Introduction to Wireless and Cellular Communication
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Lecture – 55

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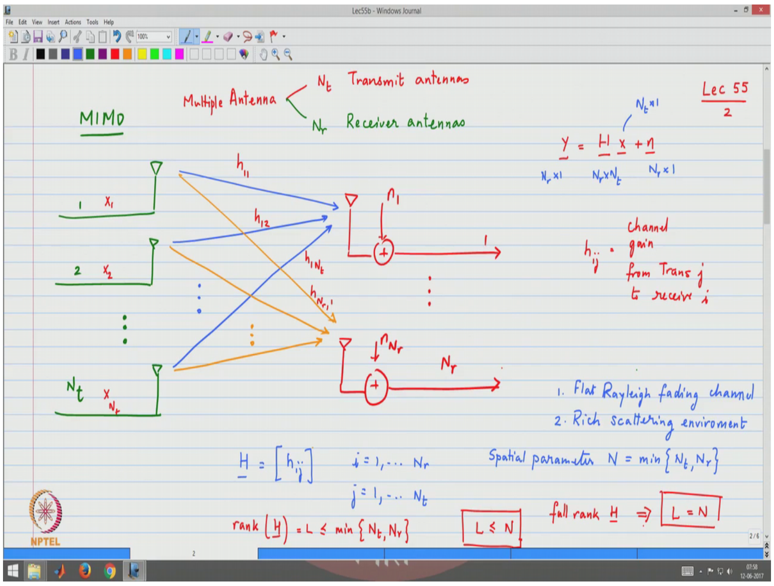

Good morning and welcome to lecture 55, todays lecture we will be covering a detailed discussion on MIMO capacity. A part of this was begin in the previous lecture and todays goal would be to complete that discussion and also to provide a brief introduction to the use of coding along with the multiple transmit and receive antennas.

So, let us the framework is the MIMO framework that we have been looking at the past few lectures we have a set of Nt transmit antennas, follow at the receiver we have Nr receiver antennas and we have rich Rayleigh scattering environment and this gives us a MIMO of framework which is given by the following equation, where you have the receive vector is equal to H times x plus noise and the different dimensions Nt cross 1 Nr sorry n this is Nr cross 1, Nr receive vector Nr across Nt x is got a Nt cross 1 Nt transmit antennas and the noise vector is 1 per receive antenna which will be again Nr cross 1.

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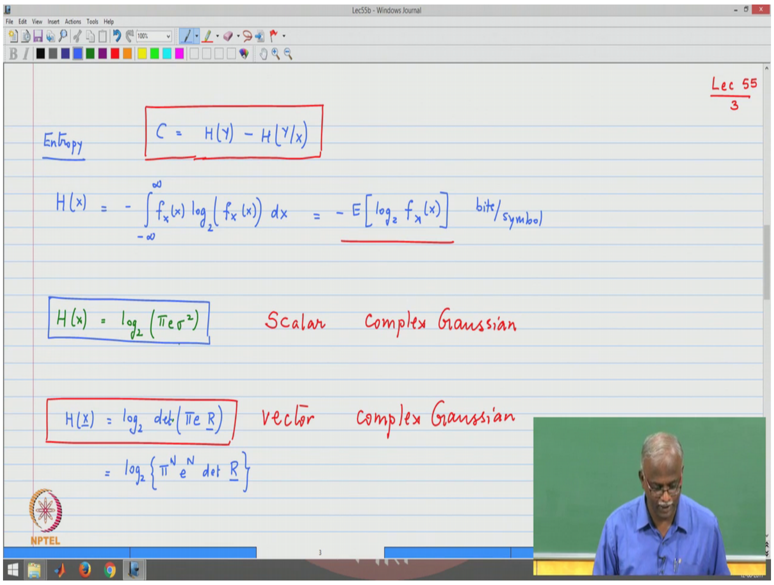

Given this framework we are now trying to look at the case of capacity in the context of a system, where the transmitter does not have knowledge of the receiver because in the case were the transmitter has knowledge about channel, then we can do the singular value decomposition of the channel which then creates the Eigen decomposition and then the parallel channel transmissions that we can do by using precoding at the transmitter and then followed by the post processing at the receiver.

Now, in the case when there is no knowledge of the channel at the transmitter, we are looking at it more in the form of the ergodic capacity. For that we would like to look at the definition of entropy and the entropy is defined in or the mutual information is defined in terms of H of y minus H of y given x. So, that is the form that we are going to be working with, and we would like to derive the results based on based on that ok.

So, now we did 2 cases in the last lecture I just like to refresh your memory and using that we will build on today's lecture. The first of the first of the results was for the case of a scalar complex Gaussian using the pdf and using the expre the understanding that the expression for the entropy is given by minus expected value of logarithm base 2 of the pdf. Using that expression for a scalar complex Gaussian we got the expression to be logarithm base 2 pi e sigma square sigma square is the variance of the Gaussian random variable then moving on from the scalar to the vector case the entropy of the vector resource is given by logarithm base 2 pi e times R determinant of pi e times R.

So, with this as the framework we would now like to continue our discussion and build build on that. So, this form can also be written in the following way, this can also be written as logarithm base 2 bringing out the pi and the e pi power N e power N determinant of R. So, given this expression where R n is the dimension n cross n is the dimension of R. So, this form is also useful for us sometimes in our discussion.

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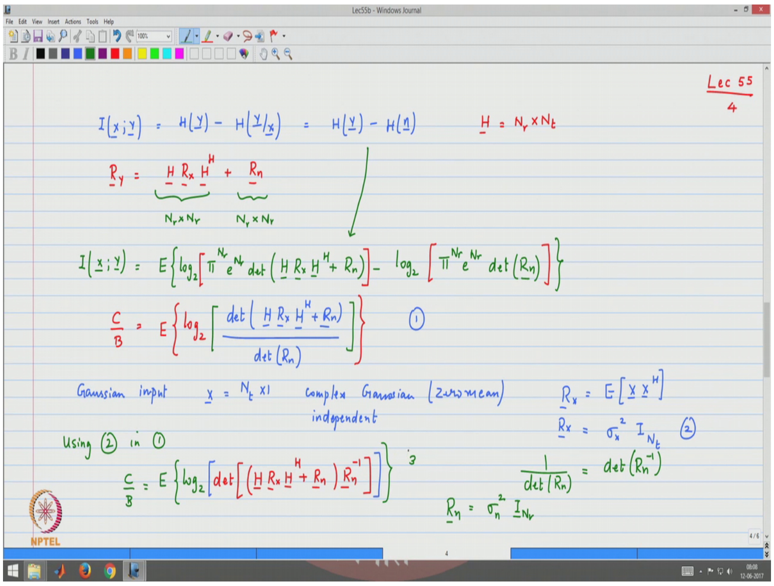

So, given this framework now, let us quickly get the key observations and results. The mutual information between the input and the output I of x vector x and vector y this is given by the entropy of the vector y minus entropy of y given x again this r basically repetition of what we are discussed in the last class this is given by H of y minus the entropy of the noise vector. And we showed a couple of results let me just mention those the dimensions of H r Nr cross Nt that is the MIMO channel representation.

And we are interested in the autocorrelation of the vector y that is given by H times R x times H Hermitian plus R n. This was the result that we had obtained in the last lecture for dimensionality this would be Nr cross Nr, again this would be Nr cross Nr and R y would also be Nr cross N r. So, this is the expression for R y substituting this in the expression for the entropy, we can now write down an expression for the capacity. So, the capacity expression that is mutual information between x and y vectors x, can be given as expected value of logarithm base 2 pi power Nr that being the dimension e power Nr determinant of the following matrix, H times r x H hermitian plus R n this is the first determinant for the logarithm the brackets are as follows. And expected value would be minus expected value of logarithm base 2 of maybe I just keep the expected value outside minus logarithm base 2 of pi power Nr e power Nr determinant of R n and the outer bracket for the expectation.

So, this is the obtained directly from this result writing the expressions for the entropy using the form given in the previous slide. So, now, simplifying this, the capacity or normalize capacity C by B given by the mutual information is nothing, but expected value of logarithm base 2 of there is a difference of logarithm. So, this ratio of these 2 it is determinant of H R x, H hermitian plus R n divided by determinant of R n, and that is the expression within the for the logarithm and then the expected value of the entire expression.

So, this is the form that we will start to work with again this is a very very useful result a very powerful result that we now have. Now if we go back and make the Gaussian assumption about the input; so, if we assume that the input is a Gaussian source where the vector components are independent then we can write down the following value. So, that is x is a Nr cross 1 x Nt across 1, Nt transmit antennas arte there it is an Nt cross 1 complex Gaussian vector; complex Gaussian zero mean vector with and the values are its in the form of an and values are independent of each other the components independent complex Gaussian zero means. So, in that case the autocorrelation matrix R x given by expected value of x x hermitian, can be written as the variance of the Gaussian random variables sigma x squared times the identity matrix, and the dimensions of the identity matrix would be Nt because the Nt n Nt cross 1 vector.

So, this is the expression for the r x now using this value in one using 2 in one, using the result for the Gaussian vector in the expression for one we now get a very powerful result with says that the normalized capacity is given by expected value of logarithm base 2, basically I am going to write determinant of R n 1 over determinant of R n I am going to write it as the determinant of R n inverse. So, if you write that in the in the write it in that form we get logarithm base 2 of the determinant of the following expression H times R x times H hermitian plus R n times R n inverse. This is the expression that we have then we have the brackets for the logarithm and then the expectation value. And basically simplifying this expression we also note that we can use the expression for R n as sigma n squared that is the variance of the noise, it is a Nr cross 1 vector each of them are independent sample Gaussian variables to this would be I times N r.

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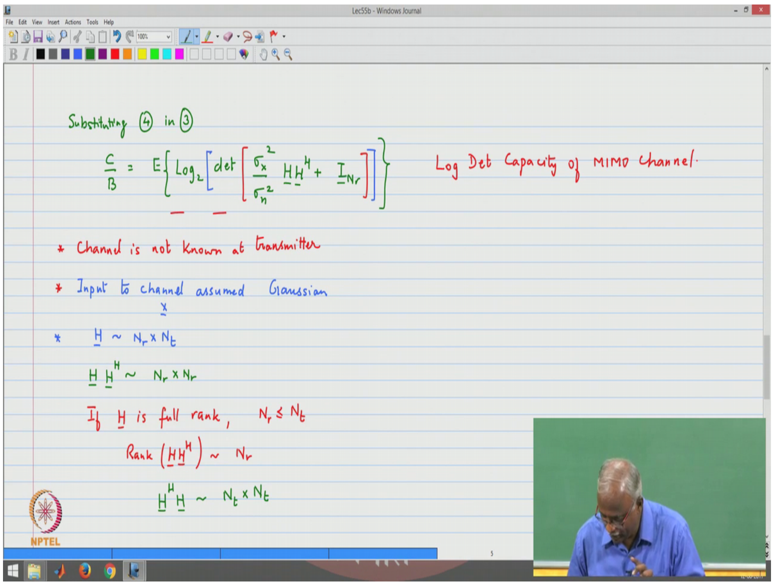

So, now substituting if you call this as equation 3 and 4, substituting equation 4 in 3, then we get a the following expression we get the following expression that C by B can be written as expected value of logarithm base 2 of determinant sigma x squared by sigma n squared H H hermitian plus I Nr, and this is the brackets for the determinant the brackets for the logarithm and then the expected value. So, this is the expression that we have and this is what is often referred to as the log det capacity of a MIMO channel. We have the logarithm of the determinant of an expression and this is called the log det capacity of a MIMO channel MIMO system MIMO channel. Now there are several very interesting results that we can derive from here and build on and get a good understanding of the characterization of the MIMO channels.

So, the just to summarize we are making the assumption that the channel is not known at the transmitter because if it were known then the best thing to do would be to do the single value decomposition the and then followed by the water filling for the power optimum power allocation. So, channel is not known at the transmitter. So, therefore, we have resorted to estimating the ergodic capacity. Now we have also in in the deriving this expression we have made the following assumption that the input to the channel is a Gaussian input to channel is the source is has Gaussian distribution. So, it is assumed that is x of n, the vector x is assumed to be Gaussian under that assumption we have simplified the expressions that we have given. And third assumption is that the dimensions of H r Nr cross N t. So, which basically means that the dimensions of H H hermitian will be Nr cross Nr. Now if H is full rank and we have Nr is less than or equal to Nt, then the rank of H H hermitian its full rank H H hermitian will be also N r.

So, in the case where Nr is less than or equal to Nt, the case where the number of receive antennas is less than or equal to the transmit antennas. H H hermitian has got the dimensionality that is consistent with the rank of H. On the other hand if you have to look at H hermitian times H this would have dimensions n Nt cross Nt right H H is got Nr cross Nt H hermitian would be Nt cross Nr and this. So, and in the case where n is less than or equal to Nt we will find that H H hermitian is rank deficient because it has a dimension Nt whereas, the dimension the rank of that is of the matrix H is only Nr and therefore, this would be a rank deficient. So, keep this point in mind because this will come back to a help us in the subsequent discussions.

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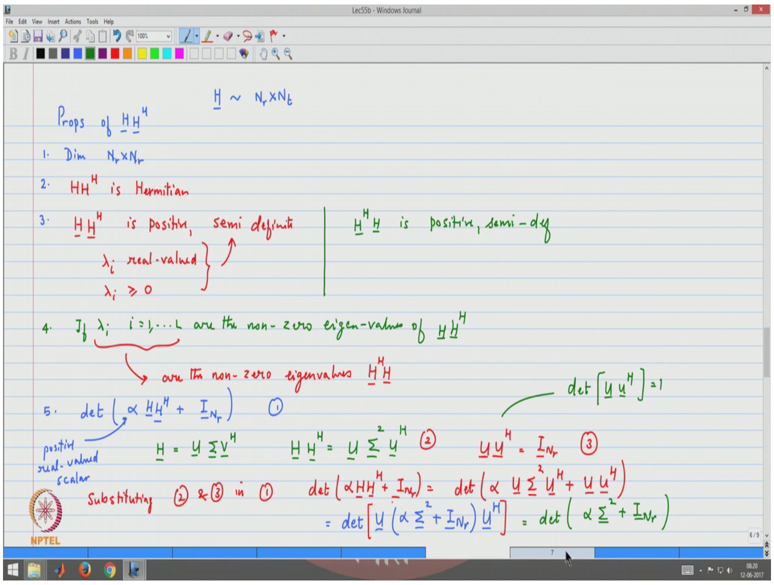

Ok. So, now, let us take this discussion ah little bit forward by looking at the properties of H H hermitian. So, the first observation is that the dimensions of H H hermitian dimension are Nr cross Nr, because H has got dimension Nr cross Nt H H hermitian would be Nr cross Nr. The second observation a very important one is then H H hermitian is by nature hermitian by virtue of its structure, if you take hermitian of it you get back the same expression and more importantly the second or the third properties what is most important H H hermitian is positive semi definite the definition of a positive semi definite matrix is known are to use. So, therefore, I will not write it down, but basically when you have this form, it means that the eigenvalues lambda i are all real value positive semi definite means they are all real valued and because and they are also greater than or equal to 0, these 2 are the properties reflected by a positive semi definite matrix.

Now, we can also using the same reasoning as we used to show the positive semi definite property of H H hermitian, we can also say that H hermitian H is also positive semi definite again these are useful properties when we go into to the analysis. So, another important observation which is helpful for us to note is that if the if lambda i, i is equal to one through l are the non zero eigenvector Eigen values are the non zero Eigen values of H H hermitian. Now what does this mean the if the rank of H is l, then H H hermitian will have a set of non zero eigenvalues and these are the non zero eigenvalues. We can show that these this set are also the non zero eigenvalues also the non-zero eigenvalues of H hermitian H. So, there are some very interesting structures that are property are present between H H hermitian and H hermitian of H hermitian times H.

Now, where does this come into play in the following property which I believe is the very useful one for us to interpret the log det capacity. Now if you have to look at the expression determinant of some positive constant alpha times H H hermitian. So, this is a positive real scalar real valued scalar, a alpha times H H hermitian and H H hermitian has got dimensions Nr cross Nr times plus I of Nr identity matrix of N r. So, basically we would like to understand what is the property of this. Because this x is this identical expression appears in the log det capacity if you have to look at the previous slide. So, I will call this as a equation one I am going to sort of build on this and draw some insights.

The first step is to look at the singular value decomposition of H, now again the we are looking at the case where the channel is not known at the transmitter. So, we are trying to understand the property of this expression not necessarily trying to do precoding at the transmitter. So, if you have to do the following property u sigma times v hermitian, that is the singular value decomposition that we have seen before then we can say that H H hermitian if you substitute will be of the form u times sigma squared times u hermitian and we also know from the property of the unitary matrices that u times u hermitian is the identity matrix and that will have the dimensions of Nr cross Nr.

So, if I call this as equation 2 and equation 3 then if I substitute 2 and 3 in 1. So, substituting 2 and 3 in one we get the following expression that the determinant of alpha H H hermitian plus I nr can be written in the in the following form that it can be written as determinant of alpha u sigma squared u hermitian plus u u hermitian. This is the form that we would do then of course, simplifying the expressions that we have this is nothing, but determinant of u within the bracket alpha times sigma squared plus I Nr times u hermitian. So, basically we have multiple multiplication of three matrices and you want the determinant of that of this expression, determinant of this which would be the product of the determinants we know that determinant of u u u hermitian if I take the determinant of u u hermitian, then I get this to be equal to one because that is the determinant of the identity matrix. So, this can be written as the determinant of alpha sigma squared plus I Nr, one step away from ah very useful and interesting result.

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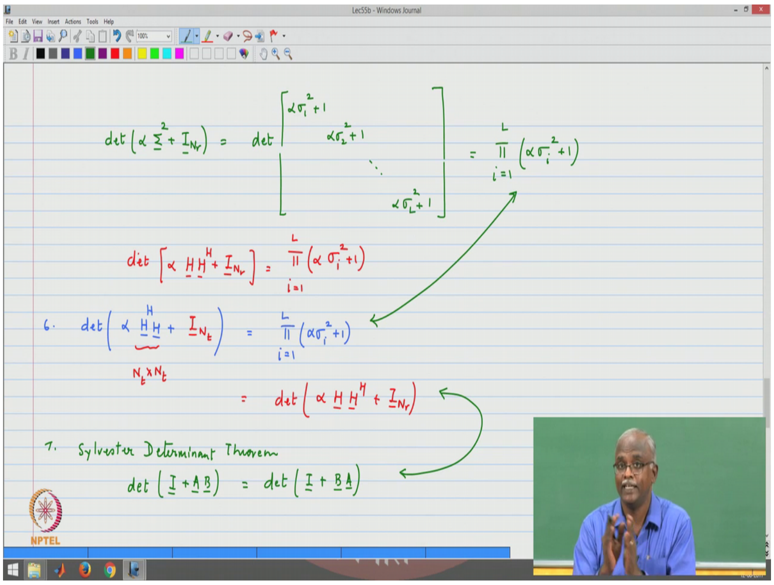

And the determinant of alpha sigma squared plus I Nr can be visualized in the following way it is the determinant of a diagonal matrix, where the first entry is alpha sigma one squared plus 1 and in the second diagonal entry is alpha sigma 2 squared plus 1. All the way to alpha sigma L squared plus 1 where L corresponds to the rank of the matrix or the number of non zero singular values, and that that would be and this would be given by product of i equal to 1 through L alpha sigma i squared plus 1. So, the summarize summarizing the result determinant of alpha H H hermitian plus I Nr determinant of this expression can be written down as the product with the number of terms response to the rank of the matrix H alpha sigma i squared plus 1 ok.

Now, the last of the properties now look at different expression, determinant of alpha instead of H H hermitian now would like to look at H hermitian times H this would this has a dimensionality which is Nt cross N t. So, this will be plus I Nt. I would like you to try simplifying this along the same meth steps that we have done previously and you can show that this is equal to product of i is equal to 1 through l alpha sigma i squared plus 1 again the same result as in the previous case. So, in other words we can write down the following result that the determinant of alpha H H hermitian plus I Nt can be written as the determinant of alpha H H hermitian plus INr right that is what we have obtained from properties 5 and 6 and of course, we have gone through and shown the steps for doing this this could have also been obtained from our knowledge of linear algebra result known as the Sylvester determinant theorem, the Sylvester determinant theorem is states the following, states that if you have the expression determinant of I plus AB. If you have a expression determinant of I plus AB and provided that the matrix operation AB and BA are defined then we can write it down that this is equal to determinant of I plus BA, BA must be define and the dimensions of the identity matrix are come in (Refer Time: 27:35) with the dimensionality of a of AB and BA.

So, given this form then we using a Sylvester determinant theorem this is the result that is obtained, and this is identical to what we have observed in the previous result as well. So, this is result or these are the properties of the main of the matrix H and the forms H H hermitian and H hermitian times H. Now why did we take so, much effort to do this for the following?

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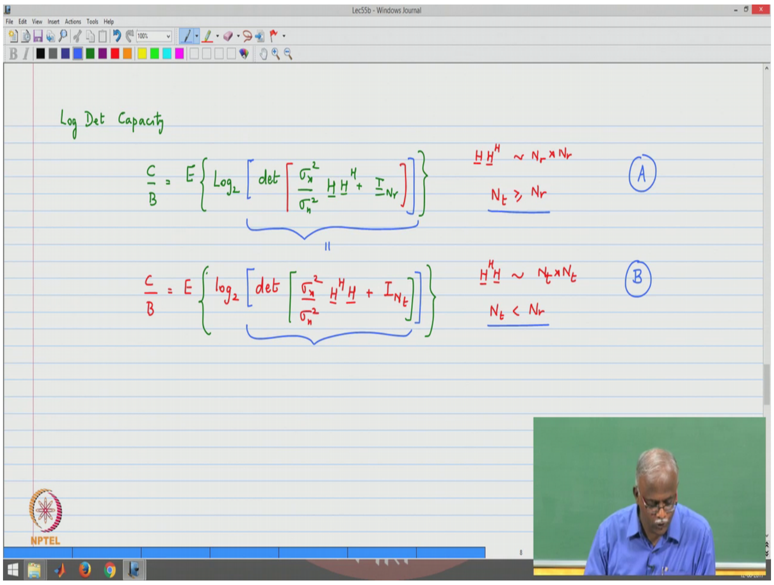

And let us see what is the benefit of that. So, the log det capacity; log det capacity for a MIMO channel we can write down in the following form the log det capacity that we have defined is C by B is equal to expected value of logarithm base 2 of determinant of sigma x squared by sigma n squared H H hermitian plus IN r. So, the determinant is for this form the logarithm is for the out of the next bracket and then the expected value now this is a form where we are using H H hermitian. So, this is of the of the order Nr across Nr. Now this is a form that is very useful for us to work with when we have the receive antennas dimensionality of the receive antennas as the limiting case in terms of the rank. So, this will happen when Nt is greater than or equal to Nr right because in that case Nr is the smaller dimension and that is going to be the limiting case in terms of the number of non zero singular values and therefore, H H hermitian, which is of the form Nr across Nr is a good ah expression for us to work with.

So, let me call this as expression A log det capacity expression A. Now the alternate form of this is can be given as C by B is equal to expected value of logarithm base 2 of the determinant of sigma x squared by sigma n squared H H hermitian times H, plus I Nt now the let me just draw the brackets, this is the determinant, this is the logarithm and the expected value. now in this case we are having the expression H hermitian times H which is of the ca of the dimensions Nt cross Nt, and this will be the limiting dimension when we have Nt less than Nr. In other words when the number of receive antennas is greater than the number of transmit antennas this would be the form and we will call this as the form B of the log det capacity.

Now, are they different from each other in a sense no, because we can we have just now shown that the determinant of this expression is the same as the determinant of this. So, basically when we look at the determinants using a Sylvester determinant theorem or the properties of the H matrix, we have already shown that this these 2 are equivalent forms in terms of the value of the determinant. So, therefore, the capacity still the same, but what it helps us is to understand and interpret the expressions in light of the dimensionality and that is why we have 2 form. One when the case where Nr is greater than the number of transmit antennas is greater than or equal to the number of receive antennas and the second case is when that number of transmit antennas is less than the number receive antennas or the receive antennas being more than the number transmit antennas.

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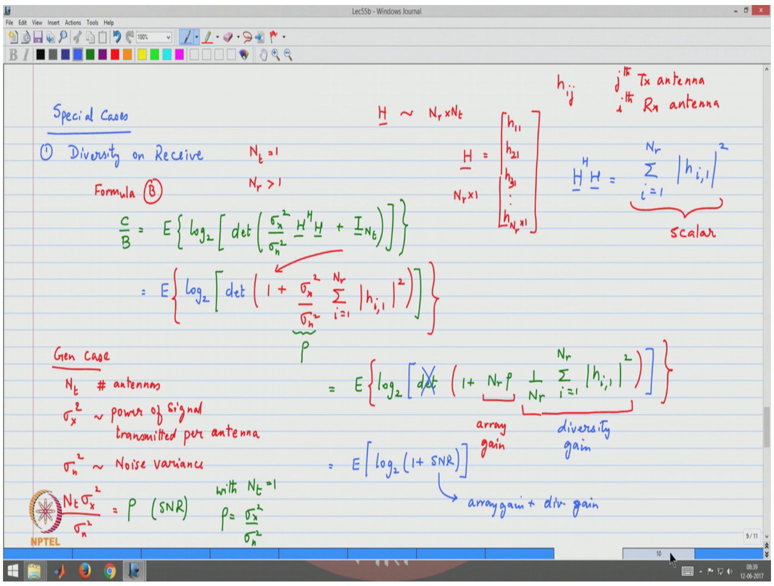

Now, the reason for this is to for us to study 2 special cases and these are very very useful for us and interesting special cases. So, special case number one, because they will reinforce what we have studied and also correlate with the earlier results that we have obtain. So, special cases 2 special cases that we would like to look at, first one is diversity on receive. Now this is the case where we were using diversity primarily to improve the robustness of the signal, where we use multiple receive antennas to pick up the transmitted signal. So, that when we combine the received signals we would then see that the effect of feeding is mitigated through the multiple copies which are picking up independence (Refer Time: 32:57) So, diversity on receive multiple receive antennas at the receiver. So, this would be the case where you would have number of transmit antennas equal to 1, number of receive antennas is greater than one typically 2 3 4 any numbers.

And since we have the number of receive antennas greater than the number of transmit antennas, we would have to look at formula B; formula in the log det capacity we will look at the version B; and the version says that the capacity is given by expected value of logarithm base 2 of the determinant of sigma x squared by sigma n squared H hermitian times H, plus I Nt close all the brackets that is the expression that we have. nNow since this is a receive diversity system I would like you to help visualize what the transm the channel matrix looks like. So, the channel matrix has a very interesting structure it is a n the dimensions of H r are always Nr cross Nt, since Nt is one we have an Nr cross 1 antenna in in this particular case. So, in the general case I just got Nr cross Nt in this particular case where there was only one transmit antenna and Nr receiver antennas, and the expression would be h 11 the first index being the received signal received the first one being the transmitted signal antenna, and the and the second index being the received antenna.

So, basically what we have is h21 h31 all the way to H Nr cross 1. So, basically there is one transmit antenna and Nr receive antennas and notice when we have h i j this indices it is from the jth transmit antenna to the ith receive antenna and there is only one transmit antenna. So, therefore, we have only one column for this and when we do H hermitian H important for us just write down that expression also. If you do H hermitian H what we will get is summation i equal to 1 to Nr mod h i comma 1 modulo square. So, this would be the expression for H hermitian H. Now substituting this result in the expression what we get is expected value of logarithm base 2 of the determinant; now this H hermitian H is the scalar. So, this is a scalar. So, the expression inside I Nt n t is one therefore, that is also. So, determinant of 1 plus sigma x squared by sigma n squared I have interchange the order I Nt comes out as 1, this one becomes summation i equal to 1 through Nr, summation h of i comma 1 magnitude square logarithm of this and expected value ok. So, this is the expression that we have.

Now I would like you to pay following a very careful attention for the following aspect of the discussion. Now in the general case the number of transmit antennas is Nt, and if the variance of the signal transmitted in each of the antennas. So, is sigma x squared. So, this is the single variance at transmitter. So, this is the number of antennas and you can think of this as these power of the signal transmitted of the signal transmitted per antenna and sigma n square is the noise variance or the noise power noise variance. So, given this the total transmitted signal power is Nt times sigma x square, the total noise power in each antenna n square.

Now, this is the signal to noise ratio based on the total transmitted power that each of the receive antenna sees divided by the noise, and this we would like to represent in terms of the variable rho as the SNR variable. Now in our case Nt happens to be equal to 1. So, with Nt equal to 1 then rho is equal to sigma x squared by sigma n square. So, this parameter is nothing, but rho and it very important for us to be able to characterize this expression. So, we will write it in the following form. So, this is equal to expected value of logarithm base 2 of the determinant of a very important form, and the important form comes out to be 1 plus Nr times rho times 1 by Nr summation i equal to 1 through Nr mod h i comma 1 whole square and the put in the brackets this is the expression for the determinant, the expression for the logarithm, and then the expected value outside. So, what we have done is basically written this expression in terms of rho, and we have also introduced the Nr and 1 by Nr and it is for a very specific purpose. Because Nr times rho the SNR of one antenna multiplied by Nr, this tells me that this is the array gain. Array gain tells you that when I have Nr receive antennas it is like boosting my effective SNR by a factor of n.

By the same token 1 by Nr times this summation is precisely what we would refer to as diversity gain. If you remember our earlier discussions this is what we referred to as the diversity gain, now the diversity gain comes in because not because it shifts the energy of the signal on average this quantity is equal to 1, but at given time because these signal levels are fluctuating what you are getting is the benefit of the stronger signals when there is fading is present. So, that is the diversity gain. So, we are able to see that the expression that we get for the log det capacity of a MIMO channel, on receive directly correlates to the expression that we have obtained when we were using receive diversity because when we were using receive diversity what was it? It was basically we can leave out the this is a scalar 1 plus. So, the determinant really does not make a difference we can leave out this. So, this is nothing, but expected value of logarithm base 2 of 1 plus SNR on receive diversity where SNR is given by the expression for a receive diversity system, where you have array gain plus diversity gain and this is exactly what we had obtained earlier and our expressions now are matching the expressions that we had obtained previously where the SNR is the effective SNR in the context of receive diversity ok.

So, diversity on receive completely explained in terms of the log det capacity, and showing and shown that it matches the expression that we have derived earlier. Now we would like to develop this little bit more for the second special case the second special case is transmit diversity.

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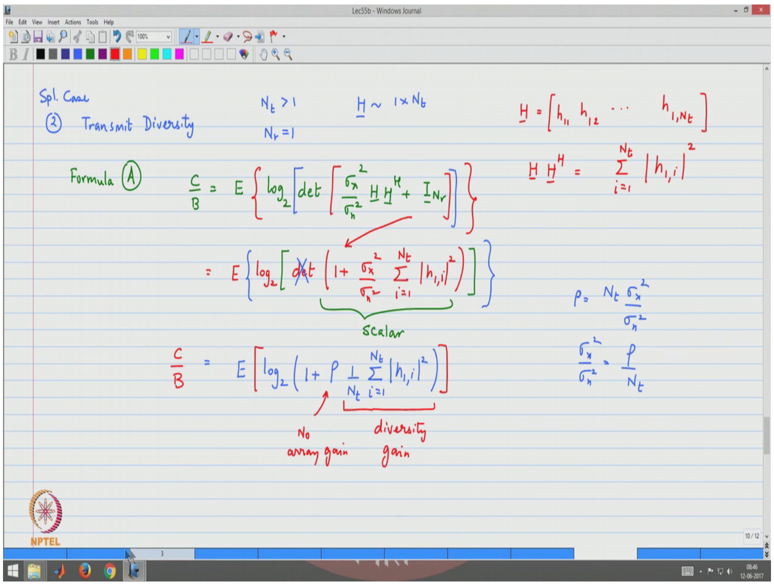

So, special case number 2and this will be for transmit diversity. Now as you know transmit diversity is a is a case where we have multiple transmitters, but possibly only one receiver. So, n Nt is greater than one Nr equal to 1 we will take that particular case. So, in that case the dimensionality of H is 1 cross Nt and the. So, based on this structure we will use the formula A for the log det capacity, because that is the one that corresponds to the case when the number of transmit antennas greater than the number of receive antennas.

So, under this sum under this formula we get C by B equal to expected value of logarithm base 2 of sigma x squared by sigma n squared H H hermitian plus I Nr. So, this is the logarithm I have forgotten to write the determinant I need to shift this. So, it is logarithm base 2 of the determinant of the expressions, and let me just write it with different colours, determinant of this form logarithm of this and this is the form that we have of the expression B. Now I would like to write down what H looks like. H is one receive antenna multiple transmit antennas. So, it looks like h11 first transmit antenna to the first receive antenna, h12 second transmit antenna to the first receive antenna all the way to h1 comma Nt from transmit antenna Nt to the first receive antenna.

And if we now do H H hermitian this comes out to be equal to summation i equal to one through Nt mod h 1 comma i mod square. So, H H hermitian takes the following form. So, using this expressions in here we now obtain that this is equal to expected value of logarithm base 2 of the determinant of the following expression, where it is 1 plus sigma x squared by sigma n squared again I have use the expression for Nr to come into the first place sigma x square by sigma n square this will be equal to summation i equal to 1 through Nt mod h1 comma i magnitude squared.

Now, notice that this is also a scalar. So, then we can simplify this expression this is determinant this is the logarithm and then outer is the expected value, and we can show that because it is a scalar the determinant does not play role. So, this can be written in the following form, this is expected value of logarithm base 2 of 1 plus; notice that if I were to now wanting to write this sigma x square by sigma n square in the form of rho then we know that rho is equal to Nt times sigma x squared by sigma n squared. So, sigma x squared by sigma n squared can be written as rho divided by Nt. So, this can be now written as 1 plus rho, 1 by Nt times summation i equal to 1 through Nt mod h1 comma i magnitude square.

So, close bracket and then you get the expectation. Now very important result that we if you recall when we talked about transmit diversity we said that transmits diversity systems will not have array gain. Notice that the SNR here is no array gain, there is no array gain but transmit diversity systems can achieve the diversity gain. So, if you notice that this expression allows or shows that there is a provision for the diversity gain no array gain, but diversity gain is present. So, this is also consistent with our understanding and derivation of the results that we had obtained for the case when we have only transmit diversity.

So, the good thing is whatever we have derived as the log det capacity can be used to derive look at 2 special cases; one with receive diversity the other one with transmit diversity and in both cases we have shown that the results are completely consistent with what we had derived earlier. So, to summarize what we have done is in the case of a MIMO channel, let me go back to the first drawing. We have the following expression y is equal to H x plus n, when we know the channel at the transmitter we been able to do the parallel decomposition. When we have the case where we do not have knowledge of the channel at the transmitter then we have estimated the ergodic capacity. We have obtained 2 versions of the expression one when the number of transmit antennas exceeds the number of receive antennas this is a special case, when we have just transmit diversity when Nr is equal to 1 and you have multiple transmit antennas.

On the other hand there is an alternate version which gives us exactly the same numerical value, but the expression is more consistent with our intuition for the case where Nt is less than Nr. This the case when you have receive diversity and we have shown that this also is completely consistent with the case where you do maximal ratio combining type of expressions. So, the 2 special cases and then finally that we now have this final expression. So, with we would now like to conclude our discussion in terms of the capacity of the MIMO channels. Now one of the important elements is how do we combine coding and multiple antennas and that is a very important element in our discussion.

So, let me introduce the final part of this particular section, where we would like to look at MIMO transmission and coding ok.

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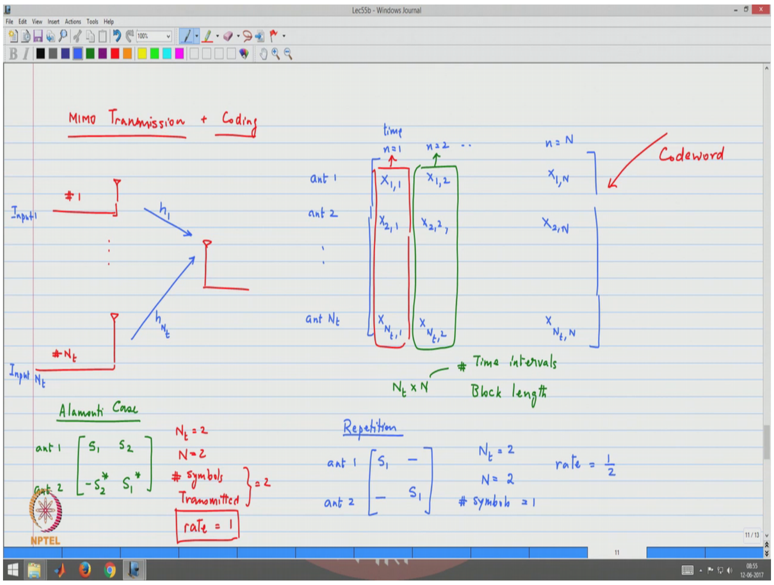

And it is a very important result important section for us and definitely something that you should pay close attention to. So, let us look at some of the MIMO transmission plus coding. Let us take a couple of simple examples, but before that let me introduce the notation. So, if you have the the following scenario, I have Nt transmit antennas that is transmit antenna number 1, similarly all way to transmit number antenna number Nt, and let us assume that we are having only one receive antenna just for simplification of the notation.

So, the first receive antenna and this will tell us that the channels now I can write it in terms of just one index this is channel h 1, and the each of these antennas will give me a channel and this is channel h Nt. Now if I were to transmit the different sets of n inputs. So, basically input number one input 1 this would be input Nt for each of the time instances. So, in other words we are going to write down the matrix where I indicate what I have transmitted on antenna 1,what I have transmitted on antenna 2 all the way to antenna Nt and the x axis will be the time this is time n equal to 1, n equal to 2 and so on.

So, this is x of the signal transmitted at from antenna 1 at time instant 1, x from antenna 1 at time instant number 2 and if there are n time instances n is equal to uppercase n this should be from antenna 1 up to time instant n. Similarly from antenna 2 it will be x 2 comma 1, x 2 comma 2, all the way to x 2comma n and finally, when we look at Nt comma 1 Nt comma 2 all the way to x of Nt comma n. So, this is our the matrix which comb which collects all the transmitted signals. So, this entire vector is transmitted at time instant number 1 and the next and vector transmitted at time instant 2 and. So, on and so forth up to n block.

So, we now have this matrix which is Nt cross n, this corresponds to the number of time intervals and we can also refer to it as the block length length of the block. So, Nt antennas and block length equal to n. So, this matrix that we transmit is what we refer to as the code word that we are transmitting; so, the code word or the information that is being transmitted. Now how is this matrix going to be generated? That is a very key element and that is what is going to give us the different cases that we are going to be looking at. So, at the receiver what we can write down in the form that we are interested in is the input output relationship, which we can then write down in the following manner.

So, let us see how to look at some of the cases that we have already study. So, first let us look at the Alamouti case. Alamouti case is one where we transmit this matrix as a 2 by 2 matrix, we transmit symbol S1 from antenna one, and in the time instant 2 we transmit S2 and. So, this is antenna 1 and this is antenna 2, and we transmit minus S2 conjugate and S1 conjugate and then this repeats. So, this is the code word or the code matrix that is transmitted. So, if you were to describe this the Alamouti case is a case where we have Nt equal to 2 and in 2 time instances n equal to 2 number of symbols transmitted is equal to 2 number of transmit ok.

So, 2 transmit antennas 2 time instances 2 distinct symbols are transmitted. So, therefore, the rate which is the number of symbols transmitted by the number of time instances is equal to 1. So, the Alamouti case is a case where you have a rate one code where you are transmitting 2 symbols over 2 time instances, and you are able to achieve the second order diversity that we have shown is present in the case of an Alamouti system. Now similar case, but a simpler one when we do repetition code; that means, first we transmit the symbol on antenna one nothing on antenna 2, and we then in the next instant of time we transmit the same symbol from antenna 2 and nothing on antenna 1. So, in this case the matrix would be S1 and nothing on antenna 2 at time instant 2. So, this is antenna ones transmission, this is antenna twos transmission time instant number one nothing on antenna 2 and then second time instant nothing on antenna 1 and S1 on antenna 2.

So, basically you have done repetition transmission on 1 and transmission on antenna 2. So, in this case we have number of transmit antennas equal to 2, number of time instances of block length equal to 2, number of symbols transmitted is equal to 1. So, this is not as efficient as the Alamouti scheme because this one achieved only a rate of one half, but of course, has the same diversity second order diversity. So, in an nutshell what we are trying to do is we would like to have the rate as high as possible and at the same time achieve good diversity methods. So, now, what we would like to do is very quickly and very briefly introduce the concept of what is one of the most popular techniques call the blast techniques.

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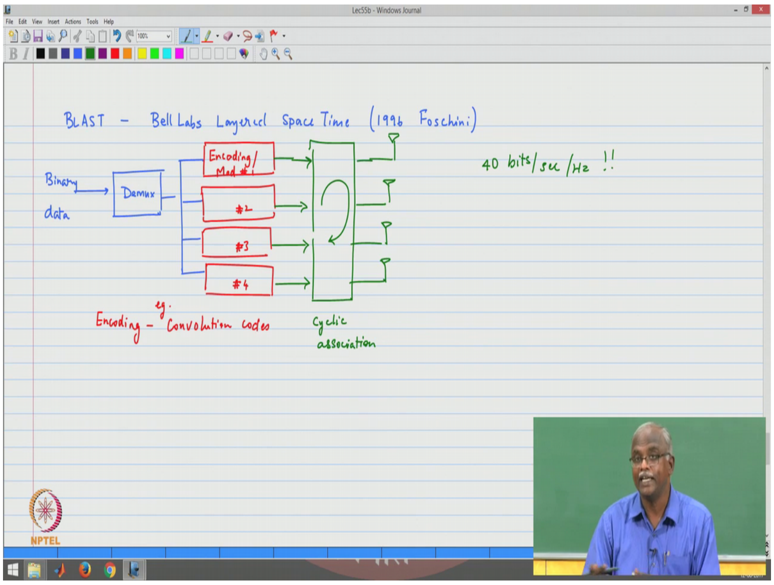

So, the blast architectures bell labs layered space time architectures. Now these were the architectures that were. So, this is what stands for blast this was introduced in 1996 by researchers at bell labs several of them one of the pioneering papers was by Foschini in 1996.

Now, here are some of the basic assumptions and the structure of a blast method. So, we have input data let us assume that this is binary data and we have 4 antennas. So, we are going to do a demultiplexing in by factor of 4; 1 is to 4 demultiplex. So, we will get 4 parallel branches coming out of the demultiplexer 1 is to 4 demultiplexer. So, this is a demux. Now each of these are going to be processed each of these substreams is going to be processed by an combination of encoding and modulation. So, this is encoding modulation block number 1, and then we have encoding modulation block number 2 block number 3 and block number 4. Notice we have already segregated the streams and each of these streams are we do modulation encoding independent of the others, and typically we can use encoding or if encoding methods such as convolutional codes.

So, it is conventional coding as example will be convolutional codes or block codes any of the standard codes that we are familiar with. So, we are using standard codes and we have separated of the data into multiple streams, and we will transmit through the different antennas. So, this is the case where we have design the structure for 4 antennas 1 is to 4 and we will see that there is a very important block in the middle which is useful for us in as part of the bell labs structure. So, this is the arrangement that we have transmit antennas and this is a form of something that does cyclic association basically it is like a switch that keeps shifting the input signals from one antenna to the other. So, this is called cyclic association. So, like circulars switch and we will show what that actually does when we do the transmission.

So, basically one of the simple forms of the MIMO transmission is the blast technique which was first introduced in 1996, showed that we could achieve very very high spectral efficiencies. In fact, was even shown that you can have achieved as high as 40 bits per second per hertz very very high spectral efficiencies using this structure. Now what we would like to do is pick it up from here and basically describe what is the encoding process what is this this cyclic association and how does it benefit as in terms of achieving the performance of the system, in terms of getting the diversity benefit, but also getting increase data rates which is what will achieve for us the high spectral efficiency that we are seen here ok.

So, the tomorrows lecture will be a brief introduction or brief introduction to the topic of the space time codes, and just an I just mention of what are some of the elegants meth methods that I use, and the powerful results that we can achieve with the space time codes which are built on the MIMO framework, that we are we have been studying. So, that will conclude our discussion on MIMO techniques, and then move as into the next topic which will be OFDM techniques.

Thank you.