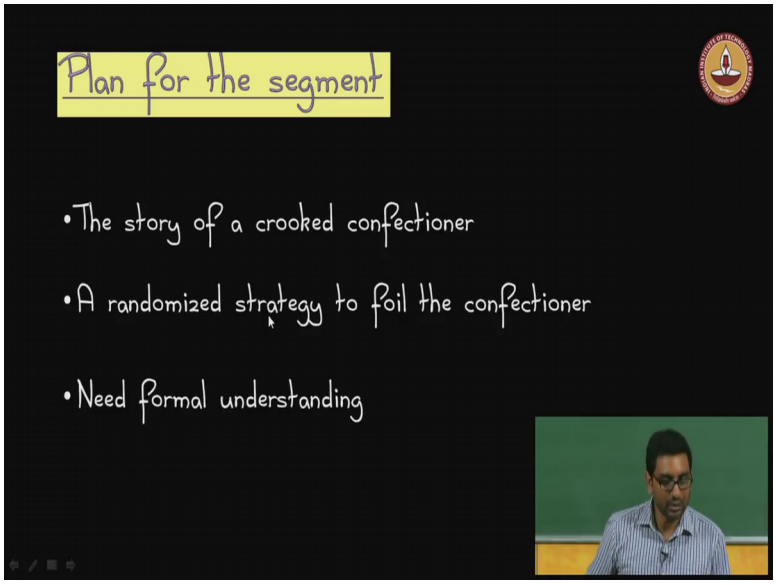

So, let us see it is a plan for the segment. We are going to talk about a crooked confectioner his goal is to sell as many chocolates as he can.

And your goal is to basically just get the chocolate that you want not buy all the chocolates, if those confectioners trying to sell you. So, the problem is you what we see is that it is hard to fight against this crooked confectioner, except if you use a random strategy ok.

And that is what we are going to see it is going to be very simple setting, but I want this to be a way to illustrate the power of randomization, and also point out the need for a formal understanding of randomization probability theory in the context of computer

science ok. That is the very goal of this course and today is this segment is just to illustrate the importance of this area.

So, without further a do let us let us see what is in the box? Where we have 100 chocolates? 50 with nuts and 50 without nuts that is the situation ok.

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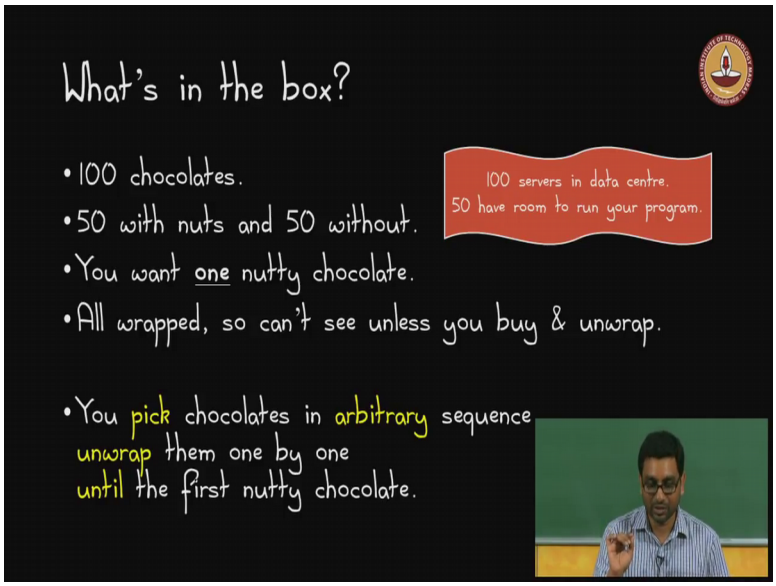

The thing is you only want 1 chocolate, 1 chocolate is all you want your health conscious, but you want the chocolate with nuts in it ok. You do not want chocolates without nuts in it ok, but the confectioner what he is done is knowing; the situation he is wrapped up all the chocolates. So, he you cannot see what is inside each chocolate?

So, what you are supposed to do is come up with a strategy to get the first nutty chocolate and that is it ok. And so, this is so, the confectioners goal on the other hand is to try and sell you as many chocolates as possible, because the moment you unwrap a chocolate you have bought it. So, all right. So, this is a situation is just in case you are getting a little worried whether this whole courses is going to be about chocolates.

Let us actually see a little bit more of a computer science he application for the exact same scenario ok. So, you have let us say a data centre, you have 100 servers in this data center and now a new job arrives it could be a web request, it could be whatever request for running a program and there are some 50 servers that have capacities within them to run your program, the problem is you do not know which ones ok.

And you want to in each attempt to put your program in a particular server is going to cost you some money. So, if you are going to repeatedly keep trying those servers that do not have enough capacity you are spending a lot of effort ok. So, it is in your interest to find a server that has the capacity to run your program ok. So, this if you can think about it is exactly the box of chocolates problem, this rephrased in a language that is more amenable to computer scientists.

So, what do you do and this is what you typically do if you come from an undergraduate algorithms course typical algorithms course I mean some algorithms course are you know covering some new topics. So, that is great, but a typical undergraduate algorithms course would have taught you deterministic algorithms and here is what you would do you would pick chocolates in some arbitrary sequence ok. Unwrap them one by one check if it is nutty or not and then keep repeating it until you find the first nutty chocolate.

What is the problem with this, well the worst case is very bad. You can check 50 different chocolates by an only sexy, you are only guaranteed to succeed on the 50 first pick right. This is your entire algorithms course right; you learn how to come up with algorithms? That in the worst case do you know gives good worst case guarantees and this is the best worst case guarantee you can have ok.

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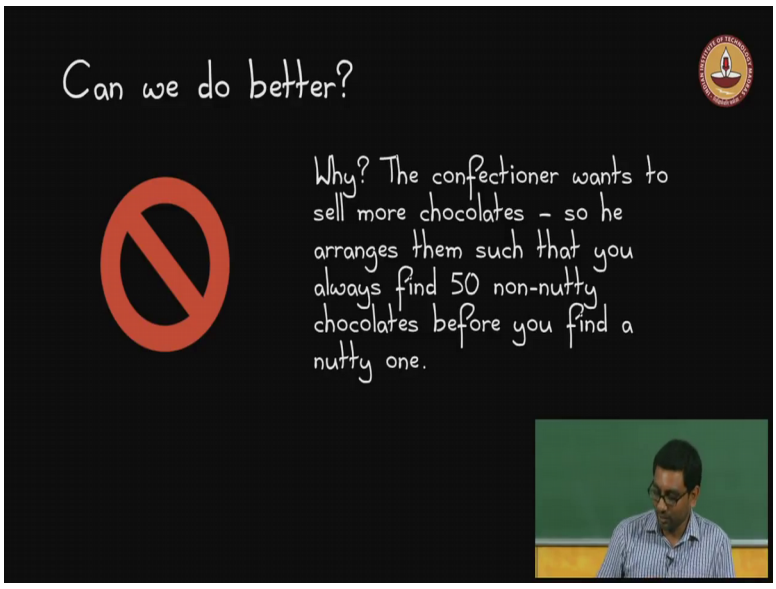

Can we do any better at all if we insist on deterministic algorithms of course; there is no other way because this is a very simple argument right.

So, now all you have to do as a member now this is a 2 player game you can think of it between you and the confectioner. All the confectioner has to do is understand the sequence at which you pick the chocolates? And therefore, just place all the non nutty chocolates first. So, in that in whatever sequence you pick and then you are then forced to pick 15 non-nutty chocolates.

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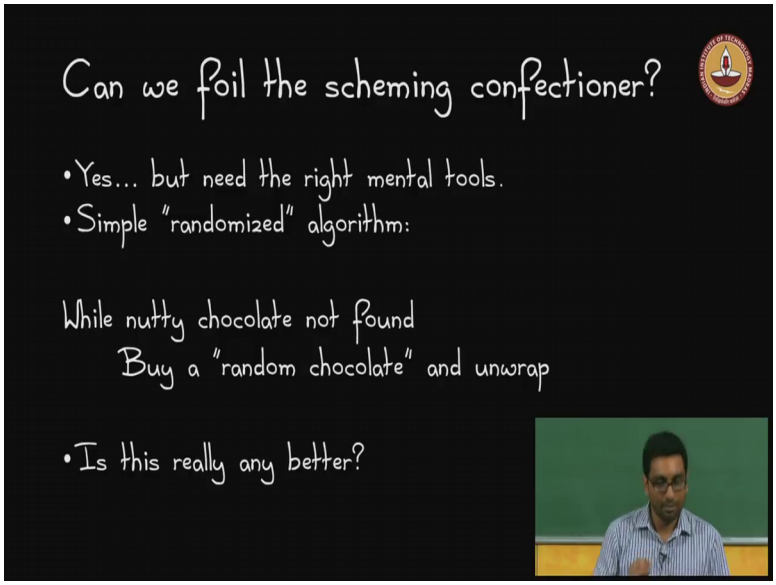

Before you get the first nutty chocolate this is where the problem lies.

So, obvious questions can be foiled the scheming confectioner. How do we do that? Well this is where we need to approach this problem in a slightly different way ok. So, whatever you learned in the undergraduate algorithms course, we need to not necessarily unlearn then, but add some tools to it and that is in particular we want to try something beyond deterministic algorithms, we want to do randomized algorithm ok.

So, here is the algorithm randomized algorithm while nutty chocolate not found by a random chocolate and unwrap.

So, this is the simple algorithm is it any better that is the question. By now any reasonable person would have a vague sense that it is better, but let us let us see.

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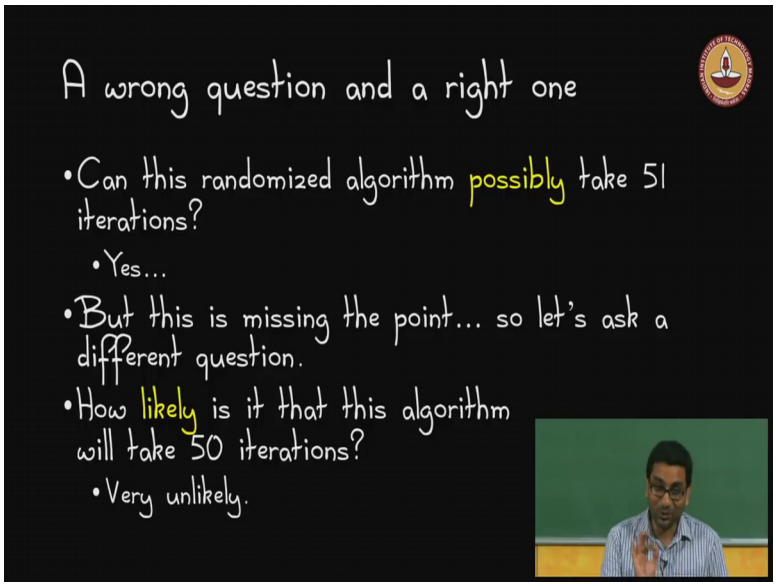

So, let us try to understand this issue first we will ask a wrong question, because sometimes to appreciate something you first have to ask the wrong question and then slowly work your way to asking the right question. Can this randomized algorithm possibly take 51 iterations? If you are talking about possibility yes it can actually take 51 iterations, but this is missing the point ok.

So, let us try to rephrase the question good, how likely is it that this algorithm will take 50, 51 iterations? There was some mathematical jargon thrown at me just now, but given this the first lecture first segment of the first module. Let us just go with something we know ok, we know it is very unlikely that is all we know for now, we do not know how unlikely right that is something we need to figure out right.

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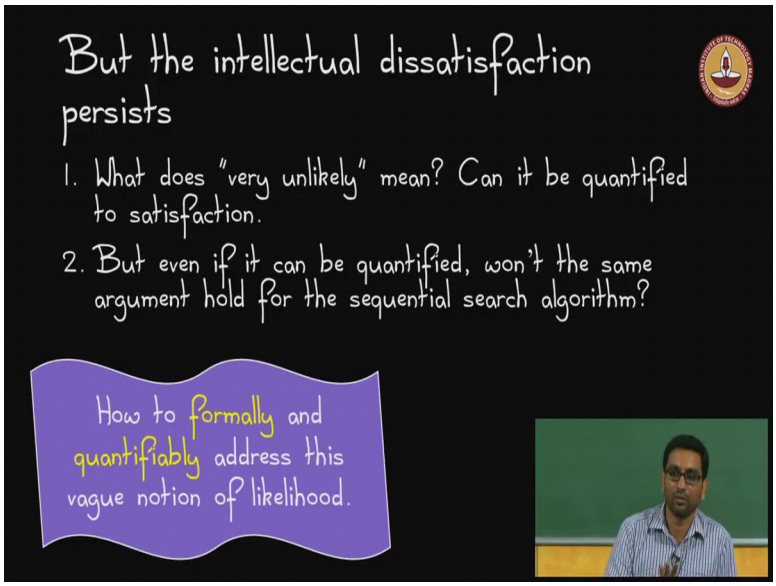

So, this is that is really. So, we have kind of reached the situation where we think we have a right solution given the level of information, we have heard. So, far we have we come up with a an algorithm, that we know, will fail we will do the bad thing basically run for too long with very low likelihood that much alone we know ok, but we need to this intellectual dissatisfaction right, because we do not know what is likelihood. We do not know you know when you say "very unlikely" we do not know what; that means, right.

So, some important thing is can it be quantified can we put a number to this and can we if we if we have 2 things, 2 possibilities that are you know one is somewhat unlikely and somewhat vary, and the other one is very unlikely we want to be able to know how different they are and we want to be able to compare quantify them.

And then the other thing is we also want to understand, whether these you know what is so, special about randomization would it you know the sequential algorithms also have the same benefits, why do they not have those benefits, that is another philosophical question that might rank will you. For now what we what we have understood. So, far is that you know there are ways to overcome these sort of pesky situations.

But we need the right tools in particular what we are of course, going towards is the probabilistic analysis tools to formally and quantifiably address this vague notion of

likelihood. Everybody has this notion of likelihood the gamblers in 19; I mean in 16th century had the notion of likelihood right.

But it took about 400 years to formalize this and a lot of probability courses will just jump to the the formalisms, but I want you to take a small you know take some time to realize that it took about 400 years you know the first attempt that probability theory was in the 16th century at least that we know of and you know. If enough material from Indian history was present we might be even able to find out that India invented probability theory, but we do not have that evidence.

So, a lot of things were invented here, which we do not know about right so, but then we know if we have clear evidence that it was invented in a there were attempts in 16th century, but what we are going to talk about in the next segment is how Koolmoe grove formalized this notion and helped us appreciate this vagueness. He gave a very concrete way to approach this vague notion of likelihood ok.

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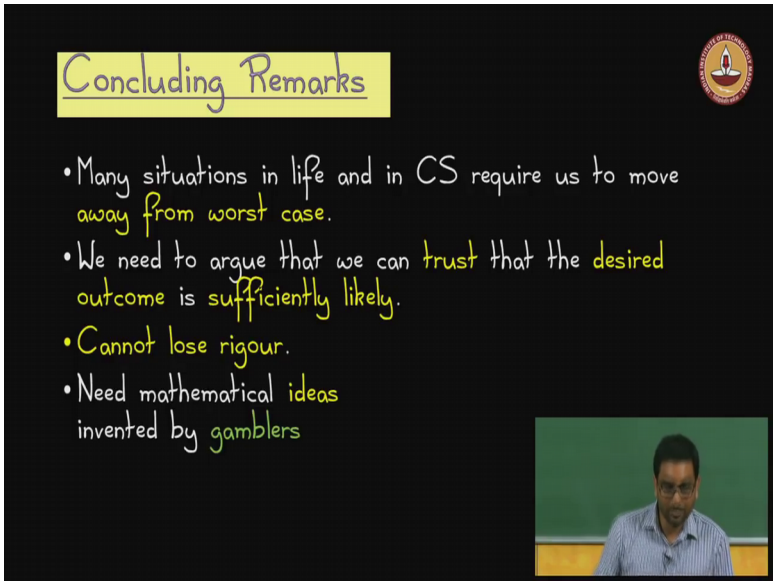

So, that is going to be the topic for our next lecture I have mentioned all the pretty much all the concluding remarks already. So, I will quickly mention them we need to move away from worst case analysis, we need to add some more in realistic ways to approach algorithm design and we need to be able to come up with formal ways to understand and appreciate it and so, that someone can trust our claims.

So, it has to be formally in proved trust that the desired outcomes is sufficiently likely and we have to do this in a very rigorous manner and we need the right mathematical ideas that is going to be the topic of this course. And of course, as I mentioned the fun fact is that what we are going to study is something invented by gamblers.

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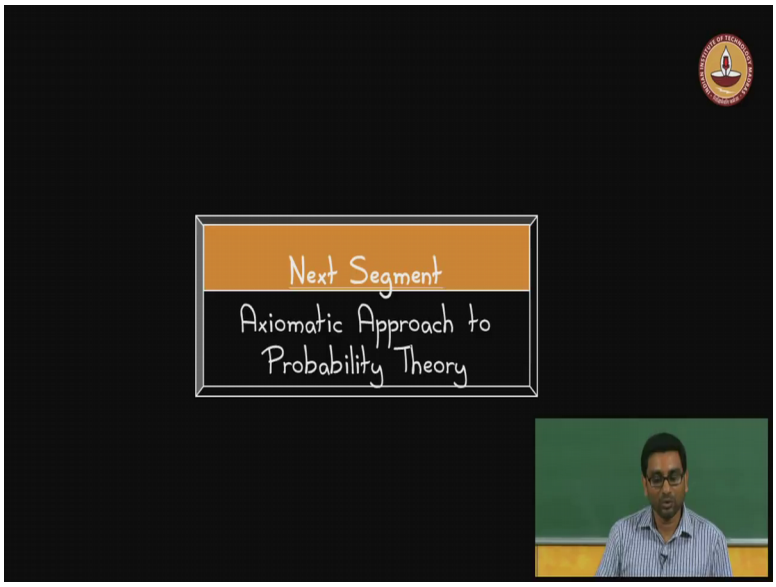

So, so the next segment will be an axiomatic approach to probability theory this is the formal way to approach probability theory that will be covered in the next segment. So,

Thank you.