

Introduction to Wireless and Cellular Communication
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Lecture - 47
CDMA Receivers
Multiuser environment

We begin lecture 48 and time permitting we will extend it for the additional hour for the as the makeup for what we should have missed on what you missed on Saturday. What we will cover today would be probably the heart of a CDMA system; we have talked about multipath environment so now we are going to go into the multi user environment. Once having understood multi user environment we will talk about the impact why is it power control so necessary for a CDMA system and if you remember when we talked about channel inversion we said that there is a phenomenon called near far problem.

So, today we will discuss the near far problem and what that has on the CDMA system capacity and in digital communications we do study a section called optimum receivers. And I hope you will be able to really put all of the understanding of optimum receiver in our discussion on CDMA because this give us a very rich framework on which to understand and reinforced what you would have studied for optimum receiver. So, the scope of todays two lectures let us quickly run through.

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L 47 Recap

Multipath channel $x(t) = \sum_{l=0}^L h_l(t) u(t - lT_c) + z(t)$

-(L+1) taps
↳ (L+1) taps Rake receiver

$$Y = \sum_{l=0}^L h_l^* y_l$$
$$= b\sqrt{E_b} \sum_{l=0}^L |h_l|^2 + \sum_{l=0}^L h_l^* y_l$$
$$E[|y_l|^2] = N_0$$
$$E[|y_l|^2] = N_0 + \frac{E_b}{Q} \sum_{j=0, j \neq l}^L |h_j|^2$$

neglect if Q large

NPTEL

So, the quick summary of lecture 47, lecture 47 we were focusing primarily on the multipath environment. So, when it comes to multipath always remember that we are working with a chips based equivalent channel. So, the received signal r of t is a summation of L plus 1 copies of different signals that I have been spaced at chip duration t minus $L T_c$. So, this is a very useful form for you to remember and of course, there is the impairment z of t . So, the multipath components themselves did not occur at chips spacing, they occurred at arbitrary spacing, but you have been able to obtain an equivalent channel which behaves very similar to what was your truth channel.

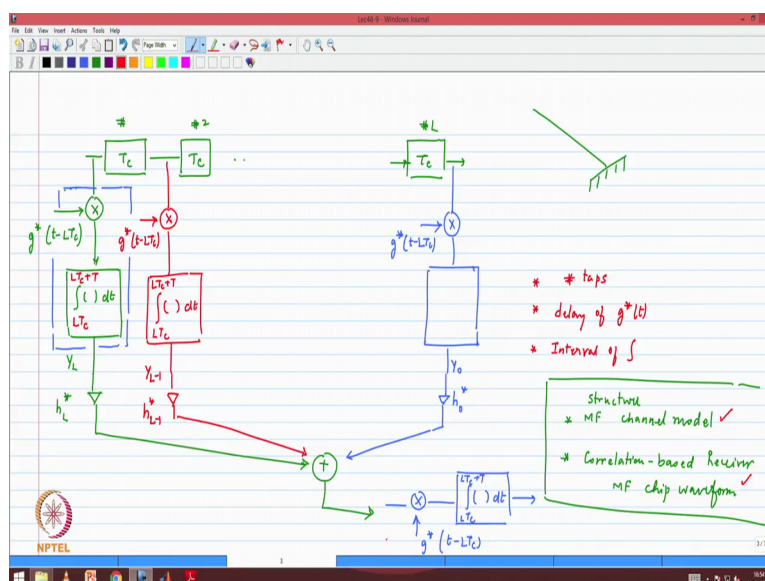
So, the characteristics of our channel are that we have included L plus 1 taps in the channel this also lead us to have L plus 1 taps in the rake receiver. So, the rake receiver is usually correlated to the number of dominant taps and for us since we have a working with a chip space model it is for each of those spaces.

The output or the decision statistic was obtained by combining the output of the different branches of the rake receiver. So, there are L plus 1 copies of the signal that we are combining using the conjugate of the chip space channel h_l conjugate y_l , this is what gives us the expression that that we are looking for. So, this basically gives us b times $\sqrt{E_b}$ summation l equal to 0 through uppercase L mod h_l squared plus summation l equal to 0 through L h_l star it n_l . So, that is our expression that that we have obtained and this is how we have got.

So, we did this for the health for the first branch, basically we did note that the components of the signal are can or of the noise can be characterized. So, basically expected value of ηL squared this was N naught white noise, but if you look at the output of the correlated there is some component which is related to the cross correlation or the autocorrelation at different lags. So, expected value of n_l , n_l squared 1 is η the other 1 l . So, this will be N naught plus E_b divided by Q summation l is equal to 0 through L minus 1, let me just do it little bit differently.

So, this will be j equal to 0 through L not equal to $l \bmod h$ of j squared and this is the term that we were going to neglect under the argument that Q is large the upper L is small compared to Q . So, Q if is large.

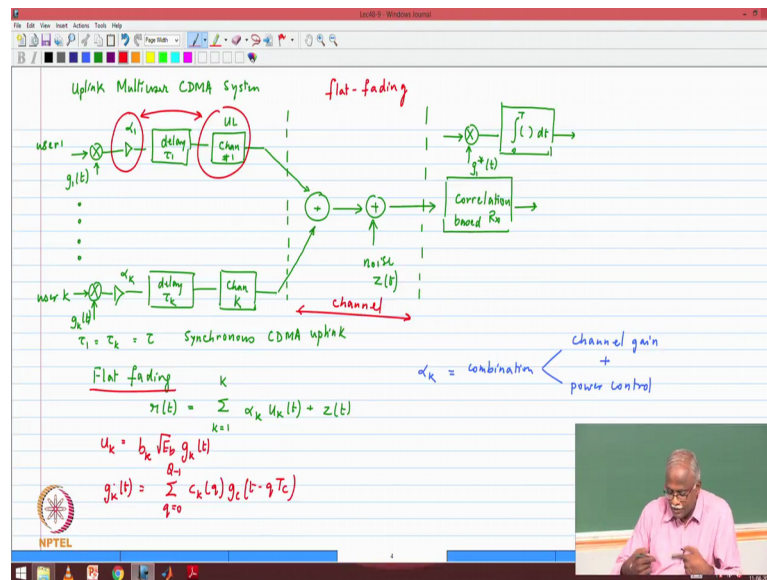
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So, that is the summary of the rake receiver basically this is the form that we had looked at there are $L + 1$, $L + 1$ taps and basically each of those taps gave us a went through correlation based receiver which is multiplied by g^* and then followed by integration then the corresponding coefficient added together.

The observation that we made was that this block which is within the blue in each of those branches because this the linear operation can be moved after the summation in which case we get a structure which is a structure where the first part where you have the delay chain followed by h_L^* conjugate h_{L-1}^* conjugate that is a matched filter to the channel. So, first part is, when you after you have moved the after you have moved a correlation and integration outside, what you have is a matched filter to the channel first followed by a matched filter to the chip waveform. So, basically you have match to the channel and to the waveform and you have been able to get a decision statistics in which we have shown that there is the diversity benefit is present in the system.

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So, that is a quick summary of the multipath where we handle multipath and several things that will become as you read and you know go through the expressions you will see that yes we have made some approximation, almost all of the approximation will rely on the fact that Q is large and once you have that these approximations are reasonable.

Let us move from the multipath environment to the focus on multi user environment I have just added a little bit more information to the figure that we had in the last lecture each user is spread by a corresponding waveform user 1 by g_1 , user 2 by g_2 , and user k by g_k . These α_k 's you can think of them as a power control basically you can adjust the amplitude of the signal. As they go through the channel right they could encounter different delays because this is like the uplink if user 1 transmits slightly differently from user 2 or if the distance of user 1 is very different from user 2 there will be a difference in the delays in which they arrive at the base station. So, to account for that we have included these delays we have made the assumption that the channel is a flat fading channel, so the channel is a flat fading channels, which means that there is no dispersion, but there can be a channel gain and that channel gain is captured in terms of a complex constant.

So, in principle this α and this could be combined to reflect a single coefficient. So, you have the transmitted signal multiplied by a complex gain which is because the

channel gave you gain or you did power control does not matter you can combine them to into a single complex gain.

Now, each of the users transmits their signal and the signal encounters this channel. Now active based station I am listening to all the signals because I have all of them are using the same frequency. So, in other words in the channel these different multiuser signals get combined. So, this is what is my channel, the channel basically combines all the individual user signals and of course, then will add some additive noise and that is a composite signal that is given to the receiver.

So, the task at the receiver is to be able to detect any one of the k users that have been (Refer Time: 09:48) transmitted the signal and so the assumptions once more, it is a flat fading channel so r of t can be written as summation k equal to 1 to K , k users $\alpha_k u_k$ u_k is the transmitted signal transmitted signal depends on the bit at that instant of time the energy level with which you are transmitting g_k is the spreading waveform and then g_k itself is described in terms of the chip waveform. So, α_k can be treated as a combination of channel gain and the power control.

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The slide content is as follows:

Correlation-based Rx

$$y = \underbrace{\alpha_1 b_1 \sqrt{E_b}}_{\text{desired}} + \underbrace{\eta_1}_{\text{single-user noise}} + \underbrace{\sum_{k=2}^K \alpha_k b_k \sqrt{E_b} \int_0^T g_k(t) g_1^*(t) dt}_{\text{MUI}}$$

Annotations on the slide:

- R_{1k} depends on $\frac{1}{Q} \sum_{q=0}^{Q-1} c_k(q) c_1^*(q)$
- Multi-user environment:**
$$N_0 + E_b \sum_{k=2}^K |\alpha_k|^2 \underbrace{E[|R_{1k}|^2]}_{\frac{1}{Q}}$$

So, that is a reference point and we said I think I believe we came to the point of writing down the decision statistic, if I want to pick up user 1 signal I will correlate with the g_1 star of t and integrate 0 to t and that gives me a decision statistic which give the combination of the desired $\sigma \alpha_1 b_1 \sqrt{E_b}$ plus the noise filter through the z

of t filtered through g_1 plus the other terms that are present because other users signals are also there.

So, R_{1k} is a cross correlation between g_k and g_1 . So, similarly you can get for all the different terms. So, this is the multi user interference part. So, your decision statistic contains that part that would have been there if it was a single user present plus some other impairment that is part of the multi user or multiuser interference. So, this is the framework and I believe we went through and justified that the expressions that we will get are of this form, but let me just pick it up from here and develop it in today's lecture. So, this is where we begin today's discussion.

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MUI Rapport (Appendix)

Gaussian (CLT)

For a given b_1

$$E[y|b_1] = \alpha_1 \sqrt{E_b} b_1 + E[\eta_1] + E\left[\sqrt{E_b} \sum_{k=2}^K \alpha_k b_k R_{1k}\right] \quad E[b_k] = 0$$

Variance

$$\sigma_y^2 = E[(y - E[y|b_1])^2] = E\left[\left|\eta_1 + \sqrt{E_b} \sum_{k=2}^K \alpha_k b_k R_{1k}\right|^2\right]$$

$$= E\left[\eta_1^2 + E_b \sum_{k=2}^K \alpha_k b_k R_{1k} \sum_{m=2}^K \alpha_m^* b_m^* R_{1m}^* + 2 \operatorname{Re}\left[\eta_1^* \sqrt{E_b} \sum_{k=2}^K \alpha_k b_k R_{1k}\right]\right]$$

$$= N_0 + E_b \sum_{k=2}^K |\alpha_k|^2 E[R_{1k}^2] \quad E[\eta_1^* \sum_{k=2}^K \alpha_k b_k R_{1k}] = 0$$

$$\sigma_y^2 = N_0 + \frac{E_b}{Q} \sum_{k=2}^K |\alpha_k|^2 \quad \approx \frac{1}{Q}$$

So, our key goal is to characterize multi user interference MUI and if you look at the reports book report it is in the appendix it says that this impairment or this presence of large number of users makes the multi user interference appear Gaussian and the argument is via central limit theorem, via central limit theorem. So, what we would like to understand today is how is this multi user interference going to affect my performance of my system what are some of the aspects and we want to. So, here is the way we would like to understand it.

So, supposing we were given the bit transmitted by user 1 for a given b_1 that means, the bit transmitted by user 1, the expected value of y the decision statistic given b_1 that is I know b_1 and given that this turns out to be $\alpha_1 \sqrt{E_b} b_1$ because I know that

that is the signal that is being transmitted plus this is the mean value. So, if you recall whenever you did the digital communications if you were transmitting a plus 1 your mean value will be plus 1 with some Gaussian noise around it. Similarly if you were transmitting a minus 1 there will be some Gaussian noise, so there is the mean value and a Gaussian.

So, basically what we are going to do is make an equivalent understanding where there is the mean value which is represented by the bit that was transmitted, there is a noise that is present because of the Gaussian noise and because our multi user interference also happens to look Gaussian so we are going to draw it in this fashion it is as if you had a Gaussian noise of slightly larger variance. So, basically that is the impact or at least a way to start to visualise the multi user interface. So, the multi user interference expected value of y given b_1 this would be expected value of η_1 plus expected value of $\sqrt{E_b} \sum_{k=2}^K \alpha_k b_k R_{1k}$. So, this is the mean value.

So, basically what we can show is that this expectation is of noise which goes to 0 this will be an expected value of b_k 's, b_k 's are expected value of b_k equal to 0. So, this term also goes to 0. So, the mean value is just like before it is if you transmitted a plus 1 it will be plus 1 minus 1 minus 1 that is the mean value now another important extension to this picture is what is the variance. So, the mean value is given by a bit value, but what is the variance and that is what is going to be the spread of the red signal. So, σ_y^2 is the variance of the output of the correlator for user 1 this is given by expected value of y minus expected value of y magnitude squared. So, expected value of y given b_1 is $\alpha_1 \sqrt{E_b} b_1$. So, if I write it down as y minus this the only the noise terms will be present. So, this will be equal to expected the signal part gets cancelled what will be present is magnitude of η_1 plus square root of $E_b \sum_{k=2}^K \alpha_k b_k R_{1k}$ magnitude squared.

So, like before expand the summation this will be expected value of the whatever is the quantity within the bracket multiplied by it is conjugate. So, the first term is $\text{mod } \eta_1^2$, second one is the term with the conjugate of itself. So, it will be $\sqrt{E_b} \sqrt{E_b}$ gives me E_b , $\sum_{k=2}^K \alpha_k b_k R_{1k}$, $\sum_{m=2}^K \alpha_m \text{conjugate } b_m \text{conjugate } R_{1m}$ plus then you have 2 times the real part of $\eta_1 \text{conjugate } \sqrt{E_b}$ again it is just expanding out

this term summation k equal to 2 to uppercase K $\alpha_k b_k R^{-1}_k$ and keep in mind that there is an expectation outside.

So, the first one is the noise term AWGN. So, the first term will be given by the noise variance N_0 , now expected value of $b_k b_m^*$ if k is not equal to m these will be independent and their 0 mean. So, that term will cancel, so what will be left with this only when k and m are the same. So, it will be $E b$ summation just a single summation now k equal to 2 to uppercase K α_k^2 expected value of R^{-1}_k magnitude squared b^2 will be equal to plus 1 and of course, this term will also go to 0 because you get expected value of b_k .

So, this is a very very important equation I just want you to sort of pay attention to it what is it that we have done - we have taken the multi user signal correlated with the waveform of user 1 applied as if your followed by the integrator and you got a decision statistics, mean value is given in the previous equation the variance of the decision statistics is given, so this is more or less going to be a foundational equations for us in our analysis. And expected value of R of 1 over k , R^{-1}_k this is approximately 1 over Q that is how we would design the codes and that would be the case when you had the p -n sequences. So, this expression is a very very key expression for us. So, which is σ_y^2 is equal to N_0 plus $E b$ divided by Q summation k equal to 2 to uppercase K α_k^2 . So, that is the variance of the decision statistic.

Now, we get into the very interesting parts of this discussion. So, this is a CDMA system where we do fast power control and so we will pick up several of these key concepts that we have discussed earlier.

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Uplink MS \rightarrow BS

Fast PC \leftarrow CDMA 2000
WCDMA

$|x_k| = \text{const} \quad \forall k \quad \text{w.o. G}_1 \quad |x_k| = 1$

$\sigma_y^2 = N_0 + \frac{E_b}{Q} (k-1)$

$\text{SINR} = \frac{E_b}{N_0 + \frac{E_b}{Q} (k-1)} = \frac{\frac{E_b}{N_0}}{1 + \frac{1}{Q} \frac{E_b}{N_0} (k-1)}$

$D_f \leq 1$
 $\text{MUZ} \uparrow \quad D_f \downarrow$

MUI

$\mu_{\text{MUI}} = 0$

$\sigma_{\text{MUI}}^2 = \frac{k-1}{Q} \frac{E_b}{Q}$

Impairment as White Noise

- Large # users $K \uparrow$
- Spreading Factor

NB System GSM

Interference 6 Tier - 1 users
is "coloured noise"

Ex

- CDMA 32 users
- 31 interfering
- DL
- 31 + other channels

So, this is the uplink. So, the mobiles are transmitting to the base station mobile to the base station and the base station is the one that controls the transmit power save it controls the power, so we have fast power control. So, since it is the uplink mobile power is controlled depending upon the system CDMA 2000, it is 800 times per second, 1500 times per second for white (Refer Time: 20:39) CDMA. So, it is we can call it fast power control for both of the systems that we are familiar with.

Now, a very key concept what is the role of power control? It is to make sure that all the users reached the base station with approximately same power. So, at least notionally we want all the α_k 's to be equal to a constant all of them to be the same constant right the only then that is what tells you that all of them are being received at the same time. So, this is for all values of k and without loss of generality we can take that constants to be equal to 1. Just this is you know you can choose what that constant is all of them to be equal to 1. So, now, comes a very interesting observation σ_y^2 is given by N_0 plus E_b by Q into k minus 1. So, basically there are k minus 1 interfering terms and each of them contributes E_b by Q .

So, now comes the calculation interesting calculation of SINR, what is the signal to interference plus noise ratio E_b is the signal component, the noise component after I have taken out the mean value that was this is the noise component. So, this is given by N_0 plus E_b divided by Q into k minus 1.

Another interesting form of this equation divide numerator and denominator by N naught E_b by N naught divided by $1 + 1/Q$ E_b by N naught in took $k - 1$, very interesting form because what it says is E_b/N naught what would have been is a single users SINR single user signal to noise ratio. But now because other users are present though they have been suppressed by a spreading factor Q they are still present they are contributing to that. So, the denominator is a number that is greater than 1. So, this is going to take you away from the performance of a single user system and by how much is it going to take you away is called a degradation parameter. So, if you call this as $1/d_g$ the degradation parameter is this quantity d_g degradation parameter and this is typically less than worst case it will be equal to 1, best case it will be equal to 1 always it is mostly it is less than 1.

What are the things that affect d_g ? So, if the multi user interference, multi user interference goes up what does that mean basically this E_b/Q into $k - 1$ when will multiuser interference go up if k goes up if k goes up, what happens to d_g d_g goes down it becomes smaller so; that means, your effective SINR is going to become smaller. So, again you can see the relationship between the number of users your spreading factor, if you reduce your spreading factor what will happen - you will hurt yourself again because the degradation parameter is going to become smaller. So, all those things can be understood in this fashion.

So, here is a couple of points regarding the multiple user interference multiuser interference MUI. MUI the mean value is 0, it is a 0 mean basically that is what we have shown and the variance sigma squared of MUI is given by this quantity which is $k - 1$ E_b divided by Q because the other one is the AWGN component this is the multiuser way comp variance component. So, this is what we would like to think of as our equivalent model. So, if you go back to our model, where did we draw that. So, that wider variance is now caused by the presence of the multiuser interference and the expressions for the interference have been given as follows. So, original variance with only the AWGN plus the multiuser interference the multiuser interference component can be captured in this form.

So, it is 0 mean and it is Gaussian in nature and it is got a certain variance that depends on the number of users it depends on your inversely proportion to your spreading factor

smaller the Q logic the variance and therefore, I want you to start thinking along these lines.

Now, couple of points to sort of set the context very clearly. So, compare this with a narrow band system, compare this with a narrow band system such as GSM. So, narrow band system such as GSM. Now what can you tell me about the interference that you will see in a GSM system, you see interference from a co channel cell it is not from your own cell it is coming from somebody else who is using the same frequency. Now if I look at here 1 users what is the maximum number of co channel cells 6. So, it is still a small number. So, the single narrow band system the interference has got a very different characteristics, interference comes from 6 tier 1 users at most tier 1 users and the interference still though there are 6 of them still looks like a GSM signal. So, which the way we characterize that we say that the interference is like coloured noise, it is got a strong spectral shape which looks like the GSM signal it is coloured noise. So, the interference management in a GSM system is has to keep in mind that the interference is moves away once it becomes an interference dominated system it moves away from AWGN, it moves into a noise which looks highly coloured it is got a strong spectral shape.

On the other hand if you move over to the CDMA system what are we saying we have large number of users in the system the multiuser interference now looks Gaussian and therefore, the our ability to work with this interference is somewhat different. So, the impairment in this case behaves like white noise or manifests itself like white noise impairment as white noise.

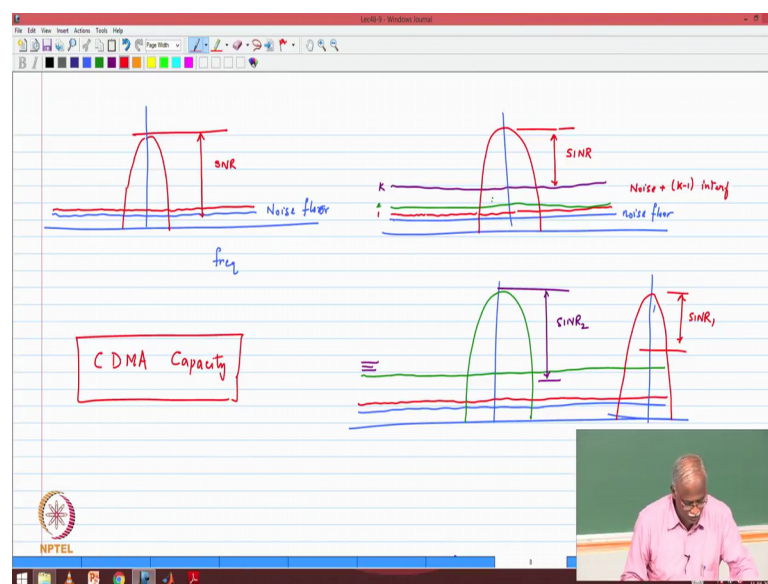
The reason for this is that we have a large number of users large number of users and then central limit theorem kicks in and then we are able to make the argument large number of users I will just say K up with an up arrow large number of users and also the fact that the spreading factor plays a role, spreading factor plays a role in the larger spreading factor the presence the spreading factor also has a role to play. Now think very carefully I have a CDMA system with 32 users in the system 32 users just as an example. So, it is a CDMA system 32 users uplink talking about uplink and the base station I want to detect user 1, how many interfering signals are present, how many interfering signals are present – 31, 31 correct very good.

So, 31 interfering signals are present. Now think and answer this question I have changed it to the downlink if 32 users are in the system how many signals is the base station transmitting 32, how many interference and mobile now receiving from the base station base station is transmitting 32 signals 32 user signals how many interference signals do I see - 31 user signals. But base station transmits something else also, it transmits control information it transmits a pilot channel it transmits at the end of the day anything that is not mine user signal (Refer Time: 29:47) the pilot channel is actually useful for me, I use it to the channel estimation, but when I want to detect my user signal what happens it is actually interference to me. So, in downlink you have to keep in mind that it is 31 plus other channels right whatever think the base station is transmitting is going to cause interference to me.

In addition other base stations transmitting also is going to cause interference to me. So, at in CDMA system the role of interference is some that we really have to get our hands on yes it looks Gaussian, but at the end of the day you got lots of sources of interference which you want to be careful about and be able to manage and control. So, this is our starting point in terms of our entry into the multiuser system.

Any questions regarding the points that we have mentioned, because just want to be sure that we are clear on this and then we build on this in the next few minutes any questions on the multiuser part.

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K users each one has got a different spreading factor there is this alpha coefficient which corresponds to power control plus channel gain and basically if I do proper power control all these signals will arrive at the base station with the same power level and which then tells me that I can characterize my noise in the following way signal E_b divide by N_0 AWGN and then the other terms assuming ideal power control will be E_b by Q into k minus 1 and that can be related to a degradation factor which then tells me that my E_b by N_0 will always be worse than what I will see in just a single user AWGN channel. Any questions?

Now we want to spend a few minutes on intuition because the intuition helps us in building up the. So, here is a very qualitative perspective this is frequency, AWGN I always will show like this. So, this is the noise floor noise floor noise floor.

Now, what happens in a in a CDMA system direct sequence spread spectrum, you transmit you take a narrow band signal and then you spread it. So, basically the nice looks like it became slightly higher because you spread the signal. At the receiver what do we dispread this signal the dispread signal looks like this my SNR is with respect to the noise floor only 1. So, this becomes my signal to noise ratio spreading disspreading what happens in spreading what happens in disspreading are you care about this.

Now, move from this into a multiuser environment I have a multiuser environment noise floor like before noise floor. Now I have user number 1 user number 1 is adding some increase in the noise floor then user number 2 dot dot dot and then finally, user k . So, user 1 is noise floor red is user number 1 green is number 2 and purple is user k . So, each of them have been spread. So, this is my environment in which I am detecting my signal

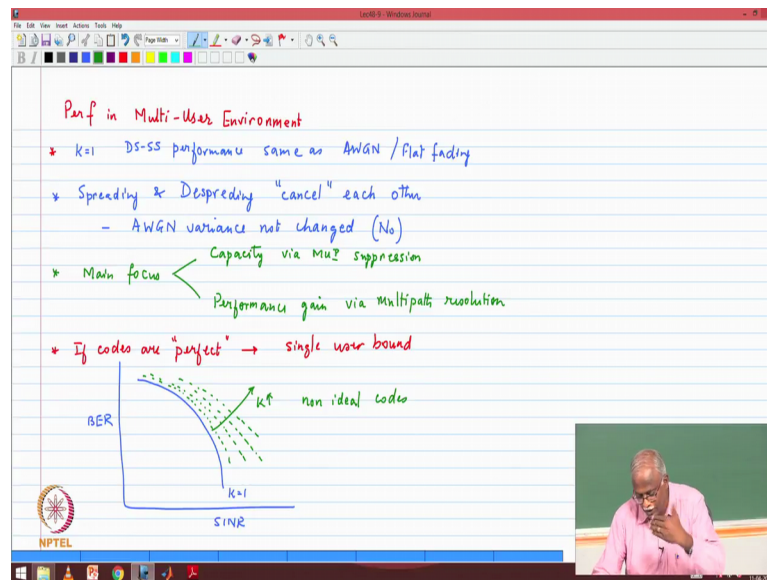
Now, if I detect red user, user number 1 user number 1 like before I dispread my signal others are not affected as part of the disspreading process notice something has happened something some change has happened with respect to the previous figure my threshold or difference with respect to the impairment is no longer SNR it is SINR and it has become smaller noise plus difference is no higher in a CDMA system. But assuming that and so basically this level will be the level of noise plus k minus 1 interference that is the reference impairment level. So, very very important that I achieve this picture and I understand.

Now, I want you to think about a anomalous situation anomalous means things are not ordinary. So, noise floor user number 1 user number 1 and user number 2 is transmitting with more power than they need to that is user number 2 all of the other users no user number 3, user number 4, users k. So, basically there are lots of other users the 1 anomalous user is the green user now if I am detecting green, detecting green notice what will happen detecting green the green part comes out of the noise floor all the others settle down he gets a pretty decent take the others to be at this level get a decent SINR. So, SINR of user 2, but what if I had to detect any of the other users what is the situation with other users let us say I want to detect the red one the red one is hear the noise floor because of the green is much higher and effectively the SINR of user 1 is actually quite bad compared to SINR of user 2. User 2 is quite happy because he does not see any problem he just see he is detecting a signal he is; but the scenario is that SINR 1 for all the other uses other than user 2 are getting affected. So, keep in mind that this is our starting point to understand the complexities of a CDMA system how 1 users, users signal actually causes interference to others and it can cause interference in ways that are very very detrimental to the overall system.

So, now another thing that you will be able to appreciate from this sort of this qualitative figure is that if I start adding more and more users eventually I will reach a point where I cannot detect my signal. So, there is a capacity in understanding that comes from the interference floor. There is another capacity perspective that you get from looking at this mismatch power if 1 user is transmitting with a lot of power he is going to kill the capacity of the system and therefore, we need to be careful with. So, this is way to start thinking in terms of CDMA capacity, but we are going to formalizes in a in a short while, but I wanted you to sort of start thinking in terms of the multi user and it is impact

Any questions on the on the multi user part that we have just now (Refer Time: 37:32) discussed. If not we will move quickly into the next part of our understanding and we will try to build on the multi user or multiuser environment, but before we go there I just want to quickly summarise the results that we have obtained so far. So, the presence of multiuser inter multiusers depends on the number of users it depends on your spreading factor we assumed that the in impairment is Gaussian in nature and therefore, we are able to characterize it we are able to work with that.

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So, now I want you to give you a reference point that is used quite often in a CDMA system. So, performance in a multi user environment, performance in multi user environment, few comments about that and then we move into the capacity discussion multiuser environment. So, the first one is if K equal to 1 number of users equal to 1 direct sequence spread spectrum performance is same as a AWGN flat fading channel, same as AWGN or flat fading there is no gain that you get for a single user system this is a point that we had made very early in our discussion.

The second aspect is spreading and despreading they cancel each other that is there their inverse operations. So, spreading and despreading are inverse operations. So, as far the signal is concerned signal comes back to it is narrow band form, spreading and despreading I will just cancel each other, but please interpret it as the inverse operations cancel each other as far as the signal is concerned cancel each other, but it does not do anything to the noise. So, it does not do. So, the AWGN variance, AWGN variance is not effective not changed. So, if it was N_0 in the narrow band system it is going to be N_0 in the CDMA system as well.

So, then what is the interest in a multi user environment. So, the main focus for the direct spectrum, direct sequence spread spectrum is the capacity, what are the benefits that we can get in terms of the capacity via the multi user interference suppression because of my spreading process and the receiver techniques that I do. And of course, there is a

performance gain if I have dispersive channels performance gain because of multipath resolution. So, that is why we are interested in direct sequence spread spectrum because it gives you multipath resolution and rake receiver will combine it giving you a form of diversity. So, the combination are that I have spreading which will help me work with multiuser interference because it is a spread spectrum system wide band signal I get the benefits of diversity.

So, here is the probably the most important statement if my spreading codes are perfect; that means, they are perfect in terms of mutual orthogonality and self orthogonality then what we will achieve is called a single user bound. What that means, is the because the code are perfect what I will achieve looks like a single user the performance of other users is not visible at all BER versus SINR I will get the performance of a single user it will it will look like what this is like k equal it will look like k equal to 1 though k is not equal to 1 this is what it will look like this is a BER that you will get.

On the other hand if I do not have perfect codes then what I will see is a slight degradation if when there are 2 users, then this degradation will become even more when it becomes 3 users then eventually it starts to become worse and worse because of the presence of multi user interference. So, BER graphs will increase with as K increases for non ideal codes which is what all practical systems will have.

Our p-n codes m sequences scrambled are the ideal codes perfect codes no they do have some leakage that will come in. So, all of our CDMA systems will have this. So, whenever you look at the performance in a CDMA system multiuser environment you will always talk about a single user bound because you always want to see how far of are you from this single user bound the closer you get a single user bound the better is your system and therefore, the good how good you have how well you have designed your system. So, single user bound serves as a reference for us for any of our discussions.

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Near - Far Problem

K users in system

$$MUI = N_0 + \frac{1}{Q} \sum_{\substack{k=1 \\ k \neq i}}^K E_k |\alpha_k|^2$$

(ith user)

Assume $E_k = E_b \forall k$

With Perfect PC $|\alpha_k| = 1 \forall k$ $MUI = N_0 + \frac{1}{Q} (K-1) E_b$

* Range of signals ~ 90 dB

* With PC errors $|\alpha_k| \neq 1$

$$SINR_i = \frac{|\alpha_i|^2 E_b}{N_0} D_{g,i}$$

(user i)

$$D_{g,i} = \left[1 + \frac{E_b}{N_0} \frac{1}{Q} \sum_{k=2}^K \frac{|\alpha_k|^2}{|\alpha_1|^2} \right]^{-1}$$

NPTEL

Let me also tie in together the near far problem and help put a contexts to the situation that we discussed. So, when there are K users in the system k users in the system then we know that the multiuser interference the variance of that this will be given by N naught multiuser interference seen by ith user is N naught plus 1 over Q summation k equal to 1 through uppcase K, but K is not equal to i because i is the decide user everyone other than i is E k mod alpha k whole square. We will make one assumption just to get the insight from this particular. So, we will assume that the E b with which we want to transmit each of the signals is the same, E k is equal to E b for all values of k that is each of the single constellations are all normalise all of them transmitting the same value.

So, under this assumption with perfect power control we already know the answer with perfect power control we will have mod alpha k equal to 1 for all values of k and my multiuser interference seen by the ith user is the same as a multiuser seen by each of the others given by N naught plus 1 by Q into k minus 1 into E b. Now just for you to have a feel for the practical impact when a base station is receiving in a CDMA system is receiving without power control that is one user is close the base station one user is far away from the base station the dynamic range can be as much as 90 d b, 90 d b. So, the range of a signals range signals is approximately 90 d b. So, your power control range is actually in a ninety d b range it can even ask somebody to go up by some amount come down the total dynamic range of your power control is very very large.

Now, when you have a practical system power control may not be perfect. So, with power control errors how is going to manifest it is going to manifest saying that all alpha K's is equal to 1 that assumption does not hold and that is going to be the practicals as understanding of the situation. Now how does it affect the SINR of each user if power control is not. So, SINR 1 that is of user 1 how does that look like. So, the SINR of user 1 will be mod alpha 1 square E b notice that it is not equal to 1, I cannot E b by N naught multiplied by D g 1 whatever is the degradation that is going to be caused by the multi user interference. So, that is, so D g 1 will be expression for that will be 1 plus E b by N naught 1 over Q mod alpha 1 squared summation k equal to 2 to uppercase K alpha k by alpha 1 magnitude squared.

So, basically I have re written the previous expression with the alpha K's present and I have just written it. So, that we can bring in explicitly the expressions for alpha K's and there is a minus 1 by the way this is the term in the denominator. Now the key step is how do we interpret and how do we understand. So, let us take a simpler case keep that equation in your you know in view and please look at it in the following a special case I am going to look at a special case of 2 users 2 users, the special case of 2 users where alpha 2 is much larger than alpha 1; that means, user 2 is transmitting with more power than he needs to.

(Refer Slide Time: 48:00)

Special case 2 users

$|K_2| \gg |K_1|$

$$D_{g,1} = \left[1 + \frac{|K_1|^2 E_b}{N_0} \frac{1}{Q} \left(\frac{|K_2|}{|K_1|} \right)^2 \right]^{-1} = \left[1 + \frac{1}{Q} \frac{E_b}{N_0} |K_2|^2 \right]^{-1}$$

$$D_{g,2} = \left[1 + \frac{1}{Q} \frac{E_b}{N_0} |K_1|^2 \right]^{-1}$$

Near-Far Problem

Q1 How do you jam a CDMA BTS

The slide includes two diagrams illustrating the Near-Far Problem. The left diagram shows a signal spectrum for user 1 (red) and user 2 (blue). User 2's signal is much stronger than user 1's, and both are above the noise floor. The right diagram shows the same scenario but with user 2's signal being so strong that it dominates the spectrum, making user 1's signal negligible. A small video inset in the bottom right corner shows a man in a pink shirt speaking.

So, basically either he is very close the base station or he is not reduced his power something α_2 is much stronger than α_1 . So, degradation seen by user 1 will be $1 + \frac{1}{Q} \frac{E_b}{N_0} \alpha_1^2$ since it is only 2 users it will be α_2 by α_1 magnitude square α_2 by α_1 magnitude square.

So, this can be written as $1 + \frac{1}{Q} \frac{E_b}{N_0} \alpha_2^2$ inverse. So, if α_2 is much larger than α_1 the degradation of SINR 1 is going to be much more significant than a degradation of SINR 2. So, you can write down the degradation of SINR 2 this will be $1 + \frac{1}{Q} \frac{E_b}{N_0} \alpha_1^2$ inverse. And now if you extend this to more than 2 users also the that the trend will follow α_2 because it is in the (Refer Time: 49:47) if you write down the degradation for α_1 for D_g α_2 will come in the denominator. So, it will dominate it will suppress the others. So, the D_g becomes a quantity that is not signify, does not affect SINR much, but all the other D_g 's will affect the other users. So, if you now link this to the scenario that we just now drew, I will just read that for reference - this is noise, this is a 2 user case user 1 is transmitting with a lot of power this is user 1 and user 2 is right there user 2 power level.

Now, when I dispread user 1, dispread user 1 user 2 is sorry if when I dispread user 2 then what I have is this is my SINR for user 2 user 1; user 2 is stronger. So, I think I have labelled it wrong user 2 is stronger. So, this is user 2, user 2 and let me draw user 1 here user 1.

So, when I try to dispread user 1 this is my SINR of user 1. On the other hand when I try to dispread user 2 you noticed that it is a very different situation noise is there a user 2 is there this is user 1, if I remove user 2 from the picture user 1 moves over here the blue signal gets dispread the SINR is seating here SINR of user 2. Now if SINR is user 2 increases it is power even more who is going to get hurt user 1 is going to get hurt user 2 is not going to get hurt user 2 is quite happy. So, this is one of the reasons why this power control is. So, important and this is precisely what we call as the near far problem, near far problem.

A user in this place you can think of user 2 as a near user who is transmitter with a lot of power and because of user 2 user 1 is getting affected and it may come to a point where user 1 cannot even be detected. So, a near user who is transmitting more power than he

needs to will affect the detection of a user who is further away and therefore, unable to communicate with the base station, it is time to the notion understanding of interference and power levels and studying and disspreading all of that put together, but the where does it actually creep in where all of them are under ideal power control it does not show up.

But when one of the users does not obey power control then it starts to affect all of the others except the one user who is not obeying power control this is probably one of the most important questions that you will need to answer.

How do you jam a CDMA base station? How do you jam how can you jam based on what you have discussed now you know the answer jam is CDMA base station how do you do that?

Student: (Refer Time: 53:29).

Go close to the base station and transmit to the full power nobody else will be able to communicate it is that simple and that is why they are so careful about power control in a CDMA system. That is illegal, do not try that because you will go to jail because you know, but the thing is it is that is as simple as that if you go close to the CDMA base station and transmit the full power everybody else is (Refer Time: 53:54) and what is the due to near far problem that is it and nothing else you can do no nobody can none of them people can do anything about what they are received.

So, we are in a very interesting point where you know from here getting to CDMA capacity is just one step we will do that and then talk a little bit about the optimum receiver and then conclude.

Thank you.