Lecture - 78
Random Coin Dropping

So, just now we saw a strategy of every node being given 100 gold coins and this nodes distribute all the gold coins to their immediate neighbours; equally. And as this process continuous and continuous the number of gold coins they get distributed not evenly although, we started evenly; they get distributed haphazardly.

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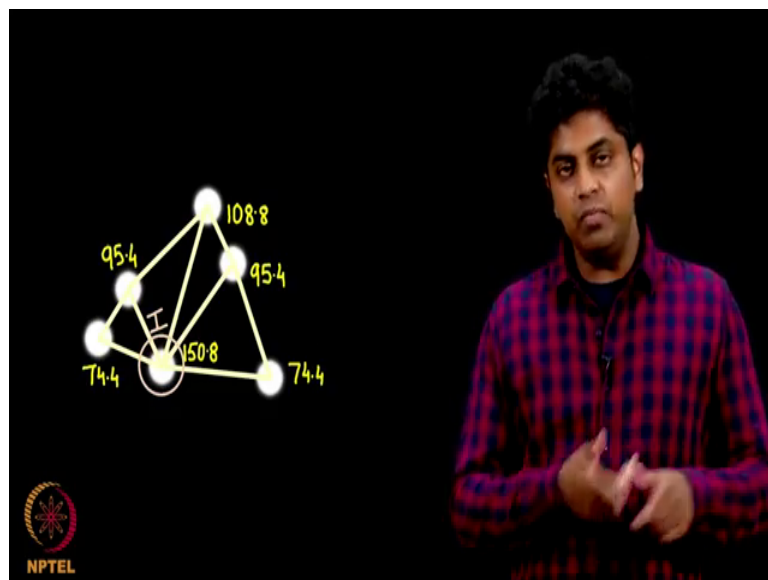
And I go and see that node which has accumulated the highest number of gold coins, call him the winner first king; first top one person; rank 1 person. Second most gold coin accumulator, I call him the second best, second king I rank him accordingly. That was one way.

I will now give you yet another way of ranking the nodes. Now, I will not use gold coins; I will use something else of course, I will use gold coins, but I will give the gold coins in a very different way.

What I will do is, I will stand on a node, some node and from 1 node assume I have 4 neighbours, I jump to a neighbour uniformly at random and give that neighbour a gold coin. From that node I will jump to yet another node, whatever it is pointing to maybe, it is pointing to 5 nodes. I will pick 1 node uniformly at random jump there, give him a gold coin. Remember the car and roads experiment I keep visiting the places I said and I explore entire city, I am doing something similar here. From 1 node is just take a random hop and go to another node. Whenever I visit a node I drop a gold coin on that node, I keep doing this, keep doing this, keep doing this, and I see, how many gold coins are accumulated by these nodes?

After some 1 million such random walks; I am walking randomly right. So, I call it a random walk. Random walk, gold coin drop; random walk, gold coin drop; random walk and I drop a gold coin. So, on and so, on and so, on and million times and I observe after a million iterations, every node has accumulated gold coins.

Although, they are not equal different nodes have accumulated different number of gold coins right. We can see that the figure says, if I take 1 million random walks which node gets how many gold coins? At the end I again, look at which node got the highest gold coin? I observe that the node which got the highest number of gold coins here was the same node which got the highest gold coins in my previous strategy previous experiment.

In fact, the second best here will also be a second best there. In fact, third best, fourth best, fifth best, all of them here we will agree with everything else there. It is very surprising that this is true for you, but mathematically if one sees it is not at all a surprise. In fact, as part of the advanced material for this chapter I am going to cover the mathematical details of why and how this is happening as well.

You are free to skip that its sort of it is not required that you go through it, but if you are curious to know what is happening, here where these 2 things the same, you may want to take a look at the mathematical material that I am going to record in the next few days, I mean in the next few hours of the video well.

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All that you need to buy from this explanation is that the strategy of 100 gold coins being distributed equally versus taking a random walk and dropping a gold coin, both this strategies rank the nodes in a same way; in exactly the same way. So, the moral of the story is sharing gold coins game and dropping gold coins game yield the same ranking of the nodes. So what? I am going to analyse this first game and the second game properly, for reasons of better explanation I am going to call the first game. The equal sharing of gold coins game, the second game the random walk drop the coin game. So, simply speaking equal sharing and random walk; game 1, game 2.

Let us slowly analyse and understand what these 2 games mean. What has happened so far? 2 games; this game is same as this game in terms of ranking the nodes. I will first

look at the first game which is 100 coins and equal sharing. Let us just think what happened here? What kind of nodes will attract the highest number of coins? A node has highest number of coins, if it is getting a lot of coins from its neighbours.
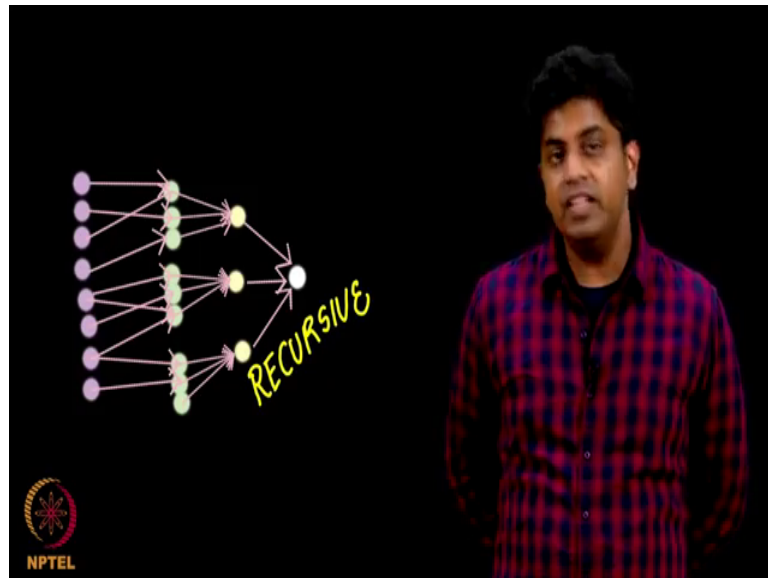
When will the neighbours give this a lot of coins? If the neighbours are getting a lot of coins. When will the neighbours get a lot of coins? If they are adjacent to whatever if there pointed by neighbours who get a lot of coins. You see this is a very recursive argument here. This captures the very notion of the following definition: you are famous, if famous people say you are famous and those people are famous if famous people say that they are famous so on and so forth. This goes on like this correct.

We discuss this in the introductory lecture in the first chapter right. You are famous if someone famous says you are famous. It is a very recursive definition. So, you see there is a definition for there is a abbreviation called GNU. If you are to computer science you will know what is GNU. GNU stands for GNU is not UNIX and now what does that GNU stand for? Again GNU is not UNIX; this is called a recursive definition.

For fun they have called it this way, but you see the all the all the time trying to say is the definition is sort of recursive here. You are famous if someone famous says you are famous and how do you know they are famous? What do you, what do you mean by famous then? Famous they are famous if someone famous says they are famous. And this famous is defined very recursively. Now you see what is happening in the first game. A

node accumulates a lot of gold coins if a lot of people, the people who are adjacent to it accumulate a lot of gold coins right. Only then you will the pass on the gold coins to me.

Please note, for me to accumulate a lot of gold coins people who point to me should accumulate a lot of gold coins and they should not have a lot of people adjacent to them. Now, what do I mean by this?

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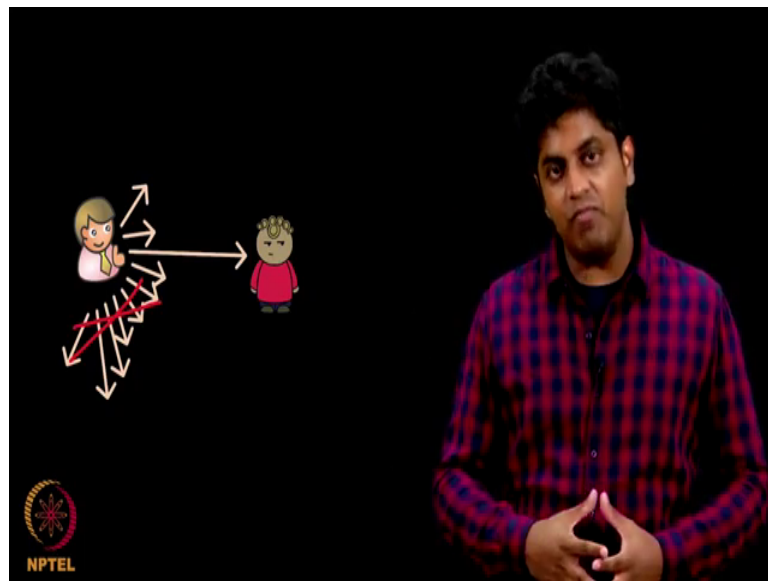
I have a rich cousin brother who helps me out whenever I want money let us say and I have 1 such cousin brother, but assume he has 10 such people to whom he is ready to help. Then I may not get all his help because his help sort of gets divided into 10. So, there are 2 aspects here; you should be having some good backups like a rich cousin brother to help you out in times of distress and he must not be ready to help a lot of people.

So, he must have very few friends outgoing and he must be rich. When will he be rich? When a lot of people point to him, so, a rich person, a rich node is someone who is being pointed at by a lot of people, lot of rich people let us say. But that person should point to a few people and one of those few people should be you I am just set up dissecting this problem and then observing it. But what I am trying to say is the moral of the story so far.
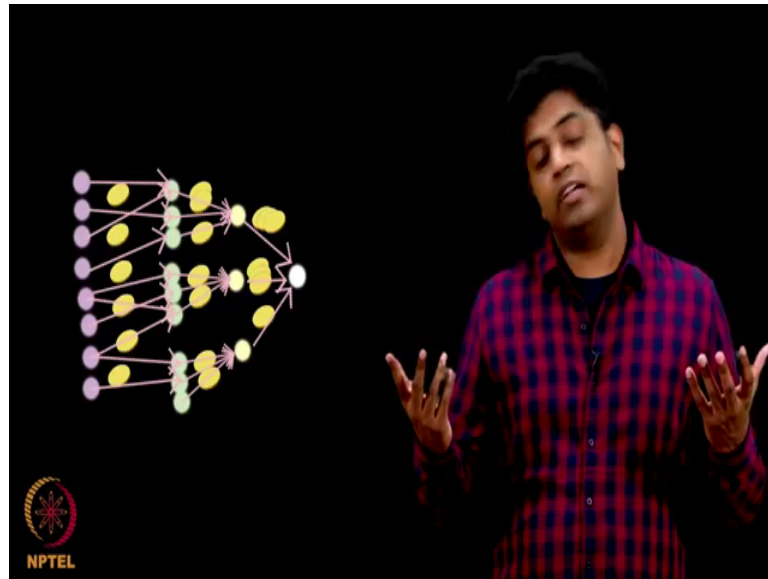
If you have understood that a node accumulates a lot of gold coins, if it is being pointed by nodes that accumulate a lot of gold coins. This much is enough and then you call a node to be highly ranked, if it is being pointed by highly ranked nodes.