Introduction to Wireless and Cellular Communication
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Lecture – 49
CDMA Receivers
CDMA Multiuser Detectors – Part 1

Good morning, welcome to lecture 50; 50 lecture mark on the honor course. We will do a quick review of the 2 lectures that were covered yesterday. Primarily looking at the multi user environment and the optimum CDMA receiver having developed the optimum CDMA receiver, we would now in this class, look at the 2 sub optimum receivers which give us was good performance, but at the same time are acceptable in terms of complexity.

So a quick recap of the key points, again the reason for this recap is I want you to see some of the similarities between the way, we have approached the multipath environment and the multi user environment.

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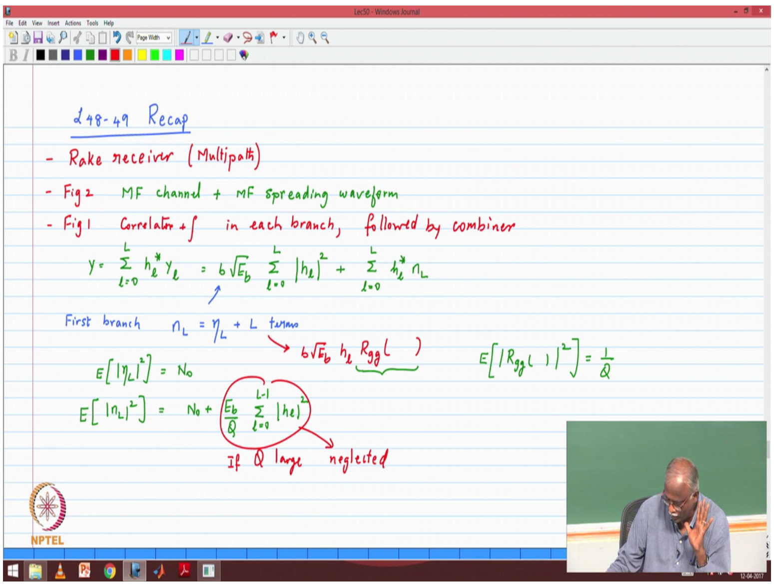

So, just as a quick summary of our refresher of the context of the multi path environment; the rake receiver was our receiver of choice that was helped us recover our work in the presence of multipath. There were 2 ways of looking at the rake receiver. Let me call that as figure 2; the figure 2 is where you have moved the multiplication by the g 1 star and the integration moved it to after the summation.

So, that is the figure 2, not a figure 1. Figure 2 gave us the understanding that the rake receiver is doing a match filter to the channel as the first operation because you have got the h 0 conjugate h 1 conjugate, you have got all of those tap conjugated. So, you basically are doing a time reversed filter conjugated filter that is a match filter to the channel followed by a match filter to the spreading waveform. So, we are doing the right thing, in the context of providing a match filter environment and using that subsequently the decision statistics to work with multipath environment.

So, the figure 2 gives us an understanding of the match filtering element; however, figure 1 is the one that helps us work with the actual implementation and how to get the quantities that we are working with. So, basically figure one is that you have a correlator and integrator in each branch; that is the first figure that we have drawn in each branch which is then followed by combiner followed by a combiner they are equivalent. So, though this was the first; the figure 2 gives us the match filtering interpretation figure 1 is doing exactly the same thing, it helps us in terms of the computation. So, the final output y, we obtain as summation l equal to 0 through uppercase L h l star of y subscript l and basically what we find is that the final expression is given by b times root E b summation l equal to 0 through uppercase L mod h l squared plus summation l equal to 0 through uppercase L h l star times N l.

Now, N l eta l definitely want you to sort of keep those 2 things in mind one point to note is that there is only one data symbol b, we do not have b 1, b 2, b 3, when it is multi user, then each of the user is transmitting a different bit here, there is only one user transmitting one bit and that bit is getting replicated, I mean that signal is getting replicated multiple times because of the multipath.

So, we can look at each of the l plus 1 branches if you look at the first branch which is what we had done most of our analysis on the first branch has got a noise term N l which is a combination of eta l plus there where l terms these are the multipath related interference terms. Now each of those terms, if you were to look at it more closely where b times root E b h l some channel coefficient times the correlation between the waveforms at different lags because you are looking at copies of the signal at different time lags.

Now, to estimate what is the impact of this, we have to focus on this quantity and we said for the typical class of spreading waveforms that we are looking at the autocorrelation at different lags. The expected value of magnitude squared should be approximately 1 over 2 that is the type of sequences that we would like to pick. So, given that expression, then we now have the following way to analyze the impairment on the first branch. So, expected value of eta l magnitude squared that is nice nothing, but the Gaussian noise that is N naught; if I were to look at the expected value of mod N l squared that is the total impairment that is present then using the expression that we derived this one is N naught plus E b divided by Q summation l is equal to 0 to l minus 1 mod h l square.

So, this was the type of the expressions that we got each of these branches has got a noise component which we will have to quantify. So, using this expression, then we and assuming that Q is large, then we said that if Q is large, then this quantity is neglected and basically or we go back and look at the expressions for the it. Now it starts to look like a diversity scheme; however, keep in mind that this will not give us the full diversity because there is some approximations that we have made and therefore, the effective noise is like likely to be higher than what we would get in a diversity system.

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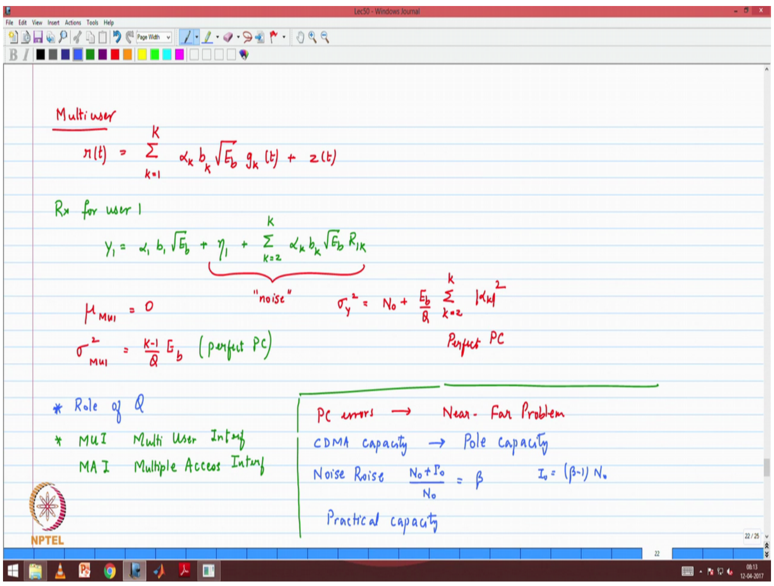

So, this is one part of our understanding of how to deal with multipath. Now to move into the multi user and to be able to compare them side by side very helpful to have that perspective. So, the received signal in the context of a multi user environment is come combination of K users K equal to 1 to uppercase K alpha K b K that is a bit transmitted by each of the users square root of E b g K of t plus z of t that is the noise term.

So, what do we do for each user? You will multiply by the corresponding spreading sequence conjugated and followed by the integrator. So, if you were to do that R x for user one, I am just skipping the steps and giving you the final answer R x for user one, you would get a decision statistic y 1 which would be alpha 1 b 1 root E b plus eta 1 the z of t passed through the filter which is represented by the spreading waveform of user one plus the correlation with the other seek other user sequences K equal to 2 to uppercase K alpha K b K root E b R 1 K R 1 K s; what is the contribution of that particular user when pass through the correlation filter of user one.

Now, this combination; we now treat as the effective noise in a multi user environment and we go through and did the mean value, we looked at the variance of this noise term. So, we came up with the final conclusion that if you were to represent this noise as the multi user interference noise; the mean of this multi user interference noise is 0, the variance of the multi user noise; multi user interference comes out to be K minus 1 by Q times E b and that comes from basically looking at this expression and arguing that this is 0 mean and then saying that sigma y squared. In this case is equal to N naught plus E b divided by Q summation K equal to 2 to uppercase K mod alpha K squared and then of course, assuming perfect power control.

Perfect power control that alpha K is are all equal and therefore, we are able to take out a K minus 1 term outside. So, under perfect power control; so, this is under the assumption of perfect power control. So, that is the expression. So, what we find in both cases whether it is multi path environment or the multi user environment, there is a signal component and there is a noise component the noise component is AWGN plus the multipath related terms, the good thing is your desired signal now gets enhanced because you have got these mod h l squared adding up in the case of the multi user interference you just have your signal and all of the multi user interference is grouped under noise and we have characterized it and said that this is the expression for the noise one of the key things that we want to take away from this discussion is the role of Q role of Q both in multipath and multi user environment both are keys Q plays a very crucial role.

Another comment we have used the term mu I multi user interference and you will find that books sometimes also call it as multi axis interference M A I multiple users using this channel at the same time is multiple access. So, M A I; M u I is multi user interference, this is multiple access interference again in the context of CDMA; M A I and M u I are the same. So, in the context of CDMA, you can call it as multi user or multi access interference. Now in this context of multi users that we presented several important results in yesterdays lecture I will just mention it and move on we said that if there are power control errors this will lead to a problem called as the near far problem.

So, it looks like there is one user who is very close because of power control and another user is very far when it is a 2 user scenario or you have one user very close and all the other users very far that is the near far problem ideal power controls this, all it looks to the base station that all the users are at the same distance from the base station. So, near far problem basically says when I violate perfect power control then one user suddenly looks like it has come very close to the base station.

So, because he is come very close to the base station, he is this signal is very strong and therefore, the detection of the other users gets affected. So, the same concept that we use for understanding the near far problem is what eventually leads us to understand CDMA capacity; CDMA capacity basically says that I can keep on adding more and more users under perfect power control what happens is the noise floor keeps on rising. So, which means my cell keeps on shrinking eventually I get only users who are around the base station and that are the pole capacity.

Now, if I limit the noise rise. So, we also understood that we have a quantity that we can control called the noise rise and the noise rise is N naught plus I naught divided by N naught, we say that is equal to some value beta that is something that is in our control then we can say that I naught is equal to beta minus 1 times N naught that is the total interference rise that we will allow and this leads to a practical capacity practical capacity for the CDMA channel; practical capacity for the CDMA channel and we also indicated that very closely related to this phenomenon is the phenomenon of cell breathing.

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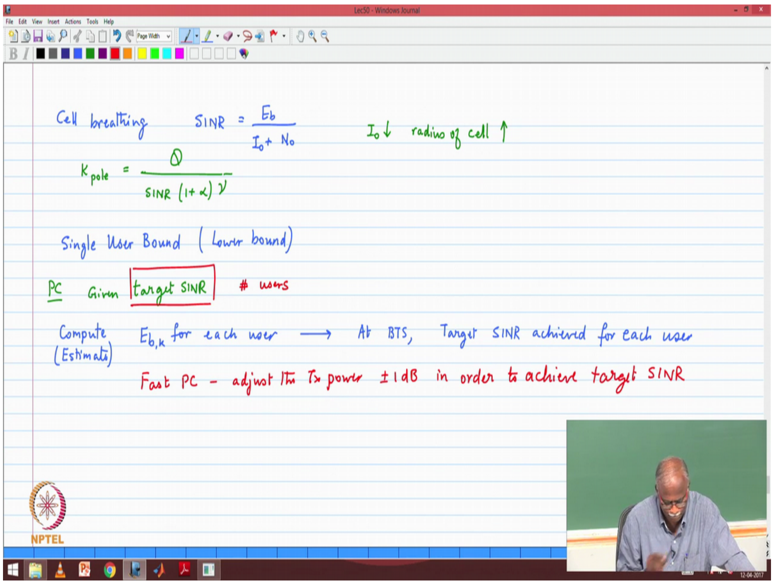

Cell breathing basically says that my effective SINR is equal to the energy with which I transmit my signal I naught the total interference in the system plus the AWGN. So, the cell breathing basically says if for any reason I naught goes down; that means, other users are not present the nominal interference is not there then the radius of the cell appears to have increased. So, at least your radius of coverage seems to have increased. So, therefore, that is a phenomenon that we observe only in CDMA systems that the boundary is not constant it is something that is constantly changing.

So, this and the pole capacity are closely related. So, if you; I forgot, write down the expression for the pole capacity the number of users pole capacity depends on Q the spreading factor the target SINR that we are utilizing what is the other cell interference that the captured by alpha and the voice activity factor which is which is captured by nu. Now this is the expression that we would use and then a similar expression we would use for the practical capacity and the cell breathing concept.

Now, whenever multiple users are present, we find that their presence affects the detection of the signal even under ideal power control. So, we always want to refer our performance to the single user bomb. Single user bound is a system where there is only one user present basically it is like other users are not there, if you had; you think of it as a narrowband system where only one user is present; what would be the performance that you would get. So, this is the lower bound always you want to compare the multi user performance with the lower bound and try to see how close we can get to the lower bound.

We had not explicitly stated this earlier, but let me just; what is the power control task in a CDMA system given that; we now have a good understanding of how power control is going to affect the performance. How it is going to affect capacity it says that given that I have a target SINR; this target SINR is very very important; you have to decide, what is the target SINR; your system can work with it may be 6 dB, 7 dB, 8 dB, some systems are designed to 5 dB because they have lot of error protection. So, again it depends on what is the target SINR you want to achieve and the number of users given the target SINR and the number of users then what you have to do the task is for the algorithm at the base station is to compute or estimate compute or estimate the energy or the signal level that it has to transmit E b for each user for each user. So, maybe we should give it a subscript K and for each user you have to tell what is the power level with which they have to transmit such that at the base station at the base station the target SINR is achieved for that user for in this case for each of the users target SINR achieved for each users for each user.

So, it is one of those things where if you find that the one of the users is not achieving the target SINR you say you increase by you increase. So, the method that is used for adjusting E b the transmit power is a technique called fast power control which says adjust the power adjust the transmit power t x power or the energy of the transmitted signal in steps of plus minus 1 dB in order to achieve or until you achieve in order to achieve target SINR. So, that is ideal power control this is constantly operating and constantly working towards maintaining the system in terms of a very delicate balance making sure that all of the users. So, this is the big picture as far as the capacity is concerned power control cell breathing pole capacity all of the concepts that we had discussed in the last class as always, a few questions to see whether you know how comfortable you are with the concept here is a question for you to think about.

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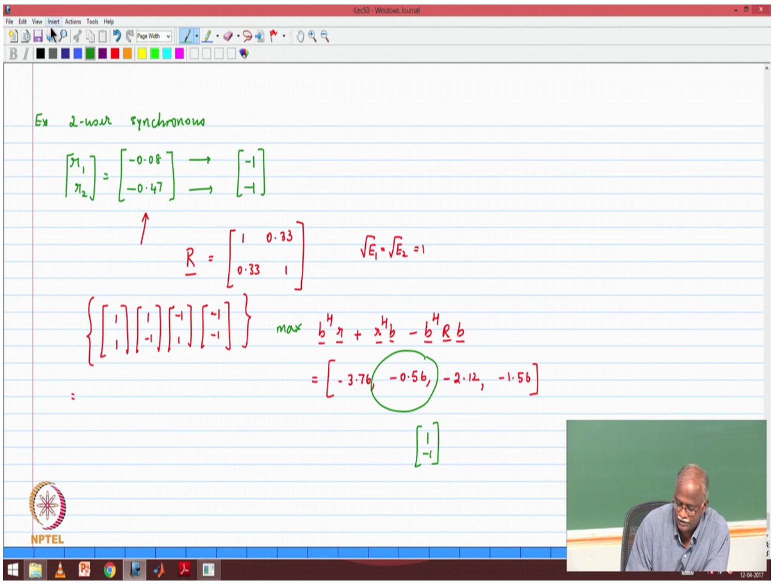

So, let us take cell breathing everyone is clear; why cell breathing occurs, everyone is comfortable with the concept interference goes down radius increases. So, cell breathing under the assumption that the radius has increased beyond the nominal value; what is the advantage of this more coverage, right, capacity; we may not be able to say because if more users come into the system again the cell will shrink. So, the fact that the radius is increasing means that users have gone down capacity in terms of area covered is correct. So, coverage area is the main advantage that we will get when cell breathing occurs and the radius increases; do you see any disadvantage; any disadvantage say why is there any disadvantage there is.

So, what happens; I have designed my let me draw a slightly differently. So, I have a base station; let me say that just I am considering only 2 base stations. Now I have designed nominally for this cell to be covered by this base station and maybe a little bit of overlap for the other base station. So, the overlap interference region now if a coverage area increases then what happens this one’s boundary goes well into this other cell and this base stations coverage now comes all the way here. So, then all the mobiles that are in this region are now in trouble because they are going to see more interference. So, it is one of those situations where yes, there is a good thing that that these the cell breathing occurs, but potentially you make user may now cause interference to 2, I am user talking to base station a let us call this as base station a and this is base station b.

Now, unintentionally I am causing interference to users and base station b because the cell radius has now increased my signal actually now reaches base station b also. So, it is one of those situations which is not so good. We need to be careful about the interference that you create or your mobiles will create to other base stations because there the coverage of the base stations has increased. So, interference; there is a chance that you actually have to look at the impact of interference. Now what is the drawback if the radius decreases is there a drawback or not nothing. So, if the radius has decreased because of heavy load on both cells then it is possible that you now have created a region without coverage.

So, basically there is a region without coverage. So, cell breathing is not a trivial thing we just have to be very very careful; how to handle that another indirect impact of cell breathing, is there any impact of cell be any link between cell breathing and soft kind of when does a mobile ask for soft handoff when it can hear 2 base stations. So, when cell breathing occurs what happens they say oh very good I can hear more base stations give me soft handoff. So, if you do soft handoff, what will happened capacity will go down. So, you have to be a little bit careful there is a link between cell breathing and soft handoff also. So, again keep all of those concepts sort of in link to each others. So, then they are not independent of each other.

So, cell breathing and soft handoff are linked because now you will hear more base stations and therefore, likely to go into a soft handoff. Any questions on the things that we had covered? These are the main things that we had looked at in yesterday’s lecture. Now to optimum detectors optimum detectors just give you a framework. So, that we can then pick it up from there and move quickly into today’s discussion.

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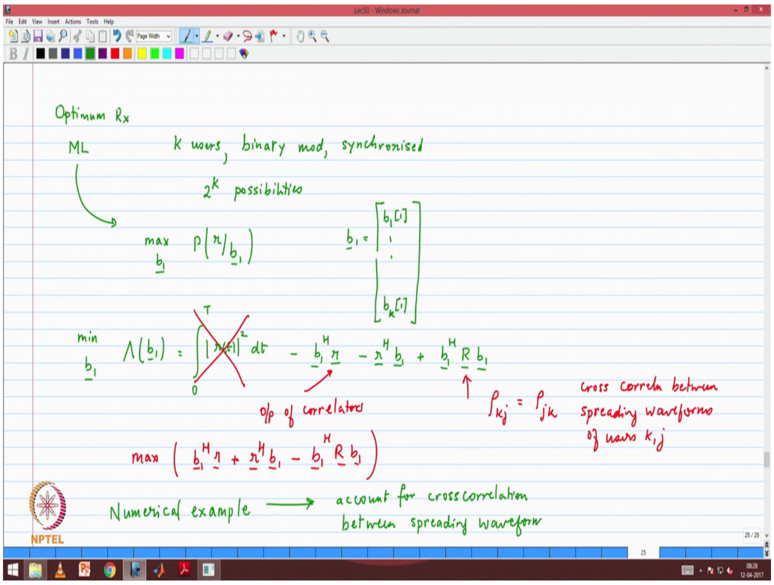

Optimum receivers; optimum receiver in our case was the maximum likelihood receiver. So, if I look at K users using binary modulation and they are synchronized binary modulation K users comma binary modulation and they are synchronized.

So, if I at a particular instant of time, all of them have transmitted their information, I have the possibility of I have to look at 2 to the power of K possibilities each of the users transmitting a plus or a minus 1. If I look at all possible possibilities the maximum likelihood detector says find that vector b that is called b 1 that maximizes the probability that R was transmitted given that b 1 R is received given that b 1 was transmitted. So, that is the maximum likelihood receiver. So, you want to maximize the probability that you receive R given b 1, we went through some steps to get the following expression b 1, the vector itself is a vector of the bits that were transmitted by each of the users b 1 through b K subscript 1.

And we said that our goal is to minimize to in order to get the maximum likelihood, we would look over all possible combinations of this vector b 1 such that we minimize this objective function and this objective function; we obtained as integral 0 to t mod R of t magnitude squared d t. The integral is only for this first term the rest of the terms; the integral has already been captured into the quantities that have been defined b 1 Hermitian times R, R is the output of the correlators; the output of the correlators. So, you take the received signal pass it through a correlator for user 1 correlator for user 2 and you get a list.

So, output of the correlators minus mine minus R Hermitian b plus b 1 plus b 1 Hermitian R b 1 and this R second R is the matrix that contains the cross correlation between the user waveforms. So, rho K j is the same as rho j K and that is the cross correlation between the spreading waveforms of users j and K correlation between the spreading waveforms reforms of users K comma j and basically we said that we can ignore this because that does not affect that. So, basically what we then would like to do is max do you have a minus sign. So, we then said lets write it as a maximization problem maximization of b 1 Hermitian R plus R Hermitian b minus b 1 Hermitian R b 1; this is what we rewrote the objective function says that you wanted to minimize something that minimization consists of some quantity minus something else.

So, basically the minus quantity has to be maximized in order for us to get the optimum solution and I hope you had a chance to look at the numerical example the numerical example was just a simple 2 user case which said that you have to account for the correlation between the spreading waveforms. So, that is the key message in the context of optimum receivers it is not just trying to correlate for user 1 and ignoring the other presence of the other user signals. So, that is not the right way to do it you have to account for the correlation between the different news spreading waveforms account for the cross correlation between spreading waveforms and once we did that we will we got feel for and the implementation of this I will just between the spreading waveforms.

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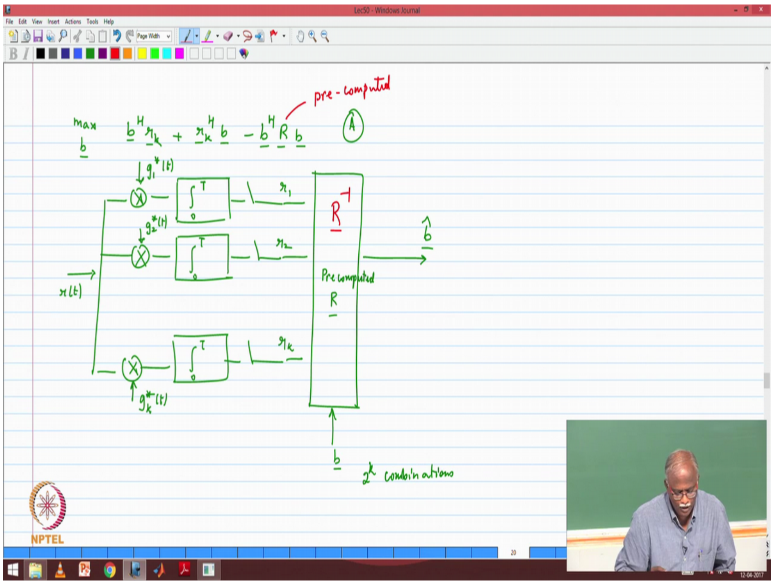

We can now think of it as the following structure where am I the following structure because what are we trying to do we are trying to implement equation a. So, basically it is b Hermitian R plus R Hermitian b minus b Hermitian R b. So, this this R is pre computed that is pre computed because I know the spreading waveform. So, that can be pre computed and fed to my computational block what I need to get is this vector of R s. So, this vector of R 1 R 2 R K is obtained by the taking R of t passing it through the match filters for each of those waveforms once I feed this then I have to try all combinations of b feed in all combinations of b, I will get a metric right all of them down find out which is the one that maximizes and then pick that corresponding combination of b in the expression.

So, that is the methods that that we have used and we found we can make the following statement that the optimum receiver will become will grow in complexity exponentially. So, the key thing of today’s lecture is; what are some sub optimal receivers.

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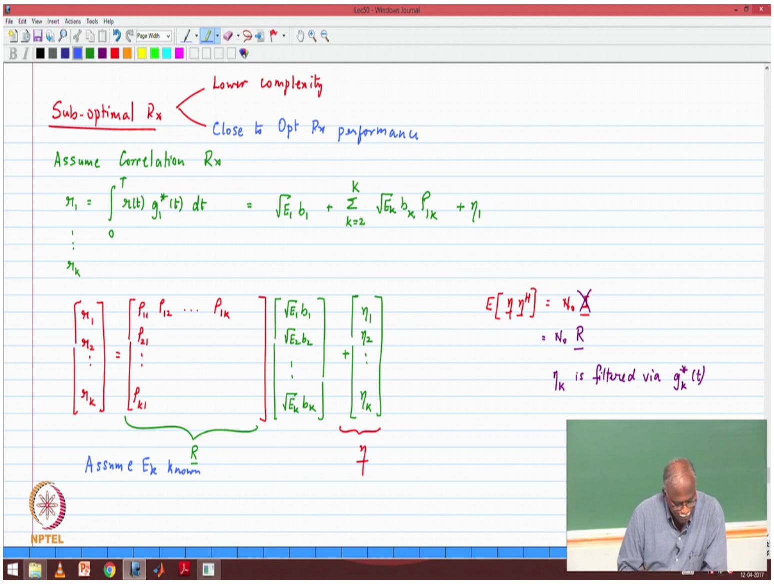

Now we call it sub optimal for the reason that it has lower complexity compared to lower complexity, but if just because it has lower complexity we will not accept any performance, if the performance must be close to optimal receiver. So, suboptimal does not mean it is a lousy performance, it is actually something that is trying to get as close to optimum or x performance, but at lower complexity and this is a very interesting type of off study.

Now, I want you to just take a moment to think about; what did we do for the maximum likelihood case, where did we start? We started by saying I will take the different combinations of the vector b, I will generate a R hat which means that that should have been the transmitted sequence and then I take the received waveform compare it with the R hat that is how it started; that is and from that we derived the optimum receiver structure and the metric that we would use for the decision now the suboptimal receiver is going to make a very interesting variation and the problem. So, one of the things that is most intuitive for us is to take R of P received signal and correlate it with each of the user waveforms.

So, assuming that it is not R of t that you are working with assuming that you have R 1 R case of t that basically the output of the correlators are given to us. So, assume that a correlation based receiver straight away assume that that is the way we are going to start that is suboptimal says do not put any constraints on the receiver you just say; what is the most likely; this one says I am going to simplify my problem. I am going to give used correlation based receiver. So, the first thing is I am going to assume a correlation based receiver correlation based receiver correlation based receiver basically says I will get R 1 through R K R 1 is obtained by integral 0 to t R of t g 1 star of t d t and like this you will do it for all the different waveforms this will give me root E 1 b 1 plus summation K equal to 2 to uppercase K root E K b subscript K rho 1 K that is the cross correlation term plus eta 1 will be the expression for the noise that is been filtered through the match filter for user 1.

So, now I have like this for all the case, if I write down this expression says I am now have R 1 R 2 to R sub K, I have these as my observations, this is related to the transmitted information in the following way, this is rho 1 1, rho 1 2, by the what is rho 1 1 waveform correlated with itself is 1. So, the diagonal elements are all one; all the non diagonal elements will be less than one because the correlation is not perfect. So, ideally you would like it to be 0, but they will be in some non 0. So, this will be up to rho 1 K and all of the this will be rho 2 1 this will be rho K one again it is a symmetric matrix. So, again you can use. So, this is my R matrix which came about in the previous in the optimum receiver as well.

Now, this is good be multiplied by square root of E 1 times b 1 square root 2 times b 2 square root of E K times b K plus eta 1 eta 2 eta K eta K. So, basically I have rewritten the formula given R of t, I need to get b 1 to b K, I said forget R of t, I am going to get this output of the correlator. So, given R 1 R 2 R K can I estimate b 1 b 2 b K. So, yes this is a estimation in the presence of noise. So, let us quickly look at the expressions that we will get. So, this is the noise term; let me call this as the eta vector eta vector is eta white noise or colored noise each of these terms, they are colored noise because they have been passed through a filter and each of these in a classical problems the noise in the different observations are independent.

Now, are these noise variances noise values independent; know the original noise AWGN would have been independent, but the noise got filtered through this correlation filters and these filters are correlated amongst each other. So, eta 1 eta 2 are not correlated they are actually correlated. So, if I were to write down expected value of eta eta Hermitian if it was white noise what will I get I get a diagonal matrix and the diagonal matrix will be N naught times I identity matrix. So, basically that would have been my noise, but this is not the case it is not I, it is actually N naught times R because the correlation between the noise terms is basically the correlation between the spreading waveforms and maybe this has a footnote this is the reason for this is because eta K is filtered is filtered via g K star of t right each of those noise terms are filtered and this g K is are correlated.

Now, look at this problem statement, if I were given a set of b s b 1 through b K and I assume that E ks are known. So, assume E ks are known assume E Ks are known to me at the receiver now for each of those b 1 through b ks. Now for me; the estimation problem now looks like a known quantity which is perturbed basically were known quantity is this one, this R matrix is fully known, anything that I vector that I assume here looks like a known quantity perturbed by some colored noise that it is now a different estimation problem. This is the one that I have. So, I would like to use a result from Proakis and this is a expression for complex Gaussian random variables.

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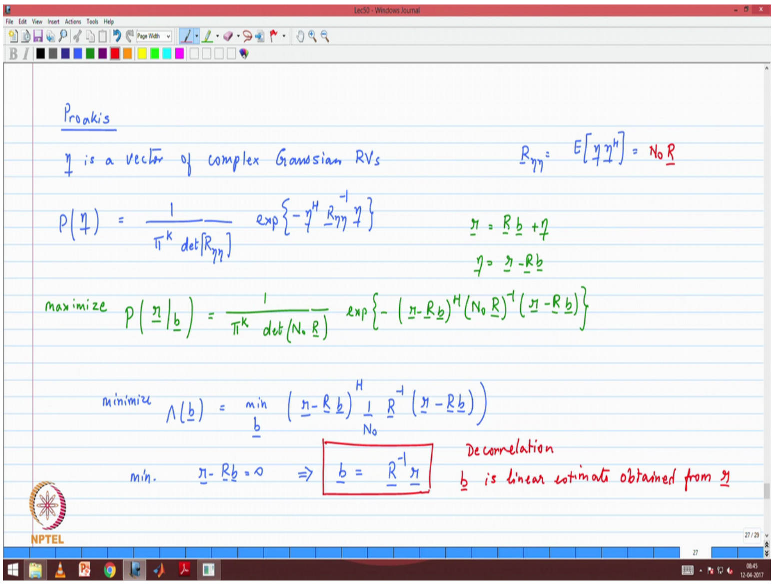

So, if I assume that eta is a vector of complex Gaussian random variables it is colored, but it is Gaussian random variables then the probability of this vector; it is not a scalar, it is a vector. So, and again basically it takes into account the statistics of the variables treated as a vector for scalar; you will get 1 over root 2 pi sigma E power minus x squared minus mu squared by 2 sigma square it is of the same form, but now in the context of vector and complex. So, basically keep in mind that there is vector and complex. So, if there are K random complex random variables there are 2 K dimensions that are present. So, basically there are 2 K Gaussian elements that are present.

So, the scale factor is pi raised to the power K the 2 goes off because of its complex; it is root pi raised to the power 2 K becomes pi power K, this is determinant of R eta determinant of this matrix, I will define what are eta eta is R eta eta is expected value of eta eta Hermitian. This is the; and we already have computed this or we already mentioned that this is equal to the noise variance multiplied by the correlation of the of the waveforms. So, this is already known to us R eta eta and taking its determinant is not a problem and the rest of the expression is E power minus you will get something squared and that comes from eta Hermitian R eta eta inverse eta simple test if you replace eta with a real random variable then what will happen you will get E power minus x squared divided by R eta eta will be nothing, but the variance 2 sigma square sigma squared will come in the denominator and it will go it will fall back to a one dimensional Gaussian random variable.

So, you can easily check the expressions are given what I am going to take this, I am going to apply it directly; why am I doing that because I now have a quantity that I am observing which is getting perturbed by this complex Gaussian vector colored Gaussian vector and I know its probability. So, the result that we are going to exploit is I want to find that value of b that I am basically; I am still interested in this maximization. I want to maximize the following, I want to maximize the probability of R. R is not the vectors that received signal, sorry, sorry, in now it is the output of the correlators R given b it is a different problems; it is not a the scalar R, it is now the vector output. So, I want to find that b which maximizes this. So, if I ask that type of a question then you say this is the R s; this is the R s; this is the input; this is a constant matrix.

So, therefore, this looks like the mean value of R perturbed by a complex colored Gaussian noise. So, the problem becomes a very simple problem in the context that if I want to maximize this probability it is like saying maximize the probability of that particular noise occurring. So, with which becomes one over pi power K, I am using the same value determinant of N naught times R N naught is a; as a constant R is the matrix E exponential eta Hermitian eta. So, it is R is equal to R times b plus eta. So, eta is nothing, but R minus R b. So, this will be written as eta Hermitian which is R minus R b Hermitian the matrix N naught times R inverse times eta which is again write it as R minus R times b; R times b. This is what I want to maximize; I want to maximize the probability and the probability of this is basically the probability of maximizing the probability of the noise. So, this just like in the classical detection cases; what happens its E power minus something and I want to maximize so; that means, I must I must minimize that exponential.

So, this is the same as saying I want to minimize the following; I want to minimize the following statistic or the objective function; this is the same as saying I want to minimize over all possibilities of b R minus R b Hermitian; this is a scalar. So, the inverse can be written as 1 by N naught R inverse R minus R b, I want to minimize this, I am going to make a argument which I want you to pay attention to this is a quadratic form which is greater than or equal to 0. So, therefore, the minimum that it can achieve is 0 and a solution that achieves the minimization minimum is achieved if R minus R b is equal to 0 which means that I now have a estimate for my decision vector in terms of R inverse times R that is the optimum for the problem statement that I have framed. How did I modify the original problem I to I; add R I modified it by saying I am going to replace R with R 1 through R K the output of the correlators.

Now, I want to minimize the; maximize the probability that this vector is received, if I transmit a particular b and that says the b that will give you that result is R inverse times R; this R inverse R is the correlation matrix when you multiply by R inverse that is why it is called a de-correlating receiver because you are doing the inverse of the correlation you are removing the correlation from R and making a decision. So, this is a interesting way because R is a matrix of a constant matrix. So, therefore, b is a linear estimate is a linear estimate which is obtained from estimate obtained from R the previous one was a non-linear estimation we did not know how the b and R were related. So, this was a estimate obtained from R.

So, it becomes a very very simple problem, if you want to do a de-correlating receiver and what is it that we will do is take the received signal R of t, pass it through a bank of correlators your R 1 through R k, this R is already known, if you can pre compute R, you can pre compute R inverse. So, the de-correlating receiver would in this case, with this step would be replaced by R inverse that is it you multiply by R inverse then you will get the estimate for the bits that were transmitted. Now of course, there is a question of did it really take care of the multi user interference all of that is very interesting. So, what I would like you to do is just work out the simple case how the d correlating receiver d correlating receiver.

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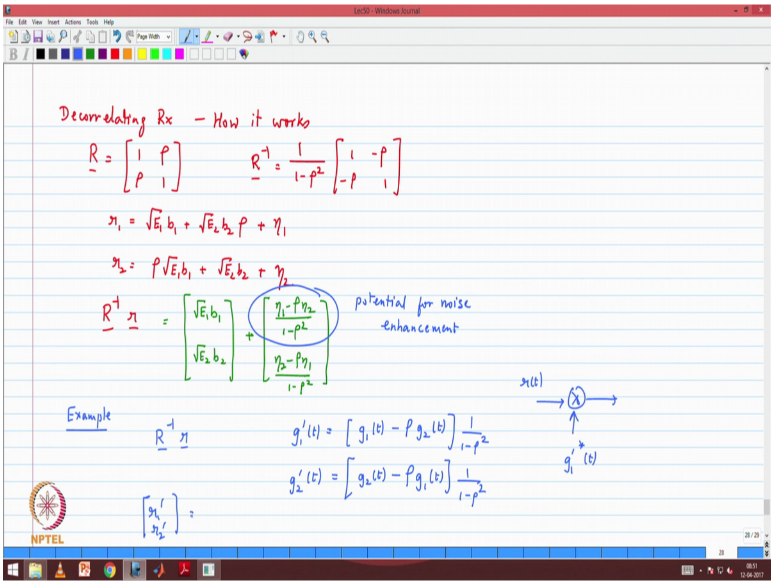

How does it work? How it works? Simplest is to look at a 2 by 2 case, the R matrix will be of the form 1 times rho rho times 1, there is only one cross correlation term and R inverse comes out to be 1 by 1 minus rho squared 1 minus rho minus rho 1 and we know that R 1 R 2 are given by the following expressions root E 1 b 1 plus root E 2 b 2 times rho plus eta 1 R 2 is given by rho times rho E 1 b 1 plus root E 2 b 2 plus eta 2.

Now, see what you get when you do R inverse times R, you will find that the result is a very very interesting one, the result is square root of E 1 b 1 square root of E 2 b 2 plus noise terms eta 1 minus rho eta 2 by 1 minus rho squared this one will be eta 2 minus rho eta 1 by 1 minus rho squared a de-correlating receiver completely separated out the multi user interference, it completely knocked it out that is why we sometimes refer to it as a 0 forcing, it completely it is not going to allow even any cross correlation if it is going to completely try to separate out because notice that I can now looking at the first variable I can make a decision on b 1 looking at the second variable is on b 2.

Now, it has completely separated b 1 and b 2 in the in the process the problem with this, 0 forcing is it did not care what it did to the noise it was focused on separating the users. So, potentially this 1 minus rho squared sitting the denominator is can be a problematic one because rho is less than one. So, if you this could lead to potentially there is a potential for noise enhancement, but you may say well I am willing to take that penalty because it is a very simple receiver, it completely separated out the users and therefore, it is its very interesting for us.

So, potential for noise enhancement yes we recognize it, but it is still a very very powerful receiver. So, de-correlation receiver is very commonly used for CDMA systems because of the strength of what it can do with for the complexity near far problem no issue because your de-correlation will remove the effect of strong even if a even if this root E 2 was much larger than root E 1 notice that it will not cause any you have separated them out. So, you in some ways you have eliminated the near far problem also. So, the example that we did look at remember we try to do a 2 by 2 example numerical example please go back and try R inverse times R and you will find that it picks the same candidate vector as the maximum likelihood vector.

So, please verify that this actually works very well the last thing that I want you to look at is if I gave you a modified waveform for user 1 which is g 1 prime at the receiver, at the transmitter you use g 1 at the receiver if I use this waveform g 1 of t minus rho times g 2 of t into 1 minus rho squared. So, what am I doing? I am taking R of t, I am going to multiply it with this g 1 prime star of t and then do the integration. So, basically instead of using only g 1 star I am going to give you a modified de-spreading sequence, please check what comes out of this what comes out at the output and you will be very very surprised.

Similarly, try for g 2 star of t which is given by g 2 of t minus rho times g 1 of t 1 minus 1 minus rho square please compute what you get; let us call that output as R 1 prime and R 2 prime. So, in tomorrow's lecture, you will tell me what R 1 prime and R 2 prime are and that will tell us why the de correlating receiver works what is the strength of the de-correlating receiver and what is that underlying principle that makes the de-correlation receiver work so effectively. So, that will be the answer to that question.

Thank you, we will see you tomorrow.