Introduction to Wireless and Cellular Communication
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Lecture - 07
Wireless Propagation and Cellular Concepts
Link budget, Fading margin, Outage

Good morning, let us begin, this is lecture number 7 and a quick recap of what we have covered in the last lecture would be helpful for us. In the last lecture we talked about antennas a little bit from the systems perspective the usefulness of an isotropic antenna.

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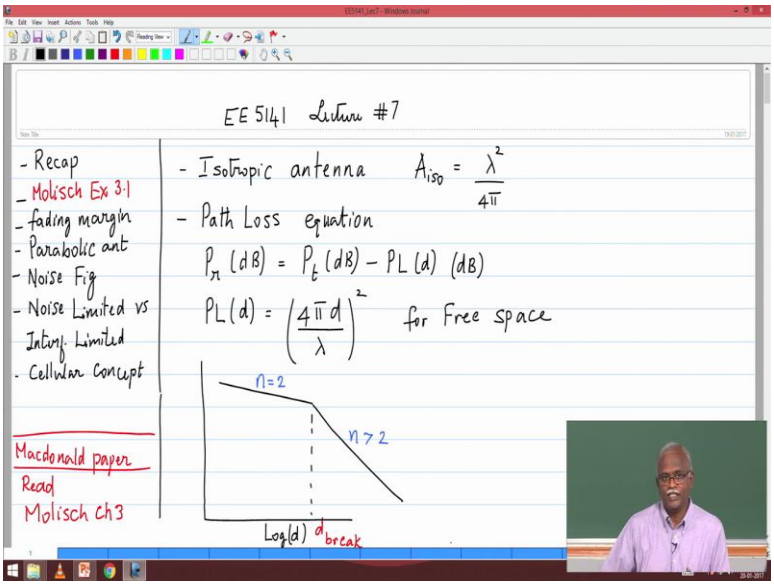

EE 5141 Lecture #7

- Recap
- Molisch Ex 3.1
- fading margin
- Parabolic ant
- Noise Fig
- Noise Limited vs Interf. Limited
- Cellular Concept

Macdonald paper

Read Molisch Ch3

- Isotropic antenna $A_{iso} = \frac{\lambda^2}{4\pi}$
- Path Loss equation

$$P_r(dB) = P_t(dB) - PL(d)\ (dB)$$

$$PL(d) = \left(\frac{4\pi d}{\lambda}\right)^2 \quad \text{for Free space}$$

The one of the things that characterizes is the scale factors which we represent as the area of an isotropic antenna again it is a conceptual term basically it is something that is helpful for us in terms of the characterizing of the wireless link. So, the area of an isotropic antenna if we take it as a reference lambda squared by 4 pi, lambda being the wavelength of the carrier frequency then we derived 2 very important expressions.

One was the representation of the transmission equation the transmitted power minus the path loss being the received signal power again this is an equation that would be represented in dB and I am sure you are comfortable with that way of looking at it because again intuitively when you look at it as path transmitted power minus path loss that gives you the total power that is received. So, if we characterized it in this fashion the path loss itself for free space propagation is 4 pi d by lambda square d is the distance between the transmitter and receiver.

And this is what we used as our baseline and that helps us characterize the transmission from the antenna to some point which we call as d break, we assume that the transmission is line of sight. So, therefore, a typical a AWGN type of transmission can be assumed n equal to 2 free space path loss the only impairment will be AWGN. Then beyond that there could be other of other impairments which cause the signal to attenuate more quickly. So, therefore, we see, so this is received signal power versus the distance we see that initially it decays as a square law and then eventually at a higher exponent.

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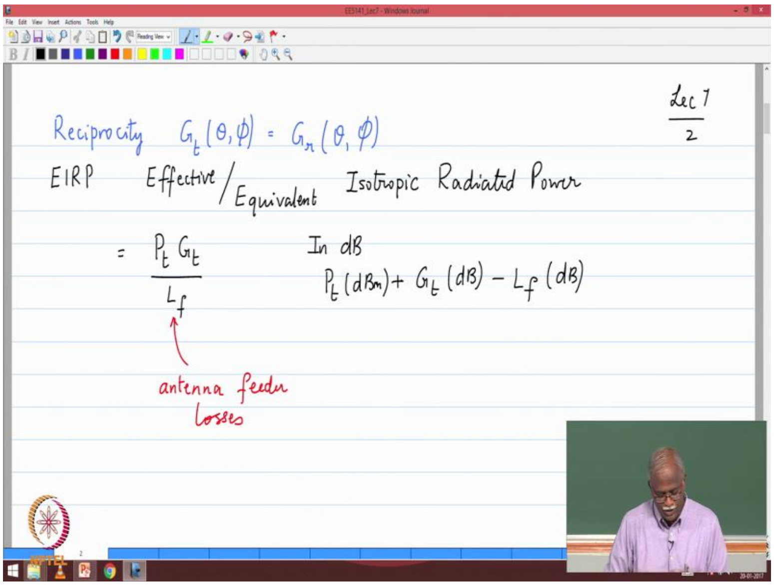

Lec 7
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Reciprocity $G_t(\theta, \phi) = G_r(\theta, \phi)$

EIRP Effective / Equivalent Isotropic Radiated Power

$= \frac{P_t G_t}{L_f}$

In dB
$P_t(dBm) + G_t(dB) - L_f(dB)$

antenna feeder losses

So, this is some of the elements that we had discussed in the last class. We also made a few other observations that typically an antenna would be characterized in terms of its elevation angle theta and the azimuthal angle phi transmitted and received gains are the same we are also interested in the pattern of radiation, we typically have for mobiles omni directional radiation this is different from isotropic which radiates in all directions omni directional is; omni directional in the azimuthal plane. Then we also mentioned that the base stations do not radiate in all 360 degrees they radiate with this specific pattern once you have a pattern then you can talk about a 3 dB bandwidth of the radiation pattern and that more or less defines how which is be the area where the radiation will be focused. So, they equation that we have for us is very much dependent on the transmitted power now there is another point another aspect that I would like you to be very familiar with and be confident of using because transmitted power is what comes out of your power amplifier.

Now, after the power amplifier if you go back and look at the original diagrams or any of the diagrams of a base station you will find that there is an antenna the antenna typically gives you some gain. So, that contributes to the effectiveness of the signal that you have transmitted. So, P t times the gain of the antenna is actually what you will see going out of into the air. So, typically we define a quantity called EIRP effective or equivalent isotropic radiated power more commonly known as effective isotropic radiated power.

So, this is like saying if I compare this with an isotropic radiator what would be the power of the radiator b that basically tells you this is this is the maximum radiation the in that we can expect from this antenna, but we keep in mind that this antenna going to radiate only in a particular direction. So, this is a way of characterizing the maximum power output of the antenna. So, P t times G t is a very useful quantity now in many cases the antenna has to be connected to the power amplifier by means of cables that will introduce some losses for you. So, before you get to the antenna there is some loss. So, that also has to be accounted for in link budgets they are called the feeder losses its L subscript f in a wonder what is where is that what is feeding what it is a power amplifier feeding the antenna the cable that is feeding it and there are losses in those. So, that will have to be taken out.

So, the effective isotropic radiated power which is what I would like you to keep in mind as we look at the link budget equation it will be the transmitted power that is coming out of the power amplifier times the gain which would be a dB term antennas typically we can have 20 dB gain or 15 dB gain depending upon the type of antenna, you have designed. So, that will be the G t that that is a positive factor and then there is a negative factor which takes away from the calculation which is the losses that we encounter in the system.

So, this is quick summary of what we have discussed in the last class, we also started to talk about Molisch chapter example 3 point one I hope you had a chance to look at it we will just quickly summarize the key factors from that example, but before that let me just ask if there are any questions on the points that we have covered. So, far radiated power

EIRP the how the losses come into the picture and how to characterize the breakpoint model.

Let us move on. So, today's lecture, we will touch upon some aspects starting from the example we will also talk a little bit about fading margin I want you to start thinking slightly differently about a fading model and it is more in terms of a statistical quantity and not as a deterministic quantity because fading is something that we cannot anticipate or we cannot tell ahead of time it is a statistical impairment then as I mentioned yesterday we will talk a little bit about parabolic antennas we do not use it on a handset or on a base station, but it is used for any type of microwave backhaul which is also part of the my cellular system.

Then a comment about noise figure again most of you are familiar with the understanding of noise figure, but I thought it would be helpful for us to also get an insight into the impact of noise figure on the system, I thought we will spend a few minutes on that after that we will talk about noise limited systems which is what we have been talking about we basically are saying that there is no impairment the only impairment is the losses that you encounter because of distance between the transmitter and receiver that would be a noise limited system compared to a interference limited system and that leads us into the cellular concept.

So, that is the role out of the plan that we have reading assignment we will definitely encourage you to look at Molisch chapter 3 because that will complete the discussion that we have had. So, far please also read Rapaport number chapter 2 because that introduces cellular concept that is the next topic that we are touching upon and as I mentioned the cellular concept was introduced by a person by name McDonald and his original paper and you know very rarely do we go back and read original papers we do not go back and read Shannon's paper or max loosely because it all happened you know years ago, but this happened quite recently in our lifetime. So, it is good to go and see what was the thinking how did he actually come up with a cellular concept what was the problem that he was trying to solve. So, I am sure you will enjoy reading it please do take the time to do that it will be uploaded in module.

So, let us begin with the discussion for today and we will again answer any questions that come about.

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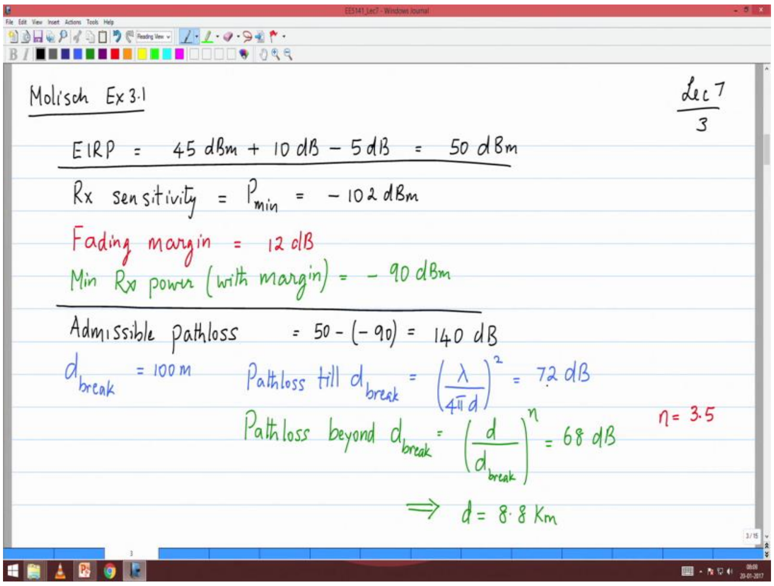

Molisch Ex 3.1

Lec 7 / 3

EIRP = 45 dBm + 10 dB − 5 dB = 50 dBm

Rx sensitivity = P_{min} = − 102 dBm

Fading margin = 12 dB

Min Rx power (with margin) = − 90 dBm

Admissible pathloss = 50 − (− 90) = 140 dB

d_{break} = 100 m

Pathloss till d_{break} = $\left(\frac{\lambda}{4\pi d}\right)^2$ = 72 dB

Pathloss beyond d_{break} = $\left(\frac{d}{d_{break}}\right)^n$ = 68 dB

$n = 3.5$

$\Rightarrow d = 8.8$ Km

Molisch chapter 3.1 example 3.1 I am assuming that you had a chance to look at it, but let us take a quick look the effective radiated power isotropic radiated power would be the power output from the transmitter I think that was given to us as 45 dBm that is a 10 dB antenna gain and a 5 dB feeder loss. So, if you take all of that into account the total EIRP is 50 dBm that is our transmit sign.

Then we move over to the receive side I am sure you would have now. So, basically we are trying to see how much path loss can we encounter. So, this is transmitted power this is received power and what is the loss in between. So, what is the receive level that I need to achieve P min is given to us as minus 102 dBm and we have told that we must account for a 12 dB fading margin. So, which way do I go with the fading margin the minimum receiver sensitivity is minus 1 or 2 I am must make sure that I am coming at a signal level higher 12 dB higher in order for me to have the 12 dB margin. So, if I add 12 dB; that means, the minimum received signal power it is not sensitivity since your receiver would still receive at minus 1 or 2, but you have to design your system. So, that the minimum received power including the margin is at minus the 90 dBm.

So, transmit minus receive is 140 dB we are given that the d break is at 100 meters. So, the path loss still d break is 72 dB notice how much of the energy you have already lost by the time you reach 100 meters and the remaining distance accounts for the 68 dB and given that n is equal to 3.5 I am sure the calculation is straightforward the distance comes out to be 8.8 kilometers.

So, EIRP received signal power 0 sensitivity margin path loss what is in between is path loss any questions let me give you a way to visualize this a whatever we have been talking about and I feel that this is a good way for us to as engineers to visualize the system.

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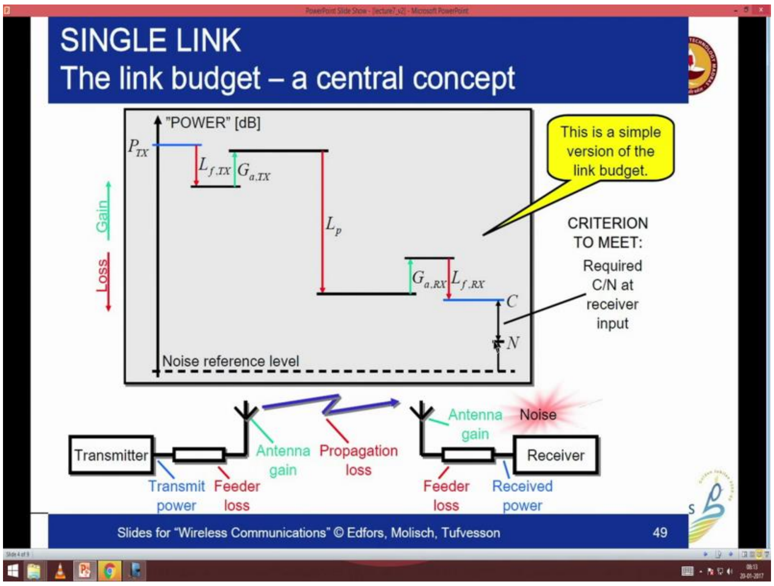

SINGLE LINK
The link budget – a central concept

This is a simple version of the link budget.

CRITERION TO MEET:
Required C/N at receiver input

Slides for "Wireless Communications" © Edfors, Molisch, Tufvesson

So, here is a simplified way of visualizing the link budget you have the transmitter that you can assume that as the output will be the power amplifier that gives you a signal at a certain level. Then you have the cables that connect you to the antenna that will cause a drop in the power because the that is the losses then the antenna gain it may boosted back to its original power or maybe boosted even beyond what the original power was that that is the level at which you transmit.

The other end you have the thermal noise floor, thermal noise floor plus the noise figure basically tells you where your noise power level is going to be located. Beyond that you know that you must have a minimum signal to noise ratio that is going to be your receiver sensitivity. Now what else can happen on the receive side? You may have an antenna gain antenna gain will boost the antenna boost the receive signal sing. So, that gives you a plus there may be antenna connector losses. So, you must account for that that will cause an additional gap most of the time they cancel each other and therefore, you do not see much of a gain; however, as I told you yesterday you may be holding the other phone in a way that actually causes additional gain. So, which means that you know there is an additional loss that you have to account for.

Now, where is the margin coming in? Margin must come above receiver sensitivity. So, basically whatever is the we after you accounted for all the losses and the gains if this is your receiver sensitivity if you now want to include a margin you say I cannot receive at this level I must receive at a level slightly higher that will be the acceptable received signal power. So, that the difference between the transmitted level and what you can accept with the margin is what we want to account what we account for as path loss though the path loss itself has got may have 2 components or 3 depending upon the model that you have used, we have used the simplest model where you have a single break point which is line a line of sight or free space propagation up to d break and then a higher path loss exponent post the d break.

So visualize it as signal level gain, loss, path loss, receive antenna, gain margin and basically you sort of go up and down, but make sure that you have got moving in the right direction. So, that then you get your path loss calculations very correctly. Any questions on why something is up why it is down and probably very intuitive, but if there was something that is not clear that is k times t Boltzmann constant times t. So, that basically sets your electronic noise floor right and the if you multiply it by the bandwidth of the receiver that tells you basically where you will have to set your threshold beyond that there is noise that your receiver will add that that gives you the that is the additional noise level. So, from the what is the ambient noise for the particular receiver bandwidth that you are looking at that gives you this threshold additional noise that your receiver electronics will add that gives you the noise reference point or Pn p N a noise power level in your the receiver. So, very important, but this is a calculation that we had done in the previous lecture.

So, basically it will be K T B that will be this line plus noise figure gives you Pn that is this line good now I want to want I want you to look at one more aspect.

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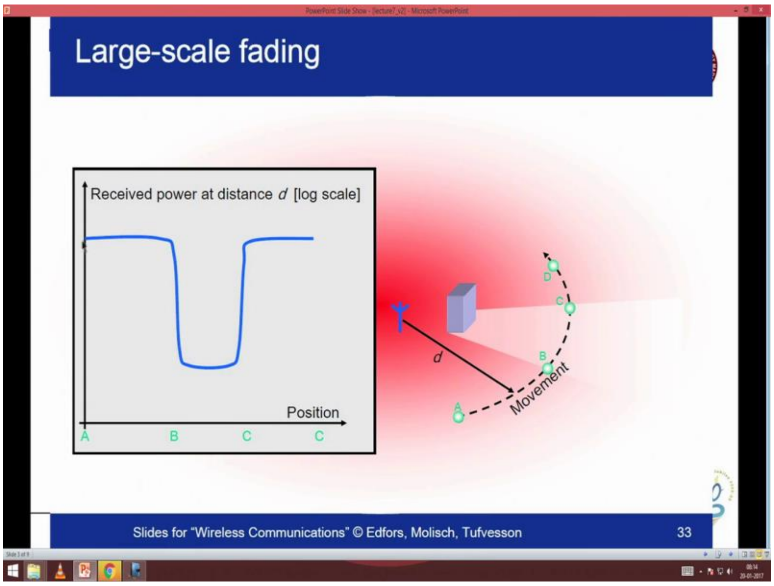

Large-scale fading

Slides for "Wireless Communications" © Edfors, Molisch, Tufvesson

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So, here is a base station you can see the base station here and let us assume that you are a mobile that is moving in an arc with a certain distance d very simple scenario except that there is a building here. So, when you are at the position A you are able to c the base station quite clearly. So, the receive signal power is high as you move to b still slightly you start to see a drop then between b and c there is a complete drop in signal level there is a obstruction then when you go back towards b the signal goes up very very simple scenario.

Now, this is what happens in reality as well because there are obstructions things that we do not necessarily know or plan for those cause a degradation in the signal. So, when I move from A to D, I cannot assume that the signal level is going to be what it is there at any one of those points I have to say that there is a possibility of fluctuation of the signal level and it is something that I have no control over. So, what I have to say is there is a notion of an average signal level P r bar average signal level, now P r is a random variable which is got some distribution around P r bar.

So, again we will introduce a notation a little bit more formally, but just sort of think about it. So, the whole idea of our discussion is to say that I have a received signal power.

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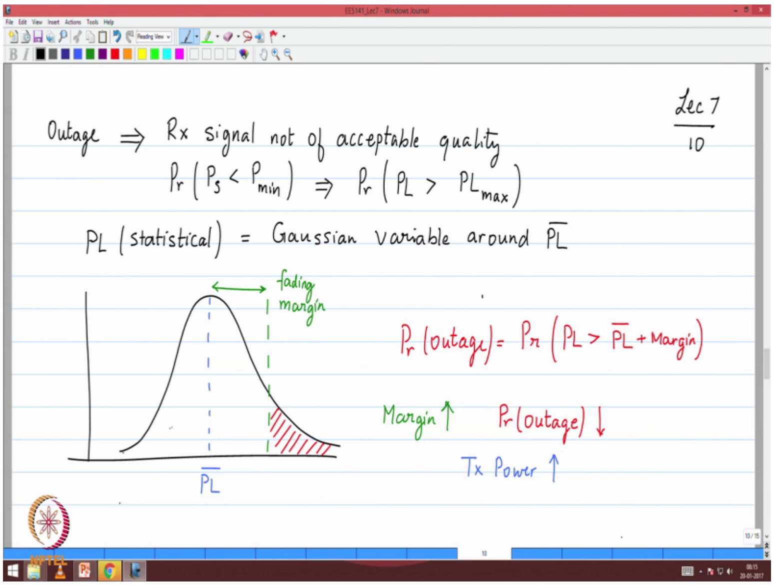

Lec 7
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Outage $\Rightarrow$ Rx signal not of acceptable quality

$$\Pr(P_s < P_{min}) \Rightarrow \Pr(PL > PL_{max})$$

PL (statistical) = Gaussian variable around $\overline{PL}$

$$\Pr(\text{outage}) = \Pr(PL > \overline{PL} + \text{Margin})$$

Margin ↑ $\Pr(\text{outage})$ ↓

Tx Power ↑

The; or you can think of it as average path loss average signal power is corresponds to average path loss as well. So, average path loss is what I expect to see, but there is a statistical variation and just for ease of visualization I have shown it as a Gaussian distribution around that in and very often it actually fits a model that is close to Gaussian. So, it is good for us to sort of use that as a baseline model.

So, if I design my system to have a path loss that is aligned with path loss bar I have designed it for average path loss and if I did tell you that the definition of outage means the signal is not of acceptable quality the outage scenario is when the signal goes below the there is receive the acceptable quality. So, outage basically means that the path loss it your actually seeing is greater than path loss max and you have designed path loss max for your scenario s PL bar what is the probability of outage Gaussian distribution what is the probability of power voltage.

Student: Integrate (Refer Time: 18:02).

Integrate from PL bar to infinity with respect to the whole thing what is it is one half. So, fifty percentage of the time we are going to be an outage now that is not a acceptable situation. So, what would you then rather do and say that no I am I cannot design my system for 50 percentage outage I must design it for ninety percentage outage or ninety percent outage, so 95 percent of the time if the signal should be of acceptable quality. So, which means that I cannot design the path loss to be PL bar I must allow for a design that is greater than PL bar. So, if I have moved my design point or the path loss allowed to this levels basically I am I have designed for this point then the lightly hood of voltage decreases because only if the path loss exceeds the green line I am going to run into outage. So, you can notice that the shaded region is what pushes me into outage.

so the whole notion of margins this is not some fix number or some rule of thumb which somebody says no its 10 dB is 20 dB, no, it is a statistical understanding of the channel and saying that the larger the margin the better is going to be or lower is going to be my probability of outage. So, again these are all interesting things for you to keep in mind, but I wanted you to start thinking about the whole notion of margins the fading aspect and how do you account for it and of course, now keep in mind that this fading comes from 2 sources one is shadowing which we saw the building type of obstruction and then the other one is the multipath and constructive and destructive addition of those signals. So, that is the second element that I want you to keep in mind.

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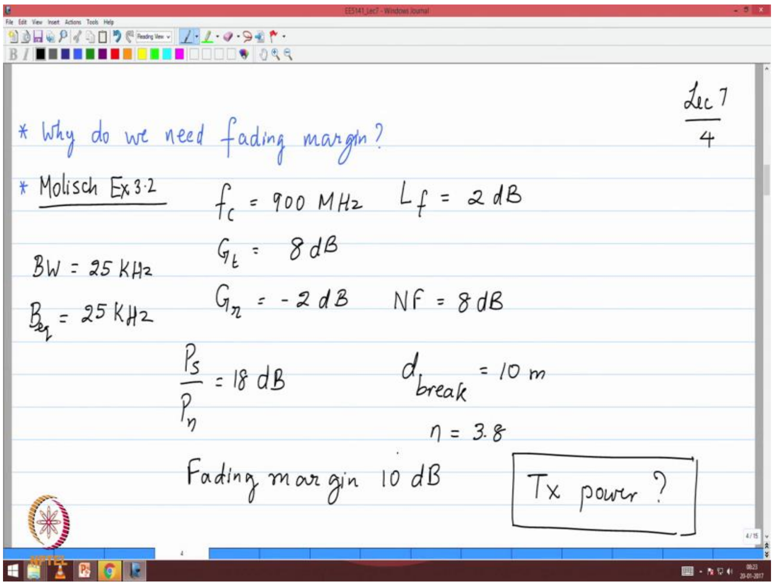

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* Why do we need fading margin?

* Molisch Ex 3.2

$f_c = 900$ MHz $L_f = 2$ dB

$G_t = 8$ dB

BW = 25 KHz

$B_{eq} = 25$ KHz

$G_r = -2$ dB NF = 8 dB

$\frac{P_s}{P_n} = 18$ dB

$d_{break} = 10$ m

$\eta = 3.8$

Fading margin 10 dB

$\boxed{\text{Tx power ?}}$

So, why do we need fading margin I would expect you to answer it in a very precise manner referred to outage its being a statistical quantity and the way to handle the statistical aspect is to allow for margins and at the end of the day what we can achieve is a is percentage outage you cannot guarantee 100 percentage, I would also encourage you to look at Molisch example 3.2 it I will just give you the parameters again I am sure you can refer to it.

This is a problem which very similar to 3.1, but it is a very good practice let me just highlight the points that are important for us and do take the time to. So, the carrier frequency is given to us as 900 megahertz very close to the previous case the transmitter gain given as 8 dB the receiver gain minus 2 dB; that means, it is an antenna that is inside and you are holding it in a funny ways is a gain is minus 2.

The feeder losses at the transmitter side we are given as 2 dB and the noise figure of the receiver is given as 8 dB we do need to understand the baud rate. So, it is a system with a bandwidth of 25 kilo hertz which also tells me that my baud rate is 25 symbols per second. So, the equivalent bandwidth that I need for my translation from e B by I am not to Ps by Pn that is taken to be 25 kilo hertz as well I want to achieve Ps by Pn of 18 dB Ps by Pn of 18 dB and we are told that us to use a breakpoint model with the d break of 10 meters. Again maybe just a quick comment on this previous example was 100 meters now you may wonder why do you take ten meters is just to check whether we are doing the math correctly no when will you get ten meters only line of sight.

Student: Urban areas.

Urban areas or your; you know mounting it in a campus environment without too much of a height the antenna. So, basically after 10 meters you are going to hit obstruction when we look at 100 meters in some rural area or you know where you put the antenna on top of a very tall mast. So, 100 meters it sees without any obstruction. So, again it is not just another exercise there is a reason behind the points that are mentioned beyond the d break we are asked to use a path loss exponent of 3.8 and introduce a fading margin of 10 dB.

The question that is being asked is what is the transmitted power EIRP that is the question and usually this is how this is a more important type of question that arises given all these scenarios what is the received signal power oh sorry transmitted signal power what I want to guarantee is the signal to noise ratio at the receiver Ps by Pn and I want to it give a fading margin as well. So, what should be my transmitted power? So, again keeping in mind that we are talking about EIRP, you can adjust the EIRP by increasing the gear the gain of the antenna and reducing the power amplifier output or it does not matter again what we are asked here is only the EIRP. So, that is enough to get a question.

Student: Need a (Refer Time: 23:51).

Well the basically it says I must change Ps by Pn after in distance when we have distance well if you were told the transmitted power you can calculate you probably need the distance is a very good question you do need the distance of the cell it is there in the in the problem (Refer Time: 24:13). So, this is very good example for us to take a look at and solve and if there are any questions we will clarify it in the class, but I think once you get the basic feel for it this should not be a problem at all.

I want to spend a few minutes talking about parabolic antennas again as I mentioned these are used in a number of applications and communications.

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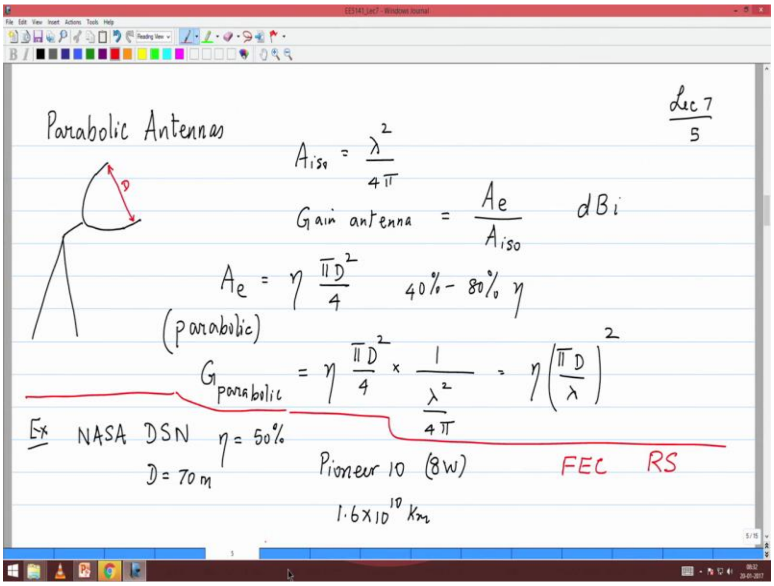

Parabolic Antennas

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$$A_{iso} = \frac{\lambda^2}{4\pi}$$

$$\text{Gain antenna} = \frac{A_e}{A_{iso}} \quad dBi$$

$$A_e = \eta \frac{\pi D^2}{4} \quad 40\% - 80\% \ \eta$$

(parabolic)

$$G_{parabolic} = \eta \frac{\pi D^2}{4} \times \frac{1}{\frac{\lambda^2}{4\pi}} = \eta \left(\frac{\pi D}{\lambda}\right)^2$$

Ex NASA DSN $\eta = 50\%$

$D = 70\ m$

Pioneer 10 (8W)

1.6×10^{10} Km

FEC RS

So, let me just take a minute to sort of get you excited about wireless because this is you know something very very good for us. So, as I mentioned earlier the area of an isotropic antenna is lambda squared by 4 pi and we always characterize the gain of an antenna in the following fashion you take the gain of your own antenna that that you have and compare it to the gain of an isotropic antenna. So, basically it is equivalent area that is what will be the area that what will be the effective received signal power or the gain of your antenna divided by a i s o this like saying you know what is your gain relative to the gain of a isotropic antenna and if you were to express it in dB you will also see a dB I represented there and it that says it is dB with respect to an isotropic radiator. So, typically the gain of an antenna will be 8 dB or correctly speaking it should be 8 dB i.

So the effective area of a parabolic antenna the cross section is a circle, so the effective area of a parabolic antenna. So, basically this is parabolic antenna this would be pi d squared by 4 and if you remember this is effective area. So, the entire area does not contribute. So, basically there is an efficiency factor that must be accounted for and typically it is in the forty efficiently forty percentage to 80 percentage range depending upon how smoothly you have done it how perfectly you have made the parabolic reflector all of that accounts for it. So, if I now were to ask for the gain of a parabolic antenna g of a parabolic antenna this would be eta times pi d squared by 4 into 1 over lambda squared by 4 pi so basically the effective area of a parabolic antenna divided by the area of an isotropic antenna. So, this basically I am sorry.

Student: What is (Refer Time: 27:13).

The eta the efficiency of a parabolic reflector would be any antenna is in that range. So, the cross sectional area times the efficiency of the antenna gives you the effective effectiveness of the antenna. So, this is given by eta times pi d by lambda whole squared. So, this is typically how you would characterize the gain of a parabolic antenna and what I want to share with you is a example of a parabolic antenna the largest parabolic antenna that we have built and some amazing stuff that has happened with then.

So, when you get a chance please do look up something known as the NASA deep space network, NASA deep space network consists of 3 antennas and they are located in 3 different parts of the world and one of the interesting elements that you would notice is the is the size of the antenna.

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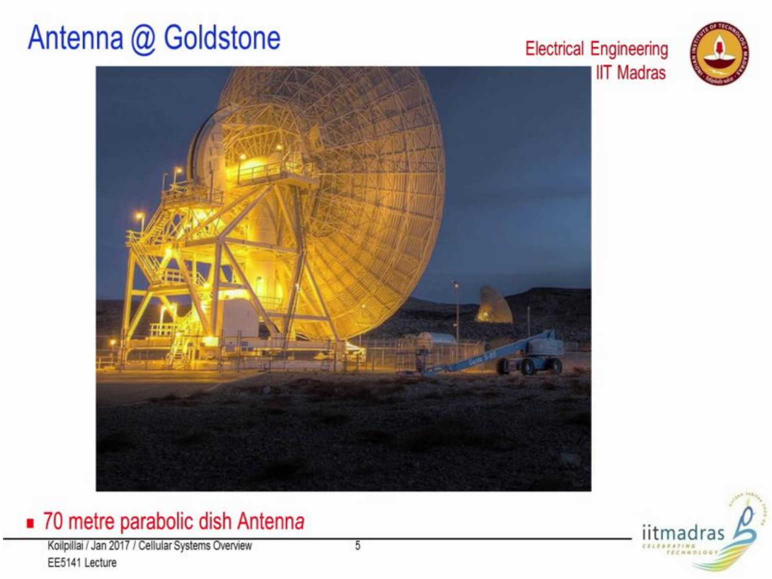

Antenna @ Goldstone

Electrical Engineering
IIT Madras

- 70 metre parabolic dish Antenna

Koilpillai / Jan 2017 / Cellular Systems Overview 5
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So, this is what the antenna looks like one of the antennas looks like it is in (Refer Time: 28:28) sitting in the desert because you know does not want to have a minimum amount of interference from other sources the diameter of this antenna is seventy meters if you have gone to the stadium and you look at 100 meter track from one end to the other this is almost the size of that. So, it is a huge antenna and it weighs 2.7 million kilograms 2 points and that is the dish is what that is how much it weighs because it is basically metal and you can think of what it takes to build this type of an antenna in 1960s basically this was when NASA was doing their deep space mission they knew that they were going to launch a spacecraft that will go into beyond out of the solar system they needed to track it. So, basically they design a 3 antenna system where you know 3 parts of the world one in Spain, one in Australia, one in the US and it is as a communications engineer when you when you when you actually go and see when you go and study these things absolutely fascinated by what it can do.

So, let us just look at the specifications of this it achieved an efficiency of 50 percent when you design such a large parabolic dish it cannot be a single sheet of metal it is individual sheets of metal then that are stitched together. So, of course, you do not get a perfect finish. So, the this is about 50 percent is what and the diameter is 70 meters that is a huge antenna and it was it can operate in a wide range of a of carrier frequencies basically it was designed for 2 gigahertz to 4 gigahertz and probably the record that its set was for a spacecraft called pioneer ten pioneer 10 look it up very interesting it is a spacecraft that went out of our solar system many many years; many many years ago, but the last communication from pioneer 10 by the pioneer 10 had an 8 watt transmitter just 8 watts and the last communications was when pioneer 10 was at a distance of 1.6 into 10 power 10 kilometers from ground station and the deep space network actually picked it up and decoded the signal.

It is one of those things where you know you say oh this is absolutely amazing because the eight watts is lower than most of these bulbs and just imagine that is the power at which you transmit from a distance of 10 power 10 kilometers and actually the space network, but the amazing thing about it is not that that is one element of it I want you to one more element of this one is the error correcting codes. Of course, this able to achieve this because of the error correcting code I am sure you have studied for there a correction right and your studied reed Solomon codes very very powerful codes, but the interesting thing is when pioneer 10 was launched reed Solomon codes were not yet invented.

So, what did they do after reed Solomon codes were invented and they were verified and they were proven to be the most powerful codes from the ground station they actually programmed the microprocessor on board to do reed Solomon code. So, in space when it was outside you know gone past mars and Jupiter they actually programmed it to do reed Solomon code. So, then it switched over from convolutional codes that it was doing to reed Solomon codes and then actually was able to communicate and that is how the last communication was with ground.

So, again wireless communications is about some absolutely fascinating stuff and hopefully you will get excited and read, but how things is a little bit less ambitious we are you know 3 kilometers 2 kilometers is what we want to communicate reed Solomon is good, but convocation course is good as well. So, I think it is we come back to ground and start talking about terrestrial cellular. So, but I thought its always interesting for you to get a perspective that this is absolutely exciting stuff you can do things which are it looks like science fiction, but you know that that is wireless communication

So, let us move on to our understanding of noise figure because this is also important because it helps us keep the practical aspects in mind can you give me the definition of noise figure what is noise figure as text book define

Student: Input SNR.

Input SNR, thank you. So, basically it is SNR at input.

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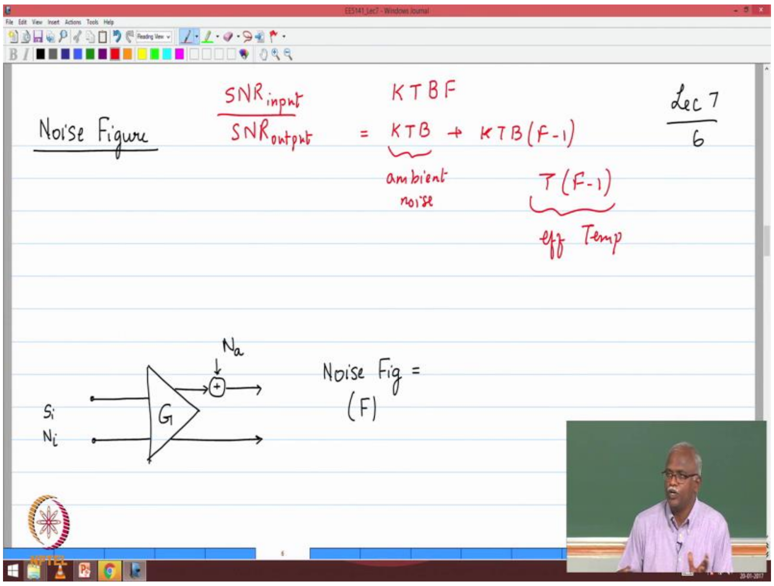

Noise Figure

$\frac{SNR_{input}}{SNR_{output}}$

$$KTBF = \underbrace{KTB}_{\text{ambient noise}} + KTB(F-1)$$

$$\underbrace{T(F-1)}_{\text{eff Temp}}$$

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Noise Fig = (F)

Let me this write it in short form SNR input by SNR output now if this number is greater than one that automatically tells you that the output SNR is less than input SNR. So, in other words somehow your receiver has added to the noise that is already present in the system. So, the total noise at a receiver which we I have understood it to be K T B F, K is the Boltzmann constant T is the ambient temperature B is the bandwidth and F is the noise figure that is how you have calculated and that is how we have done it

Now, I want you to visualize this in a very interesting and novel manner to split is as K T B that is ambient noise right ambient noise without the receiver this is what is there if you would look at the bandwidth that is present for you plus K T B F minus 1 K T B F minus 1. So, this is what is referred to as whatever is there in the ambient this is what got additionally added because of your receiver. So, this T times F minus 1 is a very important quantity because there call this as the effective temperature of your receiver effective temperature and this total K T B F minus 1 is the effective noise temperature of your receiver or the effective contribution of the noise it is as if your operate receiver was operating not at ambient, but at a slightly higher temperature and you want to sort of account for that. So, ambient noise plus this noise together is the effective temporal have total noise and at the end of the day is K T B F, so no confusions about that.

But let us see if we can understand this a little bit better in terms of the figure on the left hands corner. So, think of it as a simple amplifier amplifiers also add noise. So, this whole issue of noise figure can be easily understood in the context of an amplifier. An amplifier with gain G input signal power S i input noise power N i and there is some additional noise which shows up at the output which I call it as N a. What will be the total signal power at the output? S i times G. What is the total noise power at the output? G N i plus N a N is the power noise power that of the signal that is added at the other side. So, let us write down the definition of SNR input by SNR at the output, SNR at input would be S i by N i divided by SNR at output would be G times S i divided by G times N i plus N a.

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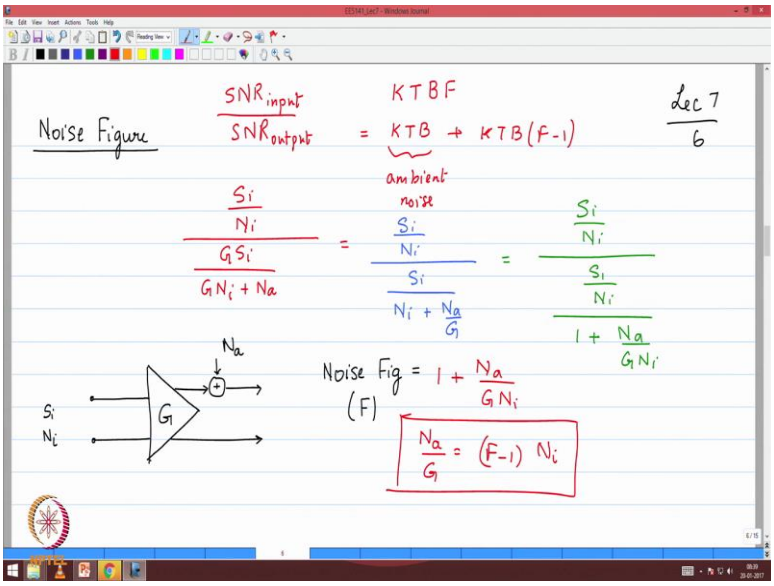

Noise Figure

$$\frac{SNR_{input}}{SNR_{output}} = \frac{KTBF}{\underbrace{KTB}_{\text{ambient noise}} + KTB(F-1)}$$

$$\frac{\frac{S_i}{N_i}}{\frac{GS_i}{GN_i + N_a}} = \frac{\frac{S_i}{N_i}}{\frac{S_i}{N_i + \frac{N_a}{G}}} = \frac{\frac{S_i}{N_i}}{\frac{\frac{S_1}{N_i}}{1 + \frac{N_a}{GN_i}}}$$

$$\text{Noise Fig (F)} = 1 + \frac{N_a}{GN_i}$$

$$\boxed{\frac{N_a}{G} = (F-1)\,N_i}$$

Now do some simple juggling of the expression factor out the g this can be written as I just change color. So, it is easy for us to see S i by N i divided by S i by N i plus N a divided by g this has 2 important implications if you wanted to reflect the noise of the amplifier to the input side what should be the power level we should be N a divided by G because that would also get amplified by the amplifier. So, this gives us a good way to visualizing is there was already N i power N i noise power present N a by g also got added and then together this thing got amplified and that is how we got this a got this a expression.

Now, take it one more step basically a factor out N i on both sides. So, it let me use one more color let me use green. So, this can be written as S i by N i, I am going to factor out N i. So, then I get S i by N i divided by 1 plus N a by G N i. So, S i by N i will of course, cancel that gives me a very very important result the that the noise figure is one plus N a by g N i or N a by G is equal to f minus 1 times N i that is a very very useful way to visualize. And again this f minus 1, why is f minus 1 coming into the picture because one component comes from ambient that is already present. Now what is a newly getting added depends on f minus 1 but together the effect of the total noise is f I can split it just for visualization in this fashion everybody is clear on this on this aspect.

So, now, we move to the realistic situation where you do not have just one amplified, but you have several cascade of components which will give you the following aspect.

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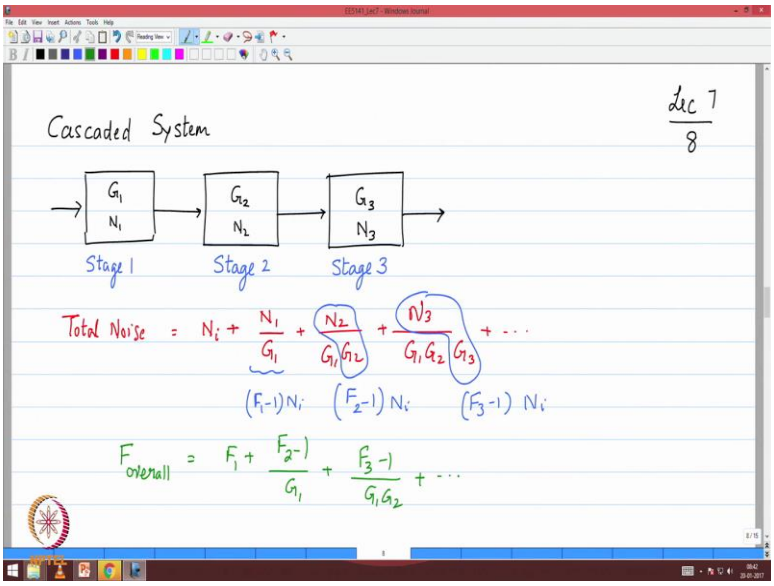

Cascaded System

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$$\text{Total Noise} = N_i + \underbrace{\frac{N_1}{G_1}}_{(F_1-1)N_i} + \underbrace{\frac{N_2}{G_1 G_2}}_{(F_2-1)N_i} + \underbrace{\frac{N_3}{G_1 G_2 G_3}}_{(F_3-1)N_i} + \cdots$$

$$F_{\text{overall}} = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots$$

So, the first system let us say has got gain G 1 with noise power N 1 G 2 N 2 G 3 N 3. So, the way to visualize this, this switch scenario is I want to look at the total noise total noise. So, total noise will be N i that is what is coming in at the input first stage if I want to reflect it to the input side it will be N 1 by G 1 then it will be N 2 by G 1 G 2 plus N 3 by G 1 G 2 G 3 and so on plus dot, dot, dot again if reflect all of them to the input side. What is N 1 by G 1? It will be f 1 minus 1 correct times N i.

What will N 2 by G 2 be? It will be f 2 minus 1 times N i and 3 by G 3 will be F 3 minus 1 times N i. So, now, if I were to rewrite the equation and you know pull out the N i factor and then look at the effective noise this is what we get and this is the famous equation that you would have been familiar with which says that the overall noise figure when I have a cascade is equal to F 1; that means, if there was no other components f one that is traditionally how we have understood receivers to function, but if there are cascaded terms the second one will contribute F 2 minus 1 divided by G 1 f 3 minus 1 by G 1 G 2 dot, dot, dot.

So, the noise the interpretation of this expression is f 1 I cannot do anything about f 1 is basically the if I did not have a cascaded system the noise figure of my front end becomes a noise figure of my receiver; however, if I have a cascaded system the noise added by these subsequent sections are down or inside downgraded to some extent by the gain of the preceding terms. Why is that? Noise at a certain level if the preceding stage has already amplified my signal by G 1 i s like saying keeping g at the signal the same level and suppressing the noise tool by G1.

So, second stage noise is suppressed by G 1, third stage noise is suppressed by G 1 G 2 if you keep the input signal level at the same point. So, this is a very very useful visualization and again this is for cascaded systems and if you do have to work with it.

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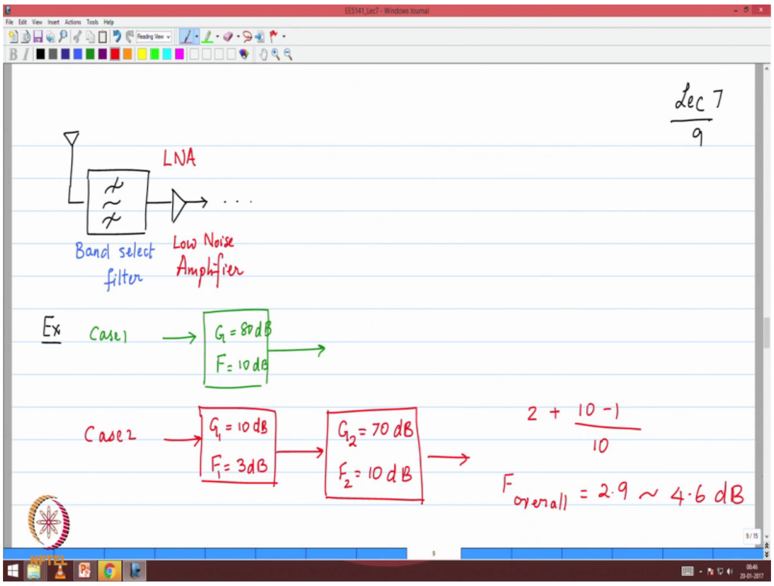

So, when you take a look at any practical receiver the whole idea of this discussion was to make sure you we come to this stage you will see that there is an antenna and this is the way we describe and RF filter 3 lines representing the high frequency, the low frequency and the middle frequency if I cancel out the high and free low I what I am left with is a band pass filter.

So, this is basically a band pass filter because it allows the middle frequencies to pass does not tell us what those middle frequencies are, but it tells you that one minutes if you look at this you know that is a band pass filter the first electronic stage that you will counter after the band pass will be a low noise amplifier and the reason for that is because this is what is going to decide the overall noise figure of the system, all subsequent electronic components are going to be suppressed in terms of their noise contributions by the gain of the low noise amplifier. So, the LNA is very very crucial in our design maybe a simple example will give us a way to appreciate that.

Let us look at case one case one I have to design a receiver with 80 dB gain; G is equal to 80 dB. So, I have to have a lot of a gain stages to get the signal to the received level and the noise figure that I have is 10 dB. So, what will it have, what will it do this is a receiver that will degrade whatever is my input noise of signal to noise ratio by a factor of 10 dB. So, again now I have a case 2 scenario, case 2; I have I am going to design it as a cascaded system I am going to have G 1 very very modest 10 dB gain only, but it is a very low noise amplifier I have (Refer Time: 44:01) to be noise figure of 3 dB followed by a second stage where I have a gain of 70 dB I have to get overall gain 80 dB. So, I have got (Refer Time: 44:13) second stage of 70 dB.

And this is less like the previous one noise figure of ten dB at a glance they do not look too different you know 80 dB; 80 dB first one 10 dB noise figure this one you may say oh first stage you did a very good design, but second stage look you messed it up by putting a 10 dB noise figure amplifier stage are you know stage. So, which means that well way dominate this is a cascaded system. So, therefore, the effectiveness or the effective noise figure is 3 dB corresponds to 2 plus 10 dB corresponds to 10. So, its 10 minus 1 divided by 10 the again of the first stage is 10. So, the effective f effective or f I think we use the term overall is 2.9. Much different from the earlier case this corresponds to 4.6 dB. So, without doing much you have actually improved your performance of your receiver by 5 dB and 5 dB is worth a lot in cellular systems those of you are doing research know that even 1 dB makes a huge difference. So, that is important element that I want you to keep in mind.

The next point that I wanted you to think we have talked about fading margin.

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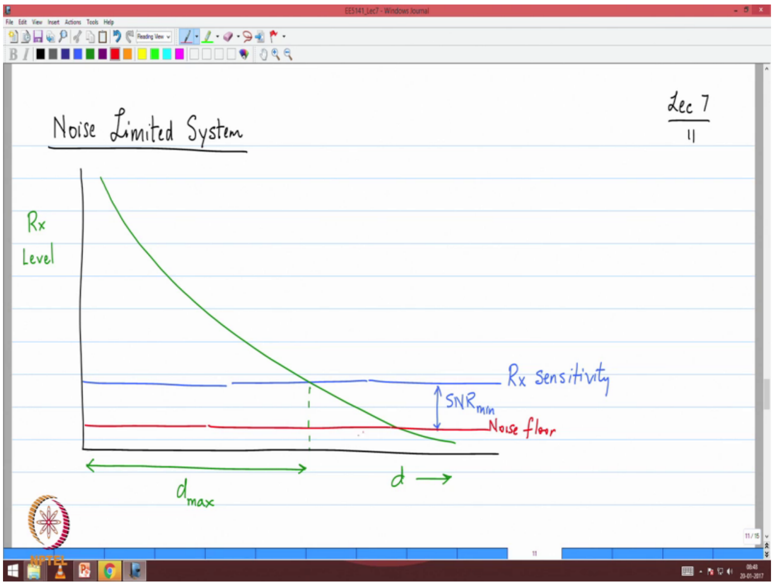

I want you to quickly introduce the concept of a noise limited system. So, all that we have learnt so far I want you to put it together in a single framework. So, the framework is that we do have a certain noise floor over which we do not have too much control over. So, notice that basically the red line tells us the noise floor means all the clever designs that we have done cascaded systems and all of that together tells us this is where my noise level is basically its K T B, K T B F on top of that you must maintain a minimum signal to noise ratio SNR min that tells us that you have to have the received signal level at least at the blue line.

On top of that if you want to introduce a margin that blue line will shift a little further up assuming that you know you know what the impact is. So, that is where you say that that is. Now the received signal power starting from the close to the base station or to the mobile or from the mobile to the base station starts off at a high level and d case with some exponent again I have just shown it as a single line it may have a breakpoint does not matter it decays with distance with some exponent now at some point this green line is going to cross over the blue line; that means, your received signal power is going to be below what you consider as the minimum received signal power for your system it is receiver sensitivity if it is there are no margins it will be higher if there are margins. But at some point it is going to cross the blue line and that is going to be the maximum range that you can cover with your system with a given amount of reliability.

This is a picture of all that we have talked about so far, is this picture clear, any anything that is not clear about this picture. Because if this picture is clear then we are ready to talk about the next concept which says that if I have another base station that is using the same frequency that is the purple line.

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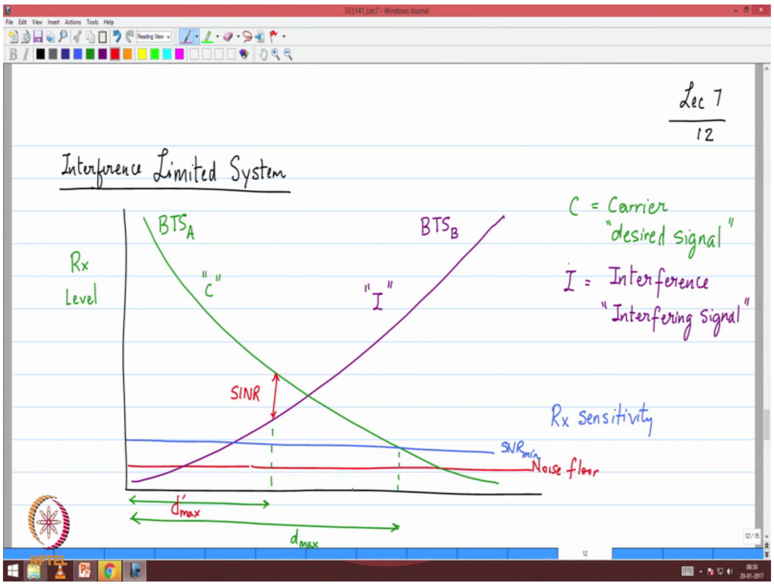

Now, the purple line is going to decay as distance from the this base station is going and it will count the distance from this base station from the right side and you find that at some point this purple base line will also cut the blue line, but probably long before it cut the blue line it actually intersected the green line. Now that point of intersection between the green line and the purple line what does that tell you, if I am a mobile sitting at that distance from I am equidistant from both base stations assuming both are transmitting in the same power level I am receiving this base station signal with the same power as this base station this is desired signal this is interference I have no idea what this base station is talking about what is my signal to impairment ratio.

Student: (Refer Time: 48:47).

1 or 0 dB almost none of our communication systems work at that level. So, which means that I this I cannot even come as far as the intersection point, I have to stop at that

point where my green line is it has to be only at a sufficiently high level above the purple line the minute it gets less no communications will not work. So, whatever was your that minimum signal to noise ratio now has to be defined in terms of signal to interference plus noise ratio SNR is signal to interference plus noise you do not care whether its interference or noise or both, but the effect of impairment the denominator term with respect to the numerator term has to be at a certain level.

So, this tells we that what used to be d max previously now may be much much smaller this is an interference limited system compared to a noise limited system. So, when I start using multiple base stations with the same frequency I run into an interference limited scenario. So, what is a limited interference limited scenario when does it arrives that comes from McDonalds concept that is called the cellular concept as he envisioned it is a paper that was written in 1979, it will be uploaded in module take a look at it we will discuss it in detail in class. It is absolutely from an engineering perspective it is brilliant piece of work very very elegant very simple and you say well I would have thought about it that is good, that is a way you should have people should write papers and come up with new concepts.

Thank you.