Social Networks
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Link Analysis (Continued)
Lecture - 114
PageRank Explained

So we are nearing the conclusion right now. As you can see, let us recollect the big question. What is the question? When you assign values, gold coins or resources whatever you call it to all the nodes and as you start sharing them, respecting the edges that leave and leave a vertex and then enter a vertex right a directed graph, you will observe that the values do indeed converge, right.

So, let us take it another example and then see it. We have already seen an example on 3 nodes, another example on 5 nodes. We will see it another example for clarity sake.

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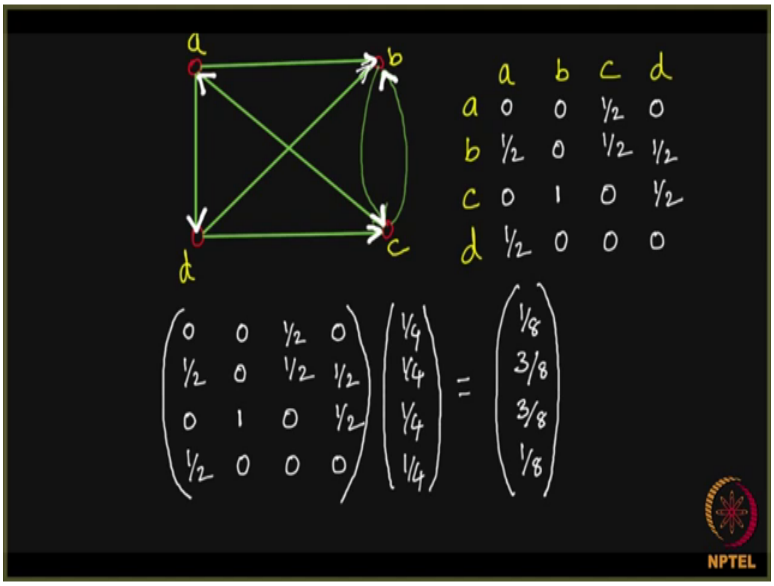

So, let us consider this graph on 4 nodes. This is the corresponding matrix; we have discussed this before right. Take a minutes' time pause the video and then check this is actually the matrix.

Now, let me write down this matrix as the matrix, then I start off with 1 by 4 1 by 4 1 by 4 and 1 by 4 and then I see where it takes this. 1 by 4 1 by 4 1 by 4 1 by 4 simply becomes 1 by 8, 3 by 8, 3 by 8 and 1 by 8 right.

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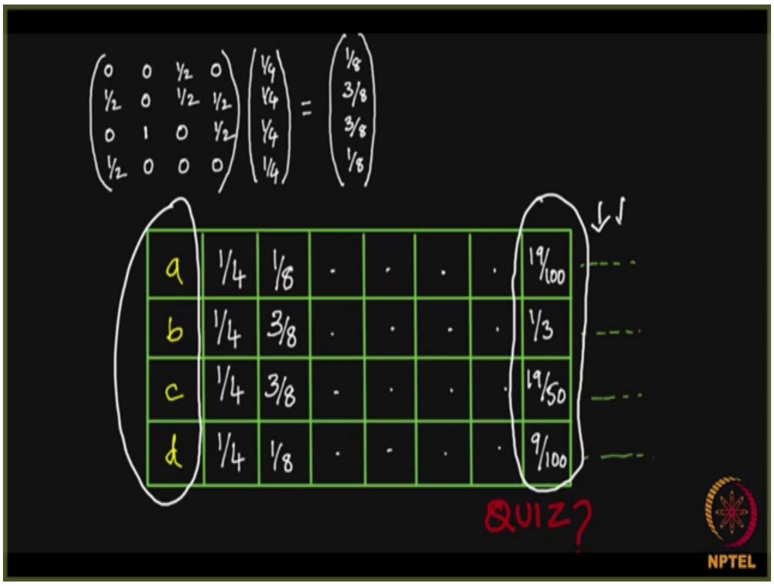

So, we know this right we saw this. Now let me write a table and then make a note of all the values here a b c d and then I started off with 1 by 4 1 by 4 and then I got this what is the next value. Where does it go like this? Where exactly does it converge?

So, here is a quiz question for you all. Take a minutes pause and then see that it indeed converges to 19 by 100 1 by 3 19 by 59 by 100. In fact, you need not even take a look at these values; you should verify that you actually arrive at these values as you proceed with this matrix operation all right. Quick quiz for you all, it is good that you check it.

Now, as this goes further you know, this assignment is now what is called as stable assignment because it does not change anymore right. You are saying these to the nodes a b c and d, you assign it to a b c and d. Assign what? Assign these values and you will observe that there is no change anymore after this ok.

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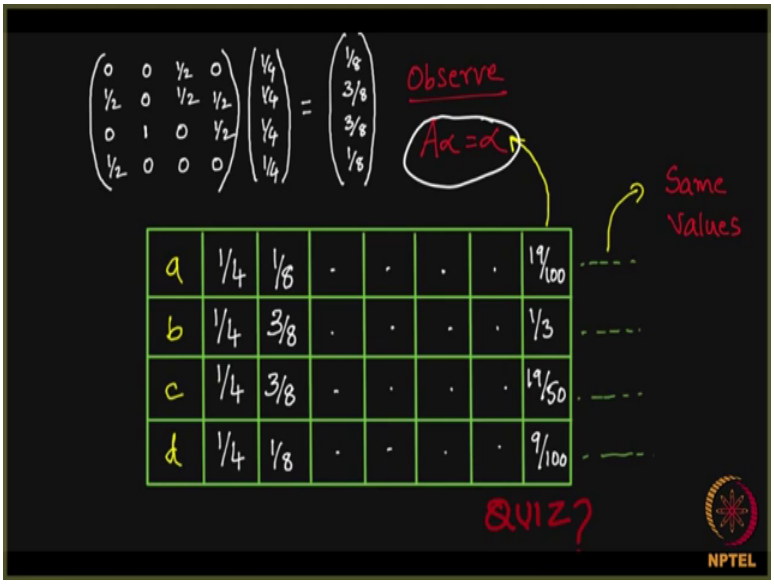

So, it remains the same correct. So, note something. Here is the situation where a alpha becomes e equal to alpha and that is why the change does not happen or rather the change does not happen because, a alpha equals alpha. It is all the same right. So, this is the matrix way of saying it right ok.

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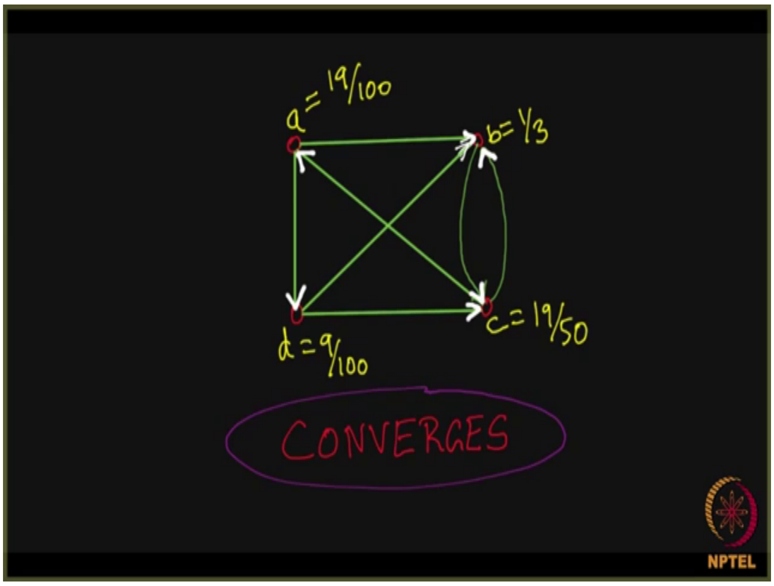

So, consider the graph. Once again and the assignment, you will observe that this is the assignment where values do not change. You can try doing it when a is assigned 19 by 100 b is assigned 1 by 3 and c is assigned 19 by 50 and d is assigned 9 by 100. The values do not change after that and such a value such as assignment is actually unique. And we all have seen it already; I am sort of free iterating it right now in this lecture video.

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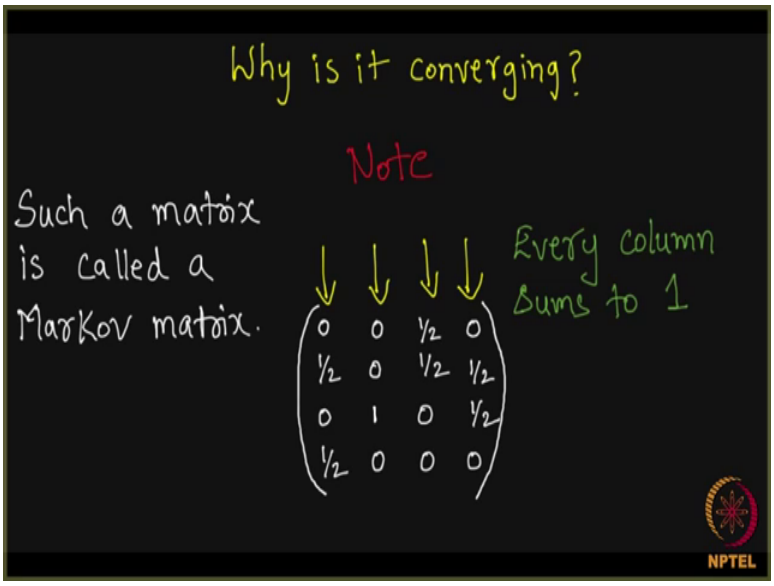

So, why does it converge? Why exactly is it converging? Now is not that obvious we have seen that enough. Take the matrix note that what is this, what do you understand by this, have you seen this matrix sometime before. If you are an engineering student, I do not think you would have passed by your semesters without noticing this matrix right.

So, any student who has finished his first four semesters of engineering would have definitely passed through a phase where he would have encountered this matrix. It is actually called the Markovian matrix right. So, observe the property here, every column sums to 1. Such a matrix is called a Markovian matrix or simply a Markov matrix.

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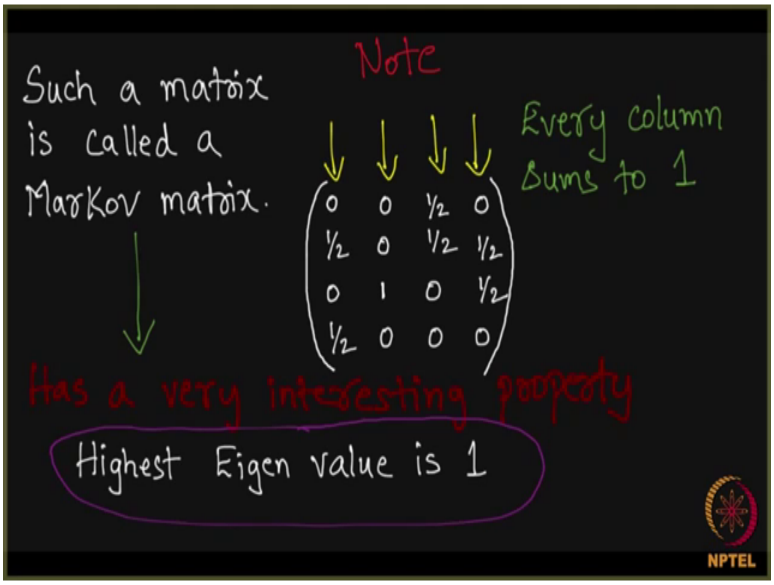

So, this Markov matrix is a very interesting property. Its highest Eigen value is simply one which means every other Eigen value is less than 1 alright ok.

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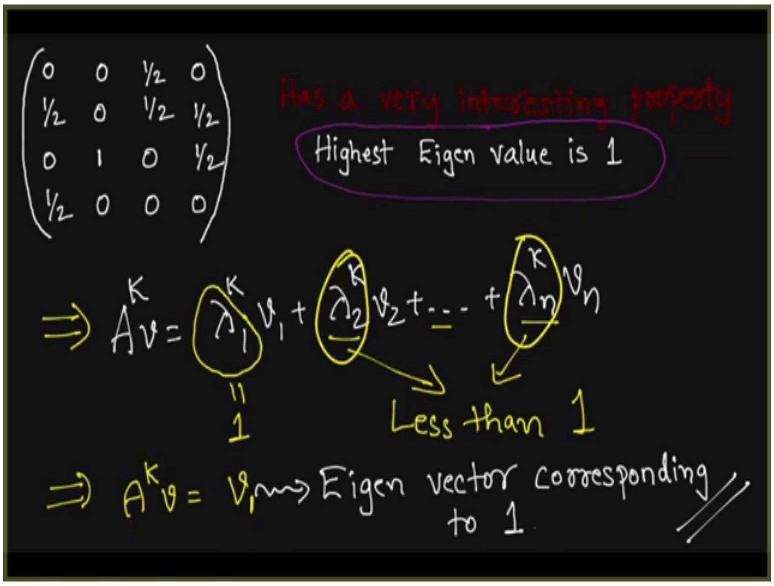

So, do you see what happens here? Let us go slowly. A to the k of 3 is simply this where v 1, v 2, v n are all eigenvectors and lambda 1 lambda 1 etcetera are all the corresponding Eigen values good. So, we can always write A's action on v can be captured with this. So, we have seen this before I am repeating it ok.

Next lambda 1 is 1 here because that is the highest Eigen value. As I told you that is the property of such a matrix and the other Eigen values are less than this Now, what does it mean? Now you would have observed a number which is 1 to the power of k will make it one itself right. Other numbers are less than 1 here. All these are less than 1 here and when you raise it to the power of k, all of them become 0, right. All of them variably become 0. So, A to the k will simply be equal to lambda 1 to the k times v 1 mind you lambda 1 is actually 1. So, this is the eigenvector corresponding to the highest Eigen value which is 1 and let me just remove lambda 1 here because it is equal to 1.

So, what do we observe? We observe A to the k of v converges to the Eigen vector corresponding to 1, perfect.

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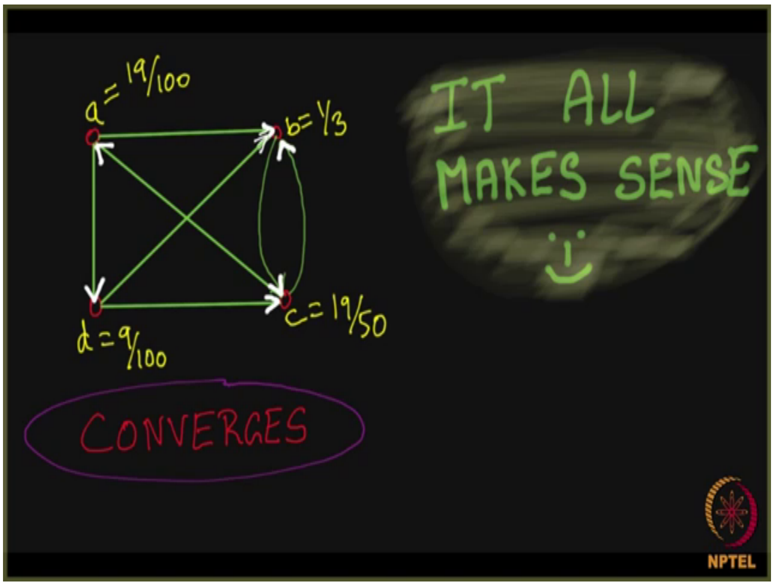

That is pretty much here we have come to the conclusion. So, this was the graph that you took. You saw this was a sort of a stable assignment which means it converges at this value and now it all makes sense as you can see, right. So, such a value assignment is unique and no matter where you start from you always converges to this. So, that is the best part.

So, that sort of concludes good amount of discussion on page rank. We have revisited page rank from our previous lectures about some 20 lectures ago, a couple of weeks ago roughly let us say 3 to 4 hours videos before we saw page rank very at a very conceptual level, we did not see the math part of it, we have now seen the math part of it. I hope it is clear to all of you. It is important for you all to take a look at it may be re-watch the videos because it takes time to sink in especially the mathematics component of a page rank.