

Introduction to Wireless and Cellular Communication
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Lecture - 09
Wireless Propagation and Cellular Concepts
Cellular system design and analysis

Good morning, let us begin lecture number 9, we will start with a quick summary of lecture 8 recap and if there are any questions; there were several interesting questions after class yesterday and if there are any doubts we can clarify them. So, to quickly highlight the main contributions that we are going to we looked at in lecture 8 where the contributions of McDonald the cellular concept which consists of 2 important parts; one is frequency reuse, the other one is a cell splitting. And between those 2 concepts more or less we been able to handle all the key elements that that we needed to work within the context of the design of a cellular system.

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EE5141 Lecture 9

- L8 Recap

- Cellular geometry

MacDonald cellular concept

- Frequency Reuse
- Cell Splitting

Interference

- Co-Channel (CCI)
- Adjacent channel (ACI) (Rx filter)

CCI more severe impairment ←

Frequency Reuse Design

k channels/cell A area of cell

N cells/cluster M # clusters (covering Geog. area)

$C = \text{Capacity (Total sim. calls)} = MNk$

Total available spectrum = Nk (fixed)

$\uparrow C$ $\frac{C}{I} \geq \left(\frac{C}{I}\right)_{\min}$ $P_r(\text{outage}) = P_r\left(\frac{C}{I} < \left(\frac{C}{I}\right)_{\min}\right)$

So, we would like to move on to understanding some of the other elements. When we talk about frequency reuse we talk about those channels which are exactly the same as the our desired user is using, those would be Co channel users then immediately on either side would be the adjacent channel and then you have the mixed adjacent channel

and then the others. The adjacent channel ones are more or less haggled completely by your receive filter.

So the receive filter takes care of all of these. So, we dually do not worry too much in our frequency cellular system planning in terms of adjacent channel assume that you have designed a good receive filter and that you have. So, Co channel interference becomes the one that we always have to worry about, Co channel interference and more or less the rest of the discussion. In today's the lecture is going to be focusing on Co channel interference now frequency reuse as we discussed yesterday will have the following components which I would like you to sort of keep in mind talking about K channels per cell and N cells per cluster.

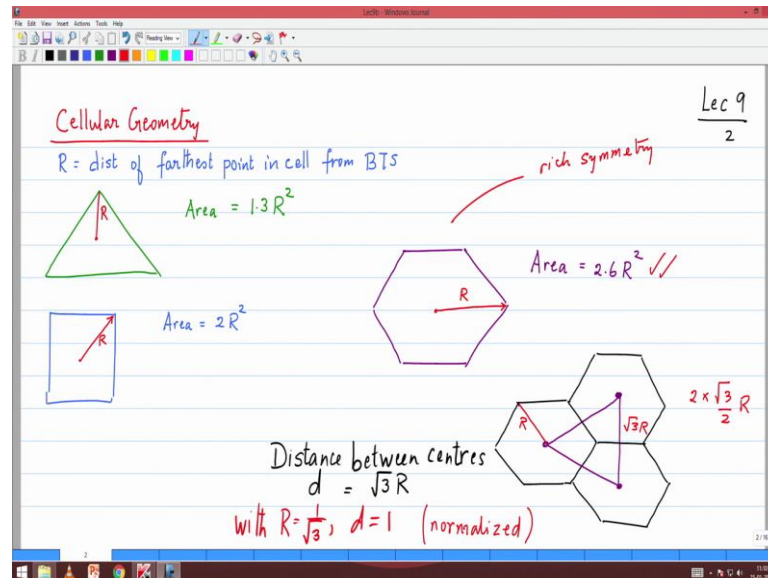
So, basically you are designing certain clusters of cells and you are able to give each of the cells K channels. So, the that basically says that a total spectrum that you have is N times K now that is what you are able to distribute and as was clarified yesterday you do not need to give K channels to each cell you could give that distribute them unequally to make sure that you have unequal capacities. But at the baseline you know designing it for uniform capacity would be a good starting point, A being the area of the cell and M is the number of clusters that you need to cover the geographical region typically it would be a city or maybe even a larger than a city as well.

So, the total capacity how many simultaneous calls that we can have is the N times K that is the number of channels available in a cluster times M that would be the total simultaneous calls that are possible in the system that is a important number for us to keep in mind and you are supporting so many simultaneous calls given a spectrum which is proportional to N K. What is most important for us is that we would like to increase the capacity now increasing capacity at the will come at a price that you will pay and, but we also have to make sure there is no possibility of outage.

We said that the in a interference limited system the outage maps to the C over I carrier to interference ratio again I am neglecting noise. So, I am not talking about interference plus noise more or less neglecting noise and saying interference is a dominant part that more or less says that the C over I that I experience for my desired user should never go below a threshold C over I min I should always make sure that is satisfied and that is what is the condition given to us on the left. C over I what I design should with the

appropriate margins should be always greater than C over I min. So, this is the broad framework of the frequency reuse concept we will now build upon it in today's lecture.

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Now, cellular geometry is the basis for the discussion today as we mentioned the we can look at any regular shape which will permit desolation and the 3 basic shapes that we looked at a triangle, square and hexagon, I hope you done the calculations basically will tell you for a given distance; maximum distance between transmitter and the farthest point of the receiver the area covered by a hexagon is the best. So, therefore, a hexagon would be the one that will require a minimum number of cells to cover a particular area and it also is a good approximation to a circle which is what you will the coverage that you will get if you have omni directional antennas.

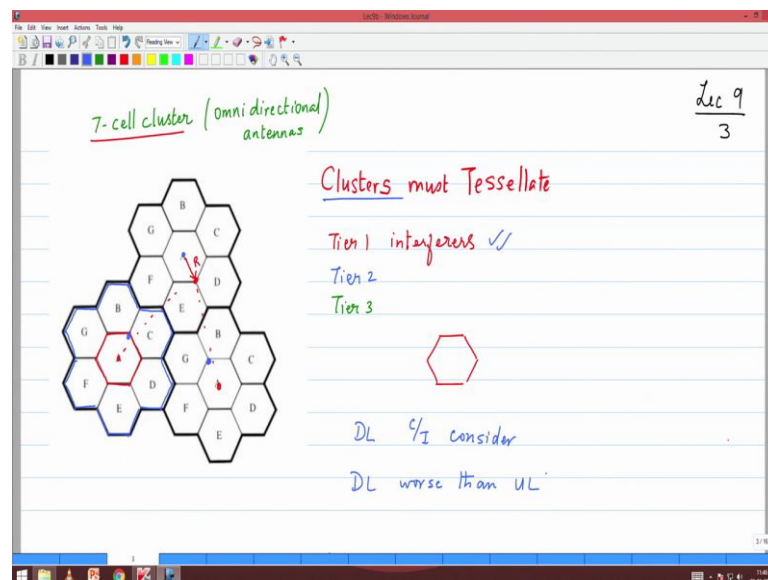
So, the other element that you will see very soon once you start working with hexagons is that it is very rich in symmetry. So, one of the advantages that we have is a lot of very very interesting symmetry because a hexagon a regular hexagon consists of 6 equilateral triangles. So, the minute you start looking at tessellations you see that the symmetry start to appear in very very different forms. So, for example, if I were to tessellate 3 hexagons touching each other at a common vertex and connect their central points that forms an equilateral triangle and that and that will repeat continuously.

So, if I have another observation if R is the maximum distance or the side of a hexagon it also happens to be the distance from the center to any of the vertices the distance

between the 2 centers basically it will be the height of those equilateral triangles twice of that. So, basically you will get 2 times root 3 by 2 R which give me root 3 R and again that is a very useful point to remember because whenever we tessellate we are looking at distance between the centers and usually in cellular geometry discussions R is a parameter that you will have to choose that will be based on the size of the cell, but for a lot of discussions we assume that we can choose R to be anything. So, if you choose R to be 1 by 3 then it basically says that you have unit spacing between the centers of your. So, this is basically a normalized values you always have to make sure that when you are specifying the distance are you specifying it in a normalized or in the real case.

So what we will do is work with the normalized diagram because its very easy for us to work there because centers between the hexagons will be unity, but when you translate it into actual design of the system you must make sure you remember that there is a radius that will define the area of each cell. So, this is a normalized representation. So, we will moving forward, we would like to keep the hexagonal geometry and keep leveraging the symmetries that are available to us in the cellular geometry.

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Now, first and foremost you must get used to drawing a hexagon. So, basically a hexagon is probably much easier to draw than the higher polygons higher order polygons once you have a basic hexagon populated with hexagons touching on all of its 6 sides that gives you a; so basically you have a single hexagon. And if you draw similar

hexagons touching all of its 6 sides you form a pattern that actually turns out to be the 7 cell cluster and that is one of the very commonly used design tools for the for a cellular system.

So basically the cluster would be along the blue lines this would be the pattern that that you would have where you have a single cell the red cell with a hexagon touching each of its 6 sides and you probably can draw it much faster than I can. So, that blue shape now becomes a cluster. Now if you design your system using a 7 cell cluster this shape would now get replicated. So, notice that the cells must tessellate the basic shape must tessellate there is additional constraint because you are going to repeat the cluster size they are not going to change the cluster shape as you more because then the geometry will get affected. So, the clusters also must tessellate right we must you cannot design some weird pattern, but basically these are hexagons blends itself to several patterns that the clusters themselves will tessellate.

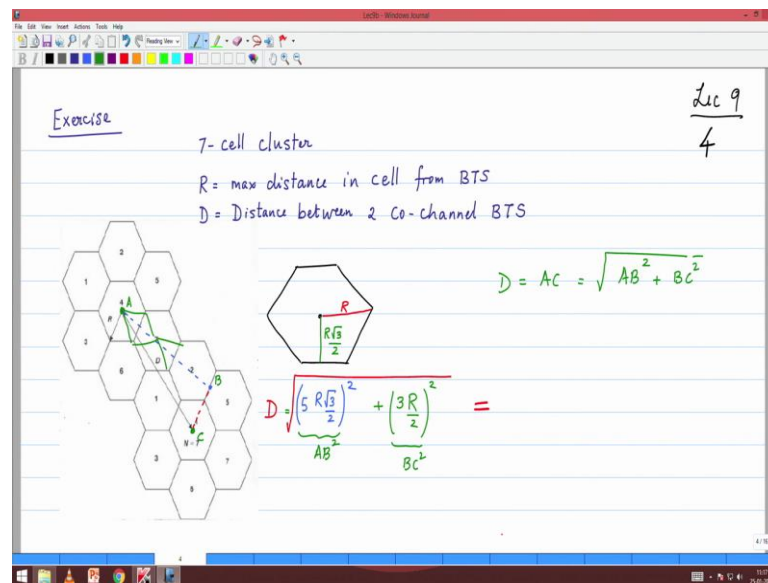
So, again keep that picture in mind that the basic cell is at shape that tessellate and the clusters also are tessellate one of the important tasks that we will do in this in this discussion is to calculate distances. Now if I were to maybe go back to the previous slide, now if I were to think of this cell and I have a antenna or my radiating element at the center of this and the farthest that user can go from that cell would be at a distance of R correct that would be R .

Now the key questions that will arise is if my user is in the most disadvantageous position; that means, with respect to my desired transmitter I am at farthest point of a away from there now at this point what is the interference that you will see. So, basically you will want to know; what is the distance of your mobile from your other base stations that can are using the same frequency and therefore, can cause interference. So, basically you are interested in all other base stations and their distances from your mobile. So, I will able to visualize. So, you in a 7 cell cluster yeah you will find that there are 6 base stations each one where you a place the structure on all sides and you will find that there are many base stations that will interfere, but there are 6 that are closest to you will have to find out those 6 and you have to measure those distances.

So, there is a name given for those they are called the tier one interference tier one interference basically those interference that are closest to you and in this shape if you

draw it you can visualize that there are 6 of them which are very close then there will be at tier 2, tier 3, tier 4, but most of the time we are concerned about the tier 1 interference we do not worry too much about the tier 2, tier 3, they are there if you want very very accurate calculations of the interference you must take into account tier 1, tier 2, tier 2, and 3, but a first order approximation which is usually very very good is by looking at the tier 1. So, the distances are very important for us to calculate. So, let me just give you a feel for how to exploit the hexagonal geometry for quick calculations.

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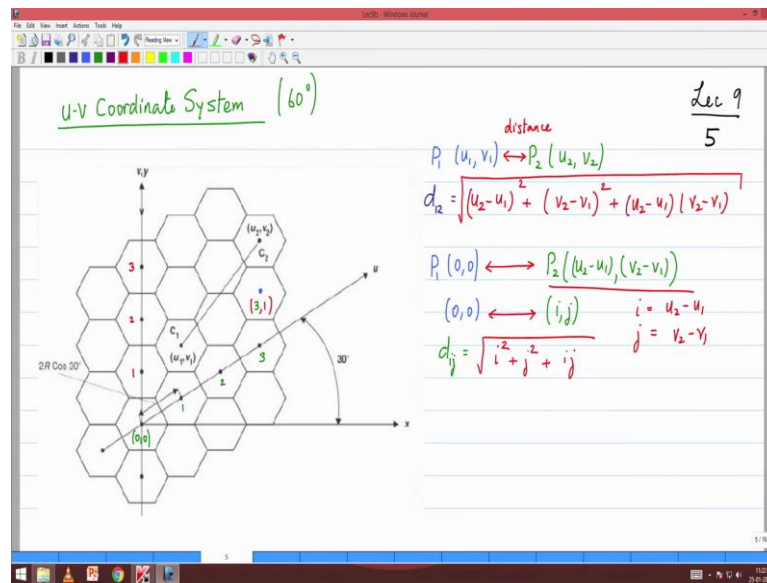
So, here is a 7 cell cluster 7 cell cluster and we are basically saying that I want to know what is this distance between these 2 cells there are there is a geometry and I have and presumably they are using the same frequencies and therefore, I am interested in knowing the distance between I am now asking for you the distance between the 2 base stations which are using the same frequency now the easiest way is to identify some pattern in the cellular geometry. So, what would easiest what would be the way is draw a line the blue line which is basically connecting along the centers of those sides until you reach that cell which is marked 2 and then from that point it is a 90 degree turn and then you can basically form a right angle and find out what is the distance of let us call that as $A B$ and C . So, basically $A B$ squared plus $B C$ squared square root will give me the distance that. So, D is equal to $A C$ square root of $A B$ squared plus $B C$ square that is that is the easiest way to do it because of there is lot of interesting symmetric.

So I want to calculate the distance A B. So, it basically traverses along the altitudes of the equilateral triangles if you notice it is traversing. So, basically if you think of an equilateral triangle here equilateral triangle here equilateral triangle basically it is traversing along the altitudes and $\frac{\sqrt{3}}{2}$ is A times R is the height of the altitude and there are 5 such altitudes that it traverses. So, 5 times $\frac{\sqrt{3}}{2}$ R whole squared that becomes A B square. So, this is A B square A C squared is going along the line connecting the center to the vertex that is R and then it basically does half a side each side of the hexagon is R. So, basically it be R plus $\frac{\sqrt{3}}{2}$ R. So, that is $\frac{3\sqrt{3}}{2}$ R that is B C. So, this is B C squared. So, again its it is once you see the patterns it is not difficult just have to be careful are you going along the altitude of the equilateral triangle or are you going along the side of the hexagon in which case it will be multiples of R if it is going along the altitude it will be multiples of $\frac{\sqrt{3}}{2}$ R.

So do an example calculation and find out what the distance is because that is a useful parameter that we will be coming back to discuss several times in the course of the understanding of the cellular systems. So, is this the method of calculating distances identifying a right angle triangle identifying it in terms of the 2 sides in terms of combinations of height of the equilateral triangle and the side of the hexagon? So, basically known quantities used to estimate the distances.

Hopefully once you do it a couple of times I am sure you will find it that it is fairly straightforward you will almost be able to see, but it is probably a good idea to draw line and do it. So, that you do not make a mistake the that you are getting the angles correctly now taking this geometry discussion to the next level is to enable us to understand the design of a cellular system. So, we are now going to introduce a coordinate system to number or label the cells.

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And notice that the; if I want to draw a line connecting the centers. So, one axis the y axis is nicely aligned in this case, but if I look at the x axis it does not go through the all the centers it goes through some centers it goes through some edges.

Now, I do not want that I would like to be able to work through the centers of the hexagons. So, which means that I cannot use the x axis I would have to sort of rotate my axis and that would be the u axis its basically the 2 axis that we have u and v are at an angle of 60 degrees not orthogonal not 90 degrees as you would have in a x y coordinate system. So, we are going to introduce a coordinate system which has an angle of 60 degrees.

Let us say and again this is not a problem is just that I am going to label them in the following fashion. So, I label them starting with 0, 0; I label the centers as 1, 2, 3 and 1, 2, 3 on this direction. So, for example, this blue dot that we have that we are seeing has got 3 units along the u axis. So, first coordinate is u 3 comma 2 it should be 3 comma 1 sorry, it is not 3 comma 2 it should be 3 comma 1. So, basically this is how we would label the u v system.

So, I will denote any point in terms of u coordinate and a v coordinate. So, basically u 1 v 1 u 2 v 2 and in any coordinate system the most important element is the distance between 2 points which are denotes. So, I want I am interested in the distance between points one and 2 which are given by the coordinates u 1 v 1 and u 2 v 2. So, I am sure

that when you did coordinate geometry when you move away from a 90 degree coordinate system the distance measure changes slightly it is no longer the Pythagorean relationship. So, let me just write it down again it is something that you are probably very familiar with.

so basically it will be $u_2^2 - u_1^2 + v_2^2 - v_1^2 + 2(u_2 - u_1)(v_2 - v_1)\cos(\theta)$, so and if it is 90 degrees so the that third term will go away, but if it is in this case it will come out to be $u_2^2 - u_1^2 + v_2^2 - v_1^2$, again if you do a simple geometric basically looking at the distance description you can confirm that this is what it will come out to be.

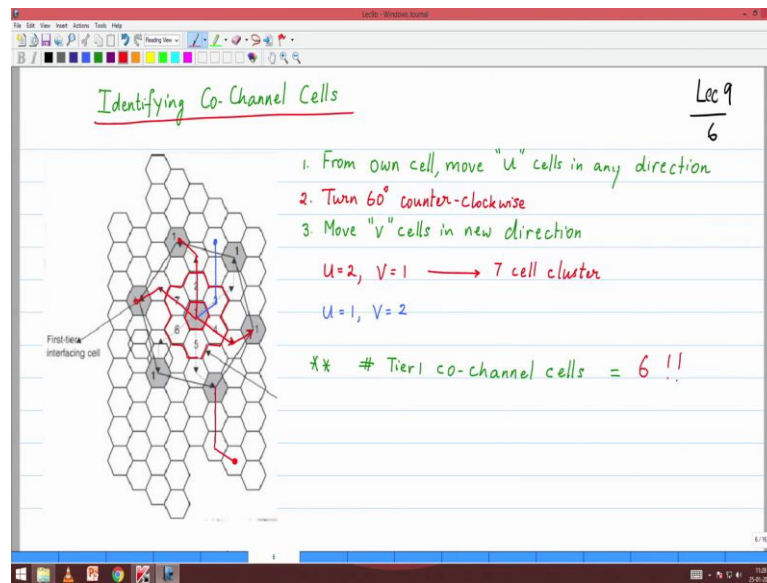
So, in this $u-v$ coordinate system if I give you 2 points if it were a orthogonal one you would just take the first 2 terms, but this is said a 60 degree angle. So, basically we would introduce a second term that is introduced that the tells me the distance measured and for ease of calculation you can always refer your distance computation to the origin; that means, if I map P_1 to $(0, 0)$; that means, P_2 then gets mapped to the points as shown as shown here.

So, an easy way for us would be used to will denote the one of the points at $(0, 0)$ and the other one as (i, j) where i is equal to $u_2 - u_1$ j is equal to $v_2 - v_1$. So, I want to note $d(i, j)$. So, that will be square root of $i^2 + j^2 + ij$. So, that is my $u-v$ coordinate system it is a slightly different from what we are used to in traditional geometry, but again given the hexagonal tessellation that we are doing its a more convenient reference system for us to work with and it gives us a certain flexibility to label these systems.

Now I now want to relate this is probably the most important reason why all this has been all these have been introduced now I want to go back and ask you to think about the following elements I want you to think about a how this tessellation is happening. So, basically I have a cluster then I place these clusters in a tessellation form now I want to know how many tier one clusters are going to be there.

So, to answer that question we have a interesting way to do that let me give you the method first and then the rationale follows. So, let us say that I have a basic cluster which is one the 7 cell cluster.

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So, let me just highlight it for you. So, this is my central cell around which the cluster has been built. So, the cluster 7 cell cluster which I am sure you are now able to draw fairly quickly we can you can a highlight sorry it should be that should be the cluster that is the central one and then the cluster around it.

Now, how to identify the location of the clusters that will come around the basic cluster, so which means that first find the center of the cluster position that and then basically place the cluster around it. So, here is an algorithm a very simple one from the hexagonal symmetry understanding which says that if you want to identify Co channel cells what you need to do is basically move along the coordinate system some integer number of locations.

Now, for example, let us take the translation u equal to 2 v equal to 1, so basically from the center point along any of the sides of the hexagon, so basically I have chosen to move along this particular direction move to 2 centers. So, the cell number 4 will be the first center. So, and the one after that will be the second. So, u is equal to 2 along that then I turn counter clockwise 60 degrees because that moves me from the u axis to the v axis over to the to the v axis and then move number of coordinates along that.

So, u equal to 2 v equal to 1 moving in this particular coordinate system tells me that I can go I will move 2 in this side and then 1. So, that tells me that I have not yet told you that this is for a cluster size 7, but I am just giving it you as a information. So, for a

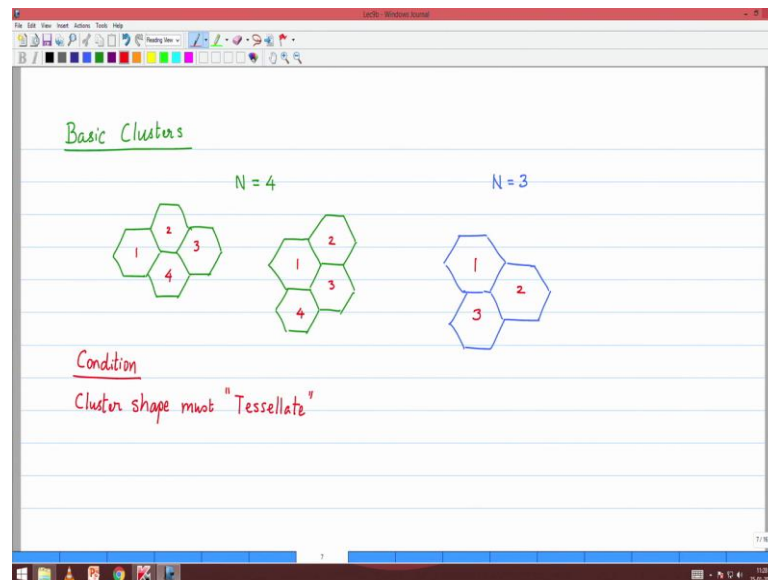
cluster size 7 u equal to 2, v equal to 1 gives us the location of the other centers now; obviously, you should be able to do u equal to 1 and v equal to 2 also. So, if you were to do u equal to 1 v equal to 2, let us try that on an in another direction. So, from here u equal to one will be one cell you move then rotate counter clockwise 60 degrees move 2 point move 2 units 2 cells that will tell you this is the location of the if you use that shift it will come there.

Notice that now you have to repeat it for all the sides. So, you we did the red one from this side then if you do the red one from the next side let us do that red one from here move 2 units counter clockwise 60 degrees that gives me this center then likewise if I did from the other side 2 units counter clockwise 60 degrees. So, you can see that basically like a wheel you get all the locations. Now if this is a valid way by of obtaining the locations of the next cell that is using the same frequency Co channel cell basically it is a method for identifying your Co channel cells we will show that this is correct a, this is a valid one.

So, if this is the way by which we do that then I have a very very important observation to make how many tier one Co channel cells will there be 6 because the only way I can locate it is by going along the side I have only 6 sides. So, this is a very very important observation that tier one will always consist of 6 interferes and of course, using this method how do I get to tier 2 from the cell use the same principle or less stick from this cell I can move 2 units turn 90 degrees as 60 degrees; counter clockwise 60 degrees that will be another cell that is using number the same frequency. So, you can basically in a in a hexagonal cluster; you can pretty much locate all the cells that are using the center (Refer Time: 24:52) and once you have those center frequency just placing the clusters on top of that is relatively straightforward.

So, the important concept is the link between the clusters the size of the clusters and the locations of these of these Co channel points and we need to be able to pull all of them together and it turns out it comes in just one step and that is the beauty of the hexagonal geometry that we are going to take up.

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But before that let me just sort of just for interest sake what are some basic clusters cluster 4 is a cluster that will tessellate. So, basically you can draw a 4 of these and then you can see that it can you can actually tessellate with this particular shape in equal to 4 there is another numbering that will also allow you to tessellate. So, basically any combination of 4 will tessellate N equal to 3 you can do and the 3 and 4 are probably the most interesting ones that we can do by hand 7 is easy, but anything beyond 7 becomes a little bit compressed cumbersome keep in mind that whatever shape of the cluster the cluster shape must tessellate I think that we already mentioned, but good for you to keep in mind.

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① Valid Cluster Sizes ?

② Verify method Co-channel cells

Cluster size

i	j	$i^2 + j^2$	$i, j = \text{integers}$
2	1	7	
1	2	7	
2	0	4	
1	1	3	
$\vdots$	$\vdots$		

$\left(\frac{D}{R}\right) = \text{Co-channel reuse ratio}$

$\frac{D}{R} = \sqrt{3N} = Q$

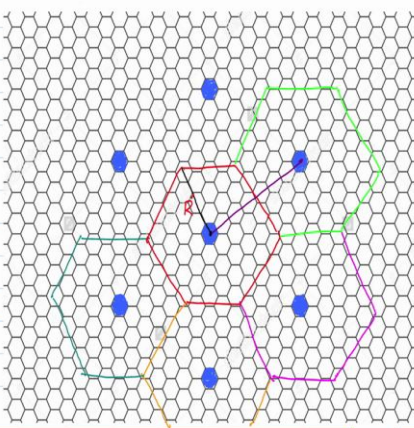
$Q \downarrow \Rightarrow \frac{D}{R} \text{ small} \Rightarrow N \text{ small}$

Capacity $\uparrow \left(\frac{C}{I}\right) \downarrow$

Now, we move on to a very important question what is the valid cluster what are the valid cluster sizes that is question number one and we have to basically verify the algorithm that we have shown that the way to clustered one is of course, to try it out that is that is a way to do it verify the method of locating the Co channel cells. So, that is the second aspect Co channel cells that is within the; so these are the 2 things that we would like to answer and we will come back to answering that in a little bit better fashion.

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Valid Cluster Sizes



Ex: $i=7, j=0 \quad N=49$

Small hexagon R Area $= 3\sqrt{3} R^2$

Large hexagon $D = \sqrt{3} R$

Area $= 3\sqrt{3} \left(\frac{D}{\sqrt{3}}\right)^2 = \sqrt{3} \frac{D^2}{R^2}$

$N = \frac{\sqrt{3} D^2}{3\sqrt{3} R^2} = \frac{D^2}{3R^2}$

Normalized $R = \frac{1}{\sqrt{3}}$

$N = D^2$

$\frac{D}{R} = \sqrt{3N}$

$N = i^2 + j^2$

So, here is a very dense picture let me take the inner the center cell the one that is shaded blue and I am going to do a using the algorithm that I did I am going to move 7 units in 1 direction and 0 in the other that that is a valid combination. So, basically along the centers notice I have gone along the centers let me draw it with a different color I go along the centers this is cell number 1 2 3 4 5 6 7 and like that I have gone on all 4 sides that gives me a location of my Co channel cells.

Now keep in mind that this is very closely tied to the fact that the number of Co channel cells is going to be 6 and that can easily be verified because I can when I go from each of the sides of the hexagon this is I get the shaded all the blue hexagons that are Co channel cells now here is a scenario that I do not know the cluster size, but I want to determine the clusters I have determined the Co channel cells. Now think of it that these clusters whatever the shape of the cluster has to be has to fit around each of these Co channel cells and if you think of a large cluster size then what you will find is that the shape that actually fits a proxy should approximate a hexagon because only then 6 of them will fit around with these 6 points as the centers of those hexagons. So, that is a sort of a maybe it is for some people it is very easy to visualize, yeah that only then it will fit, but you may say well you know some of them are going a, but approximately a hexagonal shape is what it will need to actually tessellate on this framework.

So, this actually shows us a way to validate the assumptions that we have made and let us quickly see for we can work on that. So, here is the; are the piece that we would like to work with basically we have a small hexagon and then we have a large hexagon. So, a small hexagon let us say has got radius R the area of the small hexagon is $3\sqrt{3}$ by $2R^2$ right that is the area of a small hexagon the area of a small hexagon. Now if R_{dash} is the area of the larger sort of invisible hexagon that is the one that that I am trying to estimate the cluster size. So, given that it has to be a pattern of 6 and it is got roughly a hexagonal shape I am assuming that this is the area.

So, if R if the radius of the large are the size of the larger hexagon is R_{dash} the area that I am going to work with is 3 times $\sqrt{3}$ by $2R_{dash}^2$ square which is same as 3 times $\sqrt{3}$ by $2d$ by D^2 squared by 3 ; 3 and 3 will cancel that gives me $\sqrt{3}$ by $2D^2$ squared. So, that is the area of the larger hexagon. So, the key step says the cluster size N is given by area of the larger hexagon divided by the area of the small hexagon approximately some of them are in a partly covered on one side partly covered, but actually if you look

at it the whole number has to emerge and that should be the area of the larger hexagon by the smaller hexagon.

So the area of the larger hexagon divided by that is $B \sqrt{3} \text{ by } 2 D^2$ that is area of the larger hexagon this is 3 times $\sqrt{3} \text{ by } 2 R^2$ that is the that that should be approximately the cluster size. Now let me take 2 parts to it one is going to take the path of the normalized pattern in the normalized pattern we said that we would assume R to be $1 \text{ by } \sqrt{3}$ so that we can get the right shapes. So, if I substitute R by $\sqrt{3}$ then I get the cluster size in a normalized hexagonal grid the normalized one will be equal to d^2 that is the distance between the 2 Co channel cells between the 2 centers of the 2 Co channels of the large hexagon. Now very very important what is d in the normalized framework d^2 we showed that using the $u \text{ } v$ coordinate system this will come out to be $i^2 + j^2 + i \cdot j$. So, the cluster size in a normalized framework should be $i^2 + j^2 + i \cdot j$ because you know that that is what tells we that if I were to carefully draw the those are the cluster this is what it the size would be you know making sure that I get all.

So, the important element is that we have these small hexagons which are the Co channel cells to visualize the cluster I am saying that it has to be a pattern that that surrounds it approximately a hexagon there will be some jagged edges, but effectively it should it should approximate it be an hexagon and that is the estimation that that we get now if it is not the normalized side. Let us write down the relationship on this side basically what you will get is $D \text{ by } R$ is equal to $\sqrt{3} N$ and if you take R to be $1 \text{ by } \sqrt{3}$ you know you can you can get back the original relationships.

So basically the cluster size which we for us it will be thinking of it in terms of the geometry that we have developed and the coordinate system is given by $i^2 + j^2 + i \cdot j$ and the another important relationship that emerges is $d \text{ by } R$ is equal to $\sqrt{3} n$. So, now, let us go back and look at the valid cluster sizes. So, cluster sizes basically says I can take $i \text{ } j$ then I have $i^2 + j^2 + i \cdot j$. So, if I were to take the a combination $2 \text{ comma } 1$, I get a cluster size 7 remember that that was what we had verified and of course, if I do $1 \text{ comma } 2$ also I will get a 7, I can do as long as i and j are integers right because in the coordinate system that is what we have those are the points at which the centers of the hexagons lie.

So, 2 comma 0 will give me a cluster size of 4 0 comma 2 will also give me a cluster size of 4 1 and 1 gives me a cluster size of 3 1 and 0 says it is a single cell cluster that that is where that is not really not a cluster but you can now go on and verify various combinations you can show that N equal to 12 will come N equal to 21 will come as you get as you go further the cluster sizes you know they are not too many you will basically get larger sizes.

So that is an interesting observation I would like you to be comfortable in terms of the arguments that we have presented and the ability to work with this observations. Now I want to move to a very important element from the previous discussion. So, the previous discussion we identified that there is a ratio called D by R , this is called the Co channel reuse ratio Co channel reuse ratio R is the radius of the maximum size of the of 1 cell D is the distance between the Co channel cells. So, this is the Co channel reuse ratio. So, it is a parameter that once you have designed your cluster size you can then say that D by R has got this relationship and you know D by R is equal to $\sqrt{3N}$ that is the relationship that we have we have obtained. So, let us spend a few minutes because this is a very important parameter it actually is denoted by the letter Q in some sense to denote that it is a quality measure and we will justify the fact that it is a quality measure.

Now what happens if Q is small? If Q is a small value this basically implies D by R is a small value; right D by R is small, this implies that N is small, am I right, N is small, the cluster size is small. So, the price that you will pay is that C over I will be low, but the good thing about this is that capacity will be high capacity will be high. So, the tradeoff between capacity and quality comes up and we have to. So, are the constant thing would be is what is the smallest Q with which I can work so that I can achieve the target C over I that is going to be a very very important element.

Now, it may seem kind of obvious, but let me just say it anyway Q becoming larger because the D by R ratio is directly related to N larger cluster size visualize that basically says that my Co channel cells are going to be further away geographically I am not changed the capacity, because the Co channel cells are further away the C over I is going to be better. So, again there is a important understanding of the of the impact of this D by R ratio and we will build on this a little bit more any questions on what we did as the argument for the cluster sizes or the u v coordinate system or the consequent observation of about the are the importance of D by R .

Student: sir (Refer Time: 37:58).

Decreasing R.

Student: Decreasing.

Increasing R, see increasing R what it does is increases the size of one cell if you do not change the cluster size basically what it what will happen is your Co channel reuse ratio does not get affected, right. So, increasing R without changing N will not help us in terms of capacity basically the cells will become larger.

Student: I am saying it would (Refer Time: 38:35) because in the same geographical area they are number of clusters will fall.

Exactly (Refer Time: 38:39). So, increasing the area of a cell means that your cluster area is going to increase you will have fewer clusters to cover the geographical area and therefore, the total capacity of the geographical area will go down correct that is the correct observation. So, let us move into a couple of very very important elements and basically Q is equal to D0 by R Co channel reuse ratio our interest is to calculate the carrier to interference C over I ratio Co channel interference ratio.

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Handwritten notes on a digital whiteboard showing the derivation of the C/I ratio:

$$Q = \frac{D}{R} = \text{Co channel Reuse Ratio}$$
$$\frac{C}{I} = \frac{P_s}{\sum_{i=1}^L I_i}$$
$$\frac{C}{I} = \frac{P_s}{R^{-n} \sum_{i=1}^L D_i^{-n}}$$

where $n = \text{path loss exp}$

Definitions:

- $i_0 = \# \text{ Co-channel intef}$
- $I_i = \text{power of } i^{\text{th}} \text{ interferer signal}$
- $P_s = \text{Signal power}$

So, now I want you to visualize this signal power I have only one source of desired signal that is P_s , now the interference can come from multiples. So, interferers, so let me

call the index as I equal to one through some I naught some number of interference why is my interference number not fixed depends on whether I am including tier 1 only or tier 2, tier 3 also the numbers would increase the cluster size will determine the number of interference it will also determine whether I am sectorizing the antenna lot of things will affect me. So, it is some number some integer number which is i naught times I ; i all the interference powers.

So i naught is equal to the number of Co channel interference and I have to determine which of these are significant and apply the appropriate number and include their powers and I_i is the power of the i th interferer power of the power of the i th interferer interfering signal interfering base station interfere signal and a P_s is the signal power signal power now a very very intuitive question now let us go back and visualize the scenario that we are that we are working with now let us say that I am looking at the interference scenario I have a issue of interference can a happen on the downlink and it can happen on the uplink which is the more difficult scenario for me to work with.

So, downlink means that my I am talking about transmission from the base station to the mobile correct. So, it is in the arrow in the direction of the arrow that I am transmitting and I get interference from these neighboring base stations uplink means the mobile is still at the edge of the cell it is transmitting upwards now it could be the uplink transmission can come from other mobiles right uplink can interface. So, who is who is very close to the edge of each cell and therefore, is causing interference to my base station. So, basically I can think of these 2 positions in cell which could be the ones where the mobiles sitting there could cause interference to me on the uplink.

So, you can visualize the scenario the downlink the interference comes from all the other base stations uplink comes from mobiles which are at the edge of their cell communicating to the base station. Now which of these is the more troublesome see which is likely to have a poorer C over I why do you say downlink very very important because the base stations typically transmit with higher power and therefore, are likely to cause a much worse C over I.

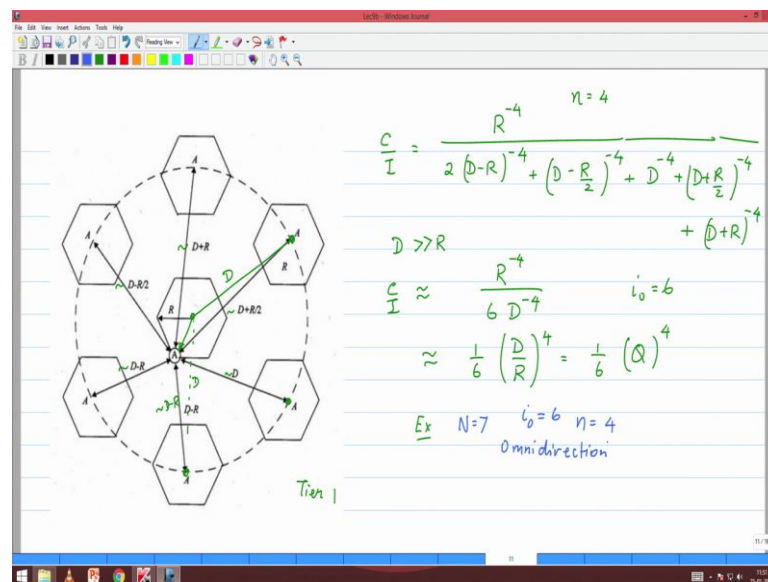
Now, uplink first of all it is a mobile transmitting and the other important factor to keep in mind is that the base station has got multiple antennas which are intended for interference suppression. So, it has ability to deal with interference in a much better

fashion than a mobile itself. So, the downlink is the more difficult situation downlink is the C over I is what we consider because that is the that is the more problematic link downlink is worse than worse than uplink from a C over I perspective. So, always we look at the downlinks scenario. So, let us build on that the assumption.

So basically we are looking at the downlink we are looking at the distance of all the interfering cells. So, here is what it this equation can be in can be written in the following fashion now the desired signal it is at a distance R from the base station. So, basically this is R raised to the power minus N where N is the path loss exponent proportional to path loss exponent and of the proportionality constant will be removed because I am going to take the denominator also is got the same proportionality constant.

Now, the number of interfering signals is the same i equal to 1 to i naught, this will be D i rise to the power minus N the distance of the base station to the mobile. So, we are basically looking at the downlink scenario. So, base station i to the; which is interfering base station to the mobile that is at, I am distance.

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Now, if you look at the diagram this is what it is going to be like if you notice that I have the desired base station which is sitting here this is the desired signal and the black arrows are all the undesired interferers that are coming in.

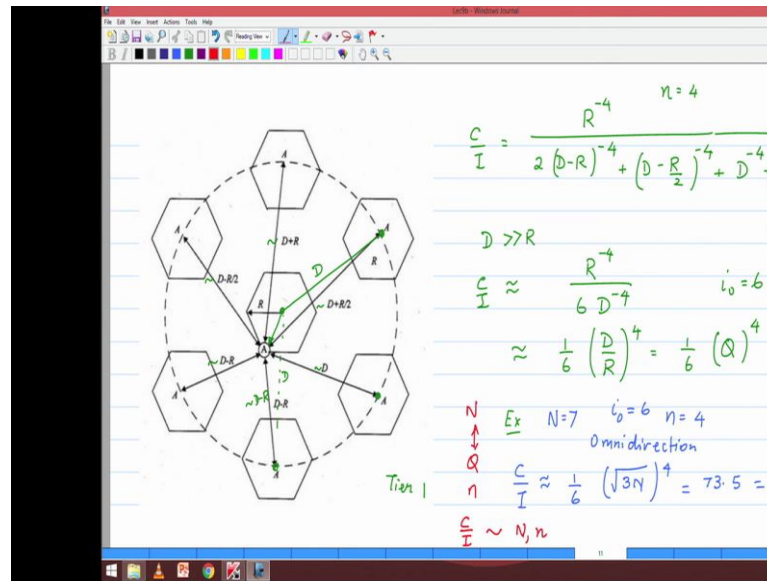
So if I were to write it down very precisely basically if you look at the cellular geometry you can then you can write down the expressions very very now D is the distance between the Co channel cells that is D , you can actually relate the distance between the base station to the mobile to something very close to D . Now in this figure what it says is this particular mobile is at a distance D from this base station from the base station from this base station it is at a distance D .

From this base station it is at a distance D minus R because D is all the way up to here right and this is d and we know that is R . So, this is approximately D minus R . So, this is approximately D , this is also approximately D minus R , this distance is approximately D plus R by 2 approximately this approximately D plus R . So, you know basically in terms of the known terms. So, if I were to ask you to write down the C over I expression for this would be I have a ; I have taken only the tier one interferers this is only tier 1 interferers. So, this would be R raised to the power minus 4 and I have taken path loss exponent of 4 divided by there are 2 base stations which are at approximately D minus R distance which will be D minus R raised to the power minus 4 there is one base station at D minus R by 2 raised to the power minus 4 plus d raised to the power minus 4 plus D plus R by 2 raised to the power minus 4 and 1 more term 2 times well no D plus R raised to the power minus 4 do I get 6 interferers 2, 3, 4, 5, 6, 5, 6, interferers.

Now, if I make the assumption that D is much larger than R which is typically the case if I have large cluster sizes then I can approximate this distance this C over I calculation as R power minus 4 divided by 6 D power minus 4. So, this is the case where the number of interferers is equal to 6 and therefore, I get 1 by 6 D by R raised to the power 4 or D by R raised to the power N notice D by R is the Co channel reuse ratio. So, this is nothing, but 1 by 6 Q raise to the power 4 and this is a quick approximation and this is a way by which we can make a quick estimate of the interference scenarios that we are going to encounter.

So, let us look at one quick example and then I will close with that, but we will pick it up from there take a cluster size let us use a different color N equal to 7 cluster size N equal to 7 only tier one user interference. So, I naught equal to 6 path loss exponent equal to 4 and omni directional radiators omni directional. So, all the base stations on all sides can create interference basically all 6 sides will create interference.

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So, if you apply the approximation C over I is approximately 1 over 6 D by R is root 2 times root 3 N raised to the power 4 . So, this can be estimated this comes out this comes out to be seventy 3.5 which is approximately 18.7 dB. So, the statement that we can make is if I use a 7 cell cluster with omni directional antennas the interference from the first tier; tier 1 interference is going to be approximately resulting in a C over I of 18.7 dB.

Now, very important for us to understand what are all the factors that are impacting the capacity or the C over I ; C over I is being affected by the Q factor D by R ratio it is also being affected by N if it is being affected by Q that that is related to the cluster size N because Q is equal to root 3 N . So, basically C over I depends on N and N very interesting cluster size and path loss exponent and in noise limited systems a higher N was your enemy correct higher N was N because what happened because the signal faded away much faster and therefore, your cell sizes shrank and therefore, your cost of your system increased what happens in the C over I situation.

If your path loss exponent increases, what happens? C over I is D by R , D is greater than R is a number greater than 1 . So, in the case of C over I a higher exponent is your frame why because your signal degrades faster; interfere signal degrades even faster. So, C over I is actually better it is very very interesting. So, in a noise limited system you do not want high path loss exponent, but in a C over I systems oh good you know I am very

happy if the exponent increases because I am going to have better C over I. We will pick it up from here in the next class.

Thank you.