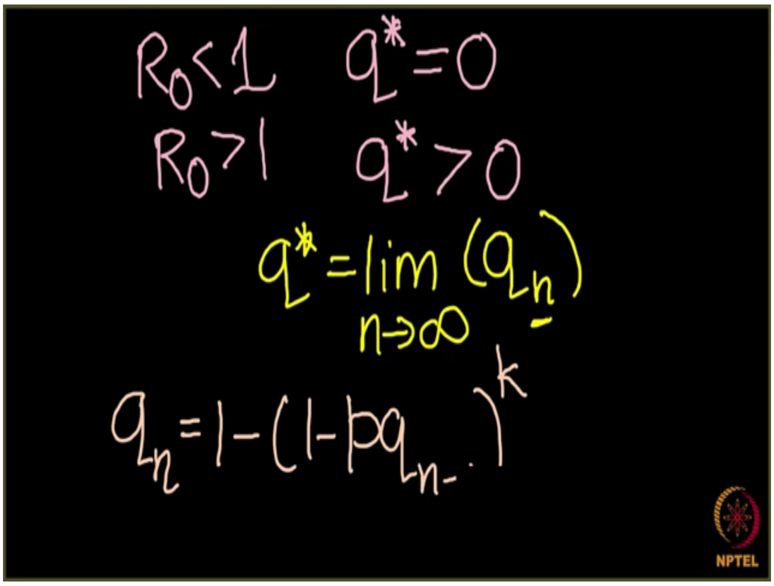
Social Networks Rich
Prof. S. R. S. Iyengar
Department of Computer Science
Indian Institute of Technology, Ropar

Rich Get Richer Phenomenon – 2
Lecture - 140
Analyzing basic reproductive number – 3

So, now in this lecture we are going to start from the same place where we left of in the last lecture. So, what did we see in the last lecture, first of all our problem statement.

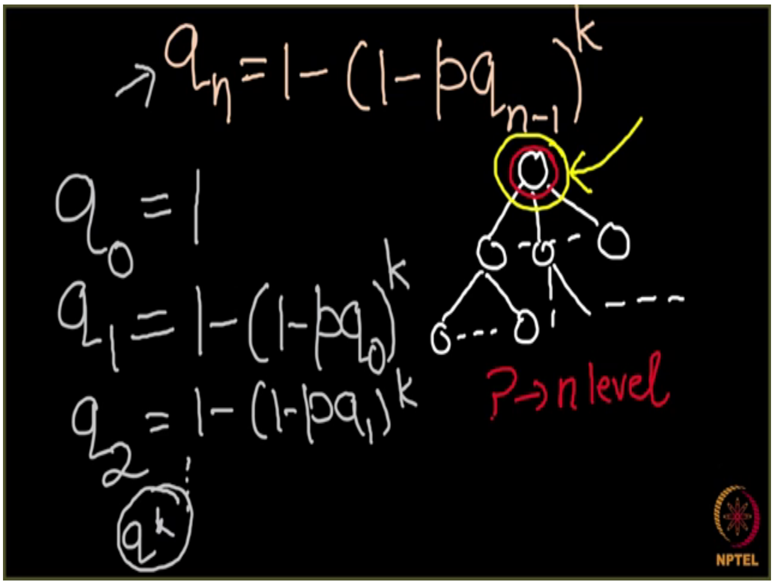
(Refer Slide Time: 00:16)



What is the problem statement? First of all if R 0 is less than 1 then we have to prove that the value of q star equals to 0 and if R 0 is greater than 1 then the value of q star is something which is greater than 0. This thing we have to prove and then we and we know what is q star right.

So, what is q star? q star is nothing but limit n tends to infinity q n where q n is the probability that your infection persist till the nth level; that is at least 1 person at the nth level is infected. And in the last lecture we have derived a formula for q n and the formula was 1 minus 1 minus p q n minus 1 to the power of k. What we are going to do in this lecture is now we are going to analyze this formula further.

(Refer Slide Time: 01:16)



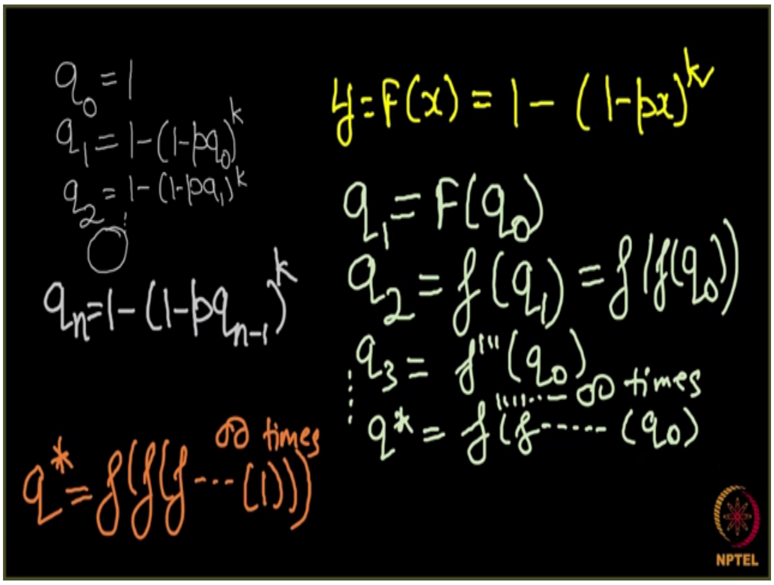
So, let me write this formula here and we will analyzing it q n is 1 minus 1 minus p q n minus 1 to the power of k ok. Now what is q 0? Let us look at q 0, q 1, q 2, q 3 and so on. What you think is q 0? You will not get it from this formula. So, q 0 is what, the probability that your infection persists till the 0th level and what is a probability.

So, what was a problem statement? This was a guy who was here having k neighbors and this person was again having k neighbors. This person was again having k neighbors and so on. And your infection started from here and the question was; what is the probability that infection reaches here to the nth level was our question.

What is level 0 here? This is level 0 and what is the probability that at least 1 person at this level is infected? It is obviously, 1 right for sure 1 this guy has to be infected here. If this guy was not infected here there was no problem we would be solving right. So, entire problem is because this person at the 0th level is infected it is infected with the probability 1. So, q 0 equals to 1. What is q 1? Finding q 1 is easy. Put the value in this formula what is q 1, 1 minus 1 minus p q 0 to the power of k, right.

What is q 2? q 2 is 1 minus 1 minus p q 1 to the power of k and what is our aim? Our aim is to find q star which will come after we will keep repeating this formula. So, we have q 0, from q 0 we can find q 1, from q 1 we can find q 2 and then we have to do this process infinite number of times and finally, we will find the value of q star. We actually we can do it infinite number of times. So, let us see how do we find out this value.

(Refer Slide Time: 03:22)

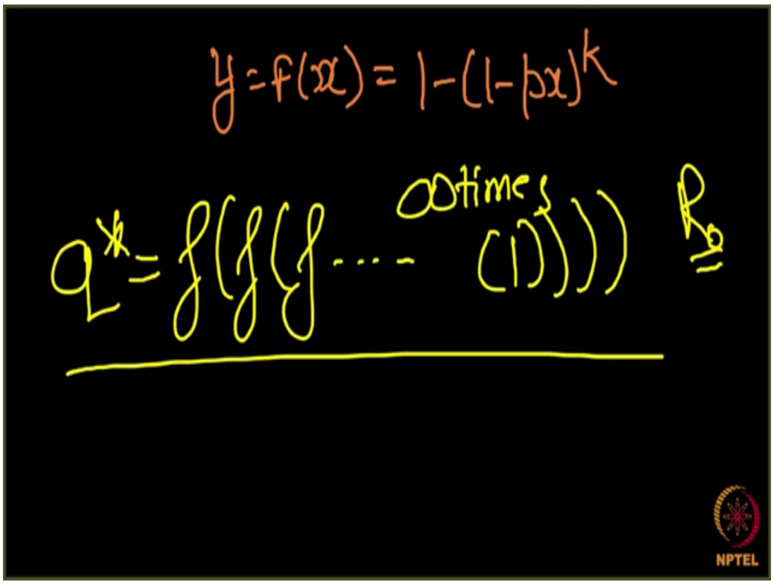


So, now we have all this we know q 0, q 1, q 2 and we know that what is q n, q n is nothing, but 1 minus 1 minus p q n minus 1 to the power of k. I try to write it down in the form of a function. It is already in form of a function. If I just take a function y equals to F of x we define this function as 1 minus 1 minus px to the power of k.

Now in terms of this function can we see what is q 1, q 0 obviously is1, what is q 1 according to this function? So, if we see what is q 1 here 1 minus 1 minus p q 0 to the power of k. This and this are actually the same. I have just written this in the form of a function here. So, we can write q 1 as nothing, but 1 minus 1 minus oh very sorry. So, we can simply write q 1 as; what F of q 0 right. What is f of q 0: 1 minus 1 minus p q 0 to the power of k which is same as this right. What is q 2 now? q 2 is f of q 1, 1 minus 1 minus p q 1 to the power of k which is nothing, but I put the value of q 1 here. It become f double dash means f of f of so, to f of f of q 0.

Similarly, what is q 3? q 3 is nothing, but f triple dash of q 0. And similarly you see what is q star going to be? q star is going to be f dash dash dash dash dash infinite times means f of f of f of f of infinite times q 0. And we also know that the value of q 0 is 1. So, I can write it down as q star equals to f of f of f of infinite times 1 right.

(Refer Slide Time: 05:37)



So, what is our overall aim now? Our aim now is we have a function y equals to f of x which we define which is 1 minus 1 minus px to the power of k and I know now the value q star which I have to find is nothing but f of f of f of infinite times 1.

And how do I find out this value? Once I find out this value my task is done, our aim is to find out the value of q star, obviously given the value of R 0 that is to come in the next lecture. Our overall aim is just to find this value of q star and we will be finding it in next lectures. Probably just 1 or 2 lecture more we will be finding what is q star.