**Social Networks**
**Prof. S. R. S. Iyengar**
**Department of Computer Science**
**Indian Institute of Technology, Ropar**

**Rich Get Richer Phenomenon – 2**
**Lecture - 137**
**Percolation model**

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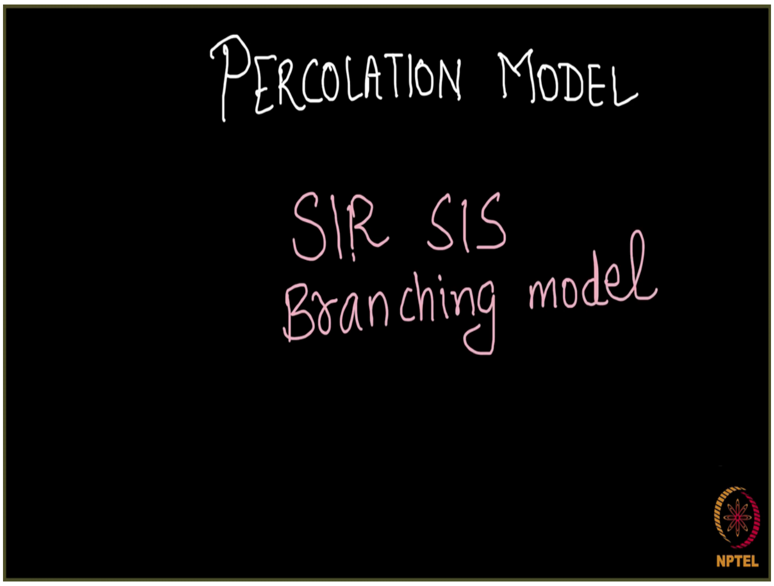


In this lecture we are going to now study a very interesting model which is known as the Percolation model. And this percolation model it is actually not a new model for you know modeling the spreading of your disease is rather it is a new angle, new way of looking at an already existing model. SIR model SIS model and for a matter of an event for your branching model, simple branching model; we are going to look at these models from a different angle. I will tell you what is the different angle.

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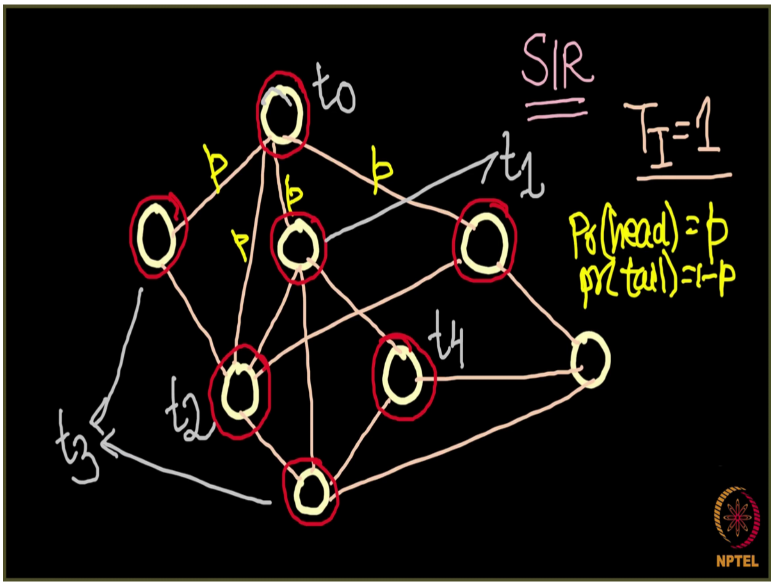


So, let us quickly see we will simulate we will quickly simulate SIR model on a network and then we will see how can we look at it in a new manner. So, let us take a network here. So, I will just quickly make a network over here, I will take some nodes and we will take some edges, (Refer Time: 01:28) edges. So, I think this should be enough ok. So, we take this network over here and now we simulate a SIR model on it. And, let us say our T I equals to 1, that is every node remains infected for 1 time period and after this 1 time period or 1 day that node stops infecting other nodes and let us say the infection starts from this node. So, this is our node which is infected at time 0, the 0 th day this node is infected.

Then now this node has 4 neighbors and it gets a chance to infect each of these neighbors. So, I am sure that you know how this process is occurring. So, each of the edges over here has a probability p of infection. So, what we are going to do is toss a coin for this particular edge and this will be a biased coin, biased coin such that probability of head on that coin will be p and probability of tail on that coin will be 1 minus p right. So, we toss a coin over here and see whether it is a head or tail, if its head we will infect this node else leave it. Similarly, for this edge, for this edge and for this edge and let us say after doing this 4 coin tosses this node over here and this node over here gets infected.

So, both of these nodes they get infected at time t 1. Now, in the next iteration both of this particular node it recovers and it is no longer able to infect other notes while, these 2 nodes over here they now go ahead and they infect the network. And, let us say that in the next time step that is at time 2 this node over here gets infected so, this node gets infected at time t 2. And, then again both of these nodes recover and then this node over here it sees and it tries infecting other nodes. And, let us say it ends up infecting this node over here and this node over here.

So, both of these nodes they get infected at time t 3 and then this node has a chance for these 2 nodes now infect further and let us say that in the next iteration this node over here gets infected. So, this happens at time t 4. So, how do we view this process? We are viewing this process as; we are viewing this process as there was this time 0, when this node gets infected day 0. Then it gets a chance to infect its neighbors and that at time 1 at day 1 these 2 nodes get infected and then time 2 these 2 nodes get infected and so on. So, we look at this process in terms of time.

Now, percolation model asks you the question that can you remove this notion of time from here. It will it seems to me to be a very dynamic process everything is running dynamically time 1, time 2, times 3, time 4 can you make static. So, let us say I want to know I was having this initial network over there ok. So, initially my network was like this and then this wait and this node over here, this node was infected. And, let us say now I am interested in knowing when this process will run whether this node over here will get infected or not.

And, then what we did for knowing whether this node will get infected or not we ran the simulation in terms of time. We looked at what will happen at time 0, what will happen that time 1, what will happen at time 2 and so on. And finally, we see that yes this node over here is infected. There is a rather of very new way a novel way to do this a very interesting one, where this process will not run in terms of time. We look at it as a very static process and that is what is called the percolation model. So, how do I make this entire process static is so, I take this network. So, this is my network and I want to see how I can make everything static over here. This is the network; this is the network I am having and no wait 1 second ok.

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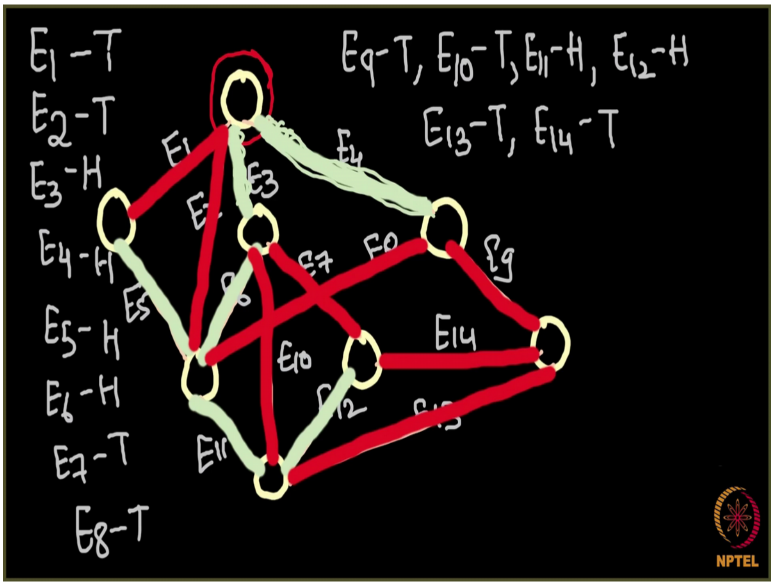


This is the network we are having and then we know that at the beginning this node over here is infected. And, we are interested to know whether this node will get infected or not and we know all this is a complete game of tossing a coin only. At time 1 we have tossed a coin for this, whatever gets infected; in the second iteration we toss a coin for that particular node whatever gets infected in third iteration we toss a coin for that particular node and so on.

Now, see what we will do is, first of all I will just quickly label all of these edges. So, let us say this is E 1 E 2 E 3 E 4 E 5 E 6 E 7 E 8 E 9 10 11 12 13. So, there are 14 edges in this network, what I am going to do is instead of running this process time wise I am now going to flip 14 coins 1 for each edge. So, first I flip a coin here for this edge E 1. So, I flip a coin for E 1 and let us say I see a tail and then I flip a coin for E 2 and I see a tail, E 3 let us say I get a head. E 4 let us say I again get a head, E 5 let us say head, E 6 let us say head we are getting so, many heads.

E 7 I got a tail, E 8 I got a tail and then let me toss it for E 9 I get a tail. E 10 tail, E 11 let us say head, E 12 again a head, E 13 tail, E 14 tail ok. So, see what I am doing is I have flipped a coin for all the edges at the beginning itself, I am not running this process time wise. Now, what do I do in this graph wherever I see a head. So, I imagine this graph in the form of let us say pipes. So, here there are the nodes and these are the pipes through which let us say some water can flow and actually here some contagion is flowing.

So, these are the pipes through which water flows across these nodes and whenever a head occurs so, I say that a pipe is open and whenever a tail occurs I say that the pipe is closed. So, let me show let us have opened pipes in green color. So, let me open some pipes. So, E 3 is a head and then E 4 is a head, E 4 is a head 1 3, E 3 is a head, E 4 is a head and then E 5 and then E 6, E 7 I am sorry not E 7 E 6 and then we have E 11, where is E 11 E 11 and we have E 12 ok.

So, all these are the open pipes and remaining whenever I get tail I say that these are these are all the closed pipes. So, this is a closed pipe, this is a closed pipe and then closed, closed, closed, closed and these are the closed pipes. So, we can now see these nodes as connected in the form of pipes, where the green pipes are the open and red ones are the closed; now what it has to do with telling us whether some node was infected or not. So, we started our infection over here and we were interested in the question whether towards the end of the process this node over here will get infected or not.

And now, I tell you the answer; the answer says that imagine here is this water flowing from this node and we see that some of the pipes here are closed and some of the pipes here are open. So, I say that if water is able to flow from this node to this node, yes this node will get infected as this node will not get infected. And, you can match it with what we have done previously. So, whichever nodes get got infected are the ones you which you can reach with the help of these open pipes. So, if you want to see whether some node is infected or not let us say I want to know whether this node is infected or not, I will see whether there is a path of open pipes to this node from this node.

So, let us say here and then I can go here, here and here I see that there is a path. So, yes this node will get infected at the end and let us say I want to talk about this node over here. So, I see that there is no path of open pipes from this node over here to this node. So, this node will not be infected towards the end of the process. And, you see we have seen that this entire process of the infection of different nodes in the network in an SIR or let us say SIS model without using any notion of time. So, this is called the percolation model and it is very useful in the analysis, in the theoretical analysis of such processes.

And, I think that by now it would be clear why use the term percolation. We say the term percolation because it is like this water is percolating through this network. And, we can use it in the modeling of the epidemic diseases.