Introduction to Wireless and Cellular Communication
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Lecture – 46
CDMA Receivers
Rake Receiver for multiple channels

Good morning. We begin lecture 47; we are coming towards the final lecture of CDMA. On the topic of CDMA, the 3 main takeaways that I would like you to focus on in addition to the various pieces of theory, regarding the sequences, regarding the aspects of spread spectrum are the following. How does direct sequence spread spectrum system slash CDMA systems; how do they handle multipath that is a very very important element. That is what we have started in yesterday's lecture that is what we will begin to study today.

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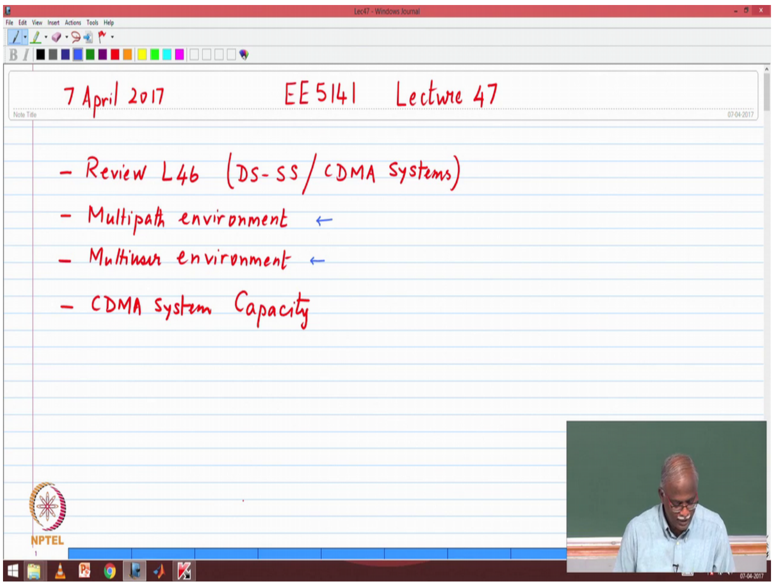

Then we also said that the strength of the CDMA system is that; it is robust to multiuser interference. So, how does it handle multi user environment. So, multipath plus multiuser is the system environment in which we are working in and what is the capacity that you can achieve with a CDMA system? How do we characterize it and how do we how does the cellular system take advantage of that is so that is the goal. These are the main things that I would like you to focus on as takeaways.

But today we build on the framework of a multi path environment; so, by way of a quick summary of lecture 46.

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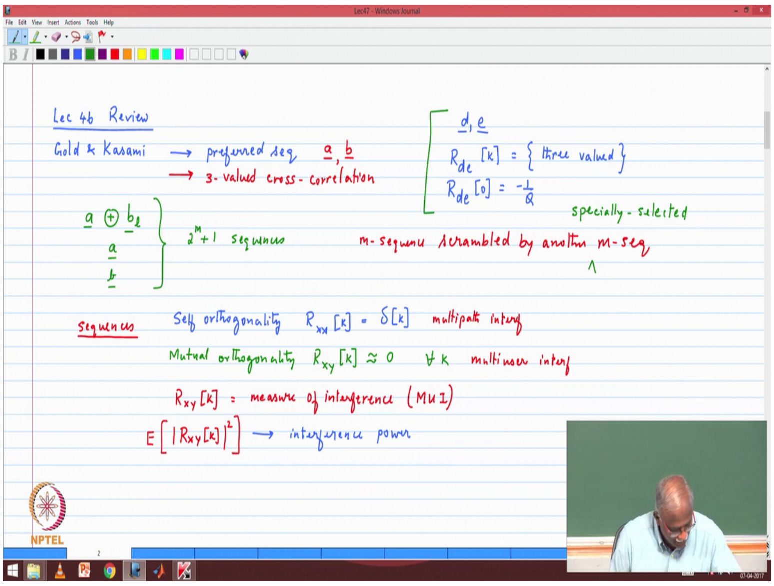

Lecture 46; we spent some time understanding the gold Kasami sequences. So, the key points are that gold and Kasami sometimes called Kasami gold basically, this is the early work in the late 60s; they came up with pairs of sequences called the preferred sequences. The preferred sequences had the property that they had a 3 valued correlation and all 3 values were good values so; that means, you could use them for a multi user direct sequence spread spectrum system 3 valued cross correlation.

So, it basically gave us several choices for the spreading sequences and the basic structure was if a and b are the preferred sequences then the gold Kasami sequences were generated as follows. You take a keep it fixed though modulo 2; addition with all cyclic shift of b then you take the code a itself and the code b itself and these together gave us 2 to the power of m plus 1 sequences and all of them have got very good properties in a cell autocorrelation and cross correlation.

So, yesterday, we made the observation that gold Kasami sequences are nothing, but an m sequence scrambled by another m sequence scrambled by another m sequence. Would you agree with that it is an m sequence scrambled by another sequence or should be qualified or should it be qualified with a specially selected m sequence? It has to be the preferred pair, right; otherwise it does not work for all of them. So, it is a specially selected m sequence. So, or the basically the preferred pair is what you would have to use. So, in summary; sequences are very important in our study of direct sequence spread spectrum systems. The most important properties that we need to take away or keep at the back of our mind are that sequences these are going to be used by users to differentiate themselves. So, we are interested in 2 properties. One is self orthogonality. So, that you can protect yourself against multipath self orthogonality basically says that if I have a spreading sequence for user for one user which is x; this should be as close to a delta function as possible only at 0 lag then you can reject the multipath then and of course, align it with a different multipath and combine.

So, the self orthogonality is what helps us against multipath and the cross correlation or mutual orthogonality is what helps us against multiuser interference and today we are going to study both of those who keep this picture in mind. So, this would be a cross 2 sequences and we would like this to be as close to 0 for all values of K and that is what will help us against the user interference. So, in summary, this is the key concepts that we want to take away from this discussion. Multi user interference; multipath also creates interference for us it is; so, it is correct to call it as multipath interferences as well and another key elements that we highlighted in the statistical analysis is the following we said that R x Y of K when we use correlation based receivers now this is the leakage from user in from a unwanted user into your decision statistic.

So, this is a measure of the multi user interference. It is a measure. It is it represents the multi user interference component measure of the interference cause by other users. So, mui is and we also said that we are interested or at least statistically in characterizing expected value of R x Y of K magnitude squared because this would represent the interference power the total interference that is leaking in addition to your AWGN; that is present. This would also cause an impairment to your decision statistic. So, this is a representation.

So, these are the points that we highlighted and the again I hope you are able to verify that if we had found the Kasami sequences let us say d comma e again has 2 different shifts. So, d is a 1 shift of the sequence b and e I hope you are able to show that R d e of K is 3 valued. The same 3 values that the preferred sequences satisfy and this is what we did in the class; I hope you had a chance to verify that.

And as an extension I had also asked you to check that you could verify that this is equal to minus 1 over Q. Please do take time to confirm that that is the case again that sort of just helps you gain confidence in the use of sequences. So, that is a broad picture of sequences and their role in spread spectrum systems our task today is 3 fold 1 is to look at how does it help us in multipath environment; how to work with it in a multi user environment and then of course, putting all of them together in a capacity environment.

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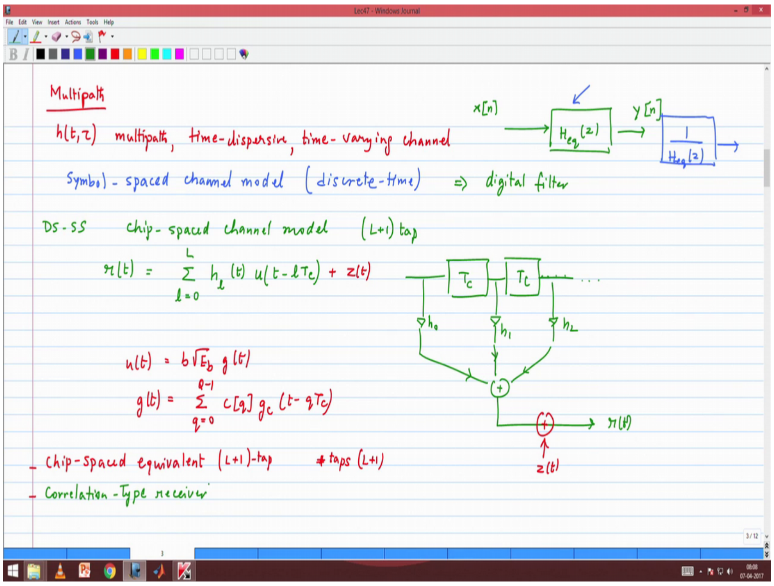

So, multipath environment; so, we said that narrow band systems; we can derive something referred to as the equivalent symbol space model for us. It now becomes a chip space model and so basically you can have large number of tabs which this is what the channel is this; what represents the channel. Now we have to build a receiver that rejects the unwanted parts and pulls out the required signal.

So, the channel model that we have is r of t with the input signal u of t convert with this. This chip space channel model which is what gives us this expression plus the impairment z of t where u of t represents the bit that is being transmitted with the spread wave form g of t represent the spreading wave form. G of t is we assuming it is upper case Q chips in duration g c is the chip waveform. So, this is the framework the spread spectrum signal has been generated. It is passing through a multipath environment which is characterized by this chip space channel which we would now need to build a receiver for that is where we are picking up.

So, the points to note are we are working with a chip space channel model. So, a chip spaced equivalent channel model and the original channel we assumed had all plus 1 tabs. Now usually when you come up with equivalent models you do not know ahead of time; how many tabs that it will be, but you know we are assuming that L is sufficiently general that it can capture the range of multipath that you are. So, the chip spaced equivalent model you must specify the number of tabs it is an L plus 1 tab equivalent; where L is adjusted based on the number of multipath components. So, the number of tabs is L plus 1 chip space tabs the second one is that we will work with a correlation based receiver.

And the strength of the correlation based receiver is that it is a matched filter and of course, we are working with sequences that have got very good correlation properties. So, we want to exploit that one is the fact that it is a matched filter. Second one is that it is these correlation properties that will help us be robust against unwanted signals like a multipath or multi user interference. So, correlation type receiver it is a match filter, but think in terms of a correlation based implementations. So, that way all of the correlation properties you know the mutual orthogonality the self orthogonality all of that comes into play and you know you are always keeping that.

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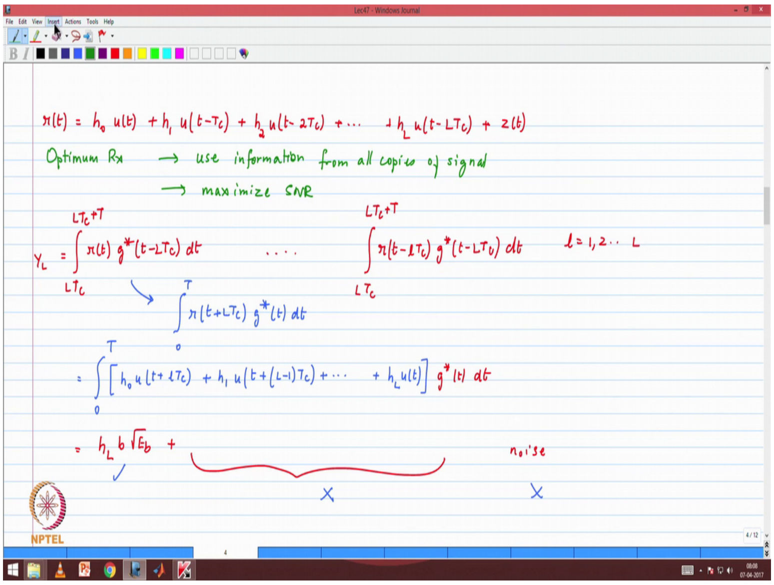

So, correlation type receiver is our framework that we are going to be studying. So, yesterday there was a question which I have thought was a very important one; I will make sure that just spend a minute to answer that question; the question was supposing I have a symbol that has been spread.

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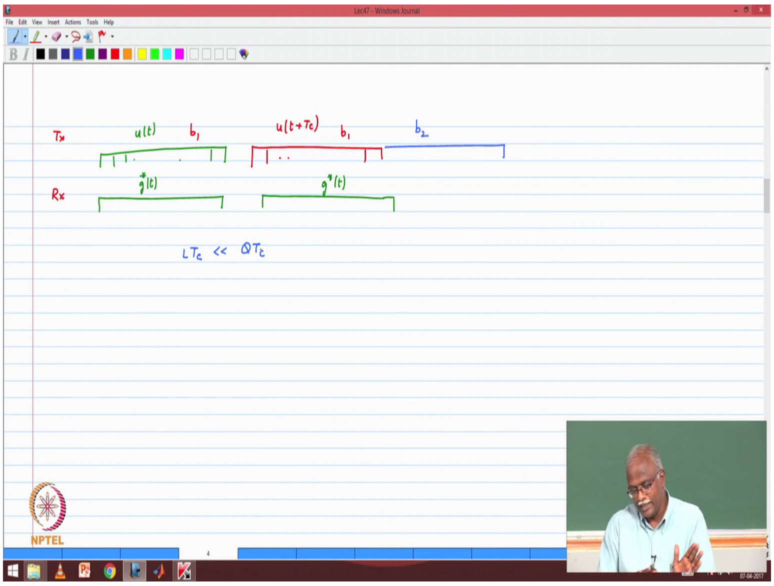

So, it has been spread by a number of chips. This is u of t; u of t. Now g of t also has the same span because u of t this; the wave form. So, when I correlate u of t with g of t that is a perfect overlap between the wave from that I used at the transmitter and the wave form that I am using at the receiver g star; let us say this is at the transmitter this is at the transmitter this is at the receiver I have a copy of g of t whatever comes in at the receive signal I correlate if and if I am if there are line they are perfect.

But what happens in the presence of multipath is that I get shifted versions of u of t and the yesterday's structure that we drew required us to have the following say that it was a shifted version. Let us say the simple case shifted by T c then my wave form g star of t is aligned in this fashion; do you see that it basically the u; this is u of t plus T c u of t plus T c correct; u of t plus T c. Now the question that was raised is this represents bit b 1 1 then there is. So, this represents b 1 then beyond this will be bit b 2, correct that next bit has already come. So, because this continuous transmission. So, this green wave form is now no longer overlapping only with b 1. It is also overlapping with b 2. So, which means that you have to take into account and that is precisely the problem with multi path is that when you when you now have copies of the signal which are going to contribute your decision statistic.

So, how do we why are we not explicitly you know why are we not explicitly taking this b into account. So, in today's in discussion, we are going to make the following assumptions that L times T c that is the total delay cause by the channel and the multipath is much less than Q times T c; that means, the total delay is still you know. So, the bulk of the overlap of g star of t is still with the bit b 1 and this additional leakage from b 2; we are going to sort of say that it is present, but we are not going to going to be is going to be a small and a part of the a discussion today will clarify this, but yes it is correct that the bit b 2 will now come into play or if you slide the other directions bit b 0 will start to come into play.

So, that is the problem with the multi path and that is the reason why you need to have equalization in a narrow band systems and that is why we need to have a special receiver in a spread spectrum system. So, again good question. So, let us pick it up from where we left off yesterday; do not want to rewrite the whole hm.

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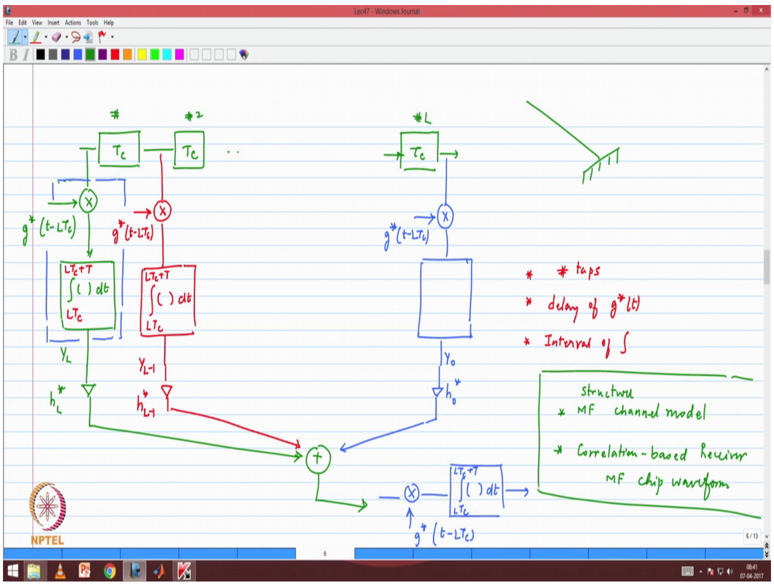

So, the structure is a tapped delay line with L delays; L delays of T c each of these branches keep in mind is being multiplied by g star of t minus L T c t minus L T c means g star of t delayed by L T c and then it is being integrated because I delayed g star by L T c. My integration limit starts from L T c to L T c plus t. Now of course, we showed that we can rewrite this as 0 to t and keeping j star of t not the shifted version. So, if you keep g star of t then you have to advance r of t. So, basically that is the structure that is easy for us to work with.

So, the starting point is this expression 0 to t r of t plus L T c; that means, I have advanced it by L times the chip duration and the equivalent expression was written down; we then multiplied it by g star of t integrated from 0 to t. The first time; we wrote down the second term, we said we need to write down the expressions for that that is where we will pick up today's discussion. So, let us quickly write down the notation and then build on the topic that we have. So, the values that we want to write down and characterize are as are as follows.

So, want to write; define a couple of terms which will help us in our discussion.

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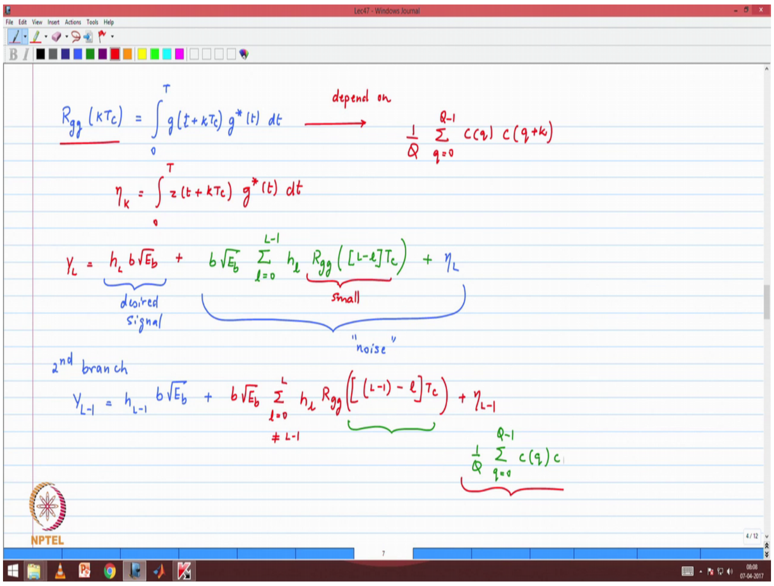

Expressions R g g of K T c; I am going to define this as follows integral 0 to t g of t plus K T c. This is the form of the expression that is coming multiplied by g star of t d t and then if you go in and write down the expressions for g of t; we find that this will; this particular quantity will depend on this parameter 1 over Q; Q equal to 0 to Q minus 1 c of Q c of Q plus K. So, basically the chip wave form does not really make a play a big role; it is that underlying chip sequence and its autocorrelation property is what is going to tell you what this value is going to be, but let us write that down and the other element is that the noise is z of t the impairment is z of t. So, that will also pass through the correlation receiver and we will get 0 to t z of t g star oh no remember we advanced the received signal. So, it should be t plus K T c multiplied by g star of t d t.

So, now using this notation and keeping that previous expression as a reference point, please confirm that you are able to get this value Y of L that is the statistic that is coming out from the first branch is given by h L b times root e b that is the first term then we get a bunch of terms where the waveforms are not fully overlapping. We will write this down as b times root e b summation; L is equal to 0 to L minus 1 h subscript L R g g; these are partial overlaps. So, the overlap depends on the different shifts L minus L times T c. I would encourage you to just write down the start from the previous expression and for each of the delays; write it down in terms of this cross correlation R g g.

And you will of course, have the noise term the noise term passing through the correlation based receiver becomes eta l. So, as we have expressed; so, this portion represents the desired signal desired signal because it has got the expression for what we are looking for this is a combination of terms which has b in it, but I need to be very careful because the correlation at a different lags are going to play a part and how they combine with h of L is also going to be important. So, I need to be careful with this. So, what we are going to do is that because of this property these cross correlation values are going to be small by design. So, what we are going to do is that because of the property of the code that we are working with.

So, more or less; we are going to say that you know it is not worth bothering with the component of b that is present here I am going to just ignore this. So, basically what is going to happen is this whole quantity is going to be treated as noise. So, what is the rake receiver doing? It takes the received signal; passes it through a delay; change each of those branches, it multiplies by g star of t minus L T c. Basically it is a correlation step followed by the appropriate integration and then we are looking at Y of L. The output of the first branch and output of the first branch is what we are written down has got a very clear contribution of the decide signal and a combination of terms which include the original noise plus some correlations terms which come from the different correlation at different lags.

So, this is the expression. This is the expression that that we have. So, what we would like to do is see how best we can work with this expression and quickly converge upon or ability to get the best out of this; best out of this system. So, here is the step that we would we would like to do. So, you can of course, encourage you to write down at least for a couple of more branches. So, for the second branch; second branch I will just write it down for the second branch and then; so, second branch; we have labeled as Y of L minus one you will notice that it will give you h of L minus one times b root e b plus it will give you a set of noise terms it will have all those h coefficients except h of L minus one because h of L minus one is now in the desired expression.

So, if you want to write that down it will be b times root e b summation L is equal to 0 to uppercase L and, but is not equal to L minus one h of L R g g. Now, it is L minus 1 minus L times T c that is the expression plus eta L minus 1 because that is the output of the correlation correlated that uses the with the with the appropriate delay now these arguments that are inside this bracket now that just interpret this if it comes out that the argument is negative what does that mean basically; it means 1 over Q summation Q equal to 0 to Q minus 1 c of Q and if the argument is negative let us say it is some minus j this means it is c of Q minus j. If it was argument was positive, it will be Q plus j minus j, but you know essentially the behavior does not change you are looking at a correlation at different lags you can you can slide one sequence to the right or you can slide one sequence to the left it does not make any difference.

Basically it is the same thing. So, these terms are all small and therefore so what is the property that we have used? We have a very heavily relied on the self orthogonality property; self orthogonality property basically says that each of those branches of my rake receiver gives me one term which is very strong in terms of the desired signal. It gives me a combination of terms which is combination of the original noise plus the desired signal, but because of the cross correlation these are suppressed. So, self orthogonality property again just for reference we are talking about the fact that R g g of K; we wanted to be as close to 0 as close to delta of K as possible not 0 for K not equal to 0.

For K naught equal to 0 and that is the; so, if you have come this far saying that each of those branches; you can write down first branch. So, basically go back to the figure here is Y of L second branch is Y f L minus 1 correspondingly write all the way to the last branch and call that Y of 0 now comes a very interesting step we say that the final decision statistic will be a sum of all the L branches of a rake receiver L equal to 0 through uppercase L h L star times Y L h L star times Y L.

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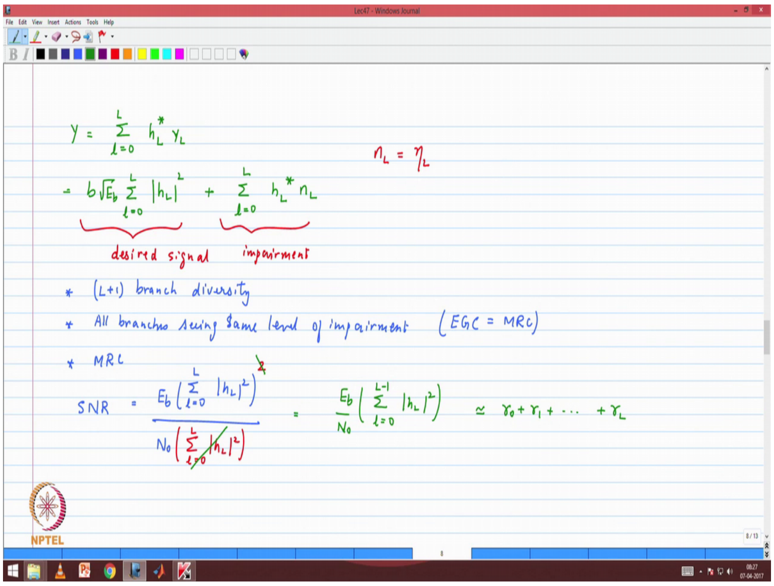

Now go back to the figure and verify Y L is being multiplied by h L star Y L minus 1 is h of L minus 1 star Y 0 h 0 star. So, basically that is what we have done; if you do that you can verify that your expressions are going to be given by summation L equal to 0 to upper case L b times root e b times mod h L squared plus summation L equal to 0 through L h L star of n L. Those are the noise terms that are coming in from the different branches.

So, this is the portion of the desired signal portion of the desired signal. This is the portion of the impairment just for insight; we are going to make one more small approximation; if your cross auto correlation property self orthogonality properties are very good, then this n L will actually be equal to eta L because that only the noise term will the rest of it would have gotten suppressed because of the cross correlation cross correlation properties.

And very important step if you now have this expression; I can hope you are seeing that this looks like L plus 1 branch diversity L plus 1 branch diversity where you have multiple copies of the signals which you have collected together and of course, you have impairment from each of those branches. So, basically if we can make the assumption that all the branches are seeing the same level of interference of impairment; all branches are seeing same level of interference same seeing same level of impairment then we can do equal gain combining equal gain combining is a same as M R C. So, basically then your; the co-phasing is done only through the conjugation of the channel tabs. So, basically MRC is what is optimal in this environment and we have used that is what we have what we have done resembles MRC. So, the SNR of a; my decision statistics comes out to be what is the signal component and the noise component. So, I will have to square it b square b is plus or minus 1. So, when I take b square it becomes one e root e b becomes e b and this term become summation L equal to 0 through upper case L mod h L square.

Noise component; the denominator if it is eta L and the noise variance is n naught only the AWGN; they have the multipath interference has been ignored for the moment then its each of those noise terms eta L is being scale by h L. So, when I look at the variance I need to make sure I take into account that the variance will be n naught into summation L equal to 0 mod h L square. So, this is going to by the way this is the scaling. So, therefore, there should be a square on; please, if I miss some of things like that you should you catch it. So, then simplifying this one will cancel the exponent.

o, this gives me e b by n naught into summation L equal to 0 through L minus 1 mod h L square that looks like gamma naught plus gamma 1 plus gamma L of a L plus 1 branch MRC system when all of them have equal noise variances and therefore, equal gain combining is the same as MRC; what are the assumptions that we have made is that we do not forget the assumptions that we had made. What are the assumptions we have ignored? The multipath interference; there is a multipath assume that if I have perfectly perfect self orthogonality, then this expression is correct, but we will never have perfect self orthogonality. So, therefore, that is one term what else have we ignored.

Well cross correlation has not come because only one sequence is involved the remember bit b , b 1; we said that the other bit is not much is coming from there as well. So, keep that those 2 assumptions in mind these are not you know very limiting in terms their impact, but we next need to be able to keep that keep that picture in mind all right.

Now, just want to take bring in a little bit more rigor and insight because when you look at n L.

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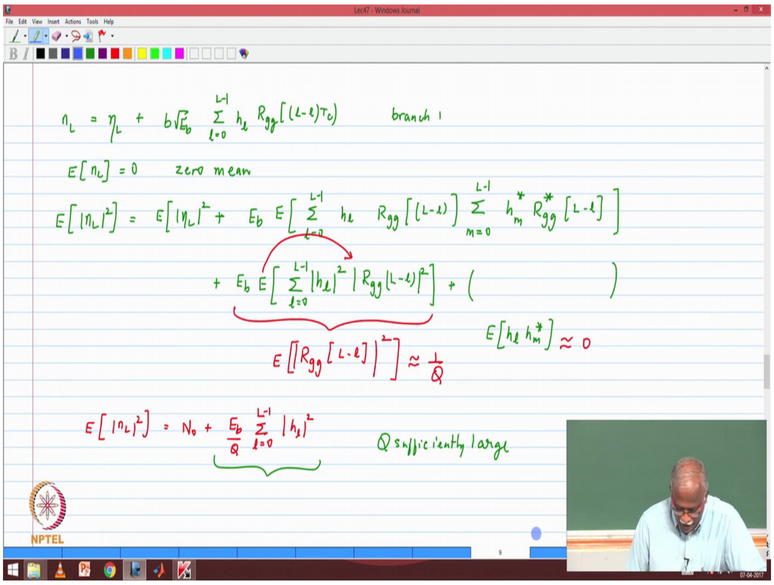

It is eta L plus a summation that we have to take into account. Let me just write down the expression. So, it is b times root e b b times root e b summation L is equal to 0 to L minus 1 h L R g g L minus L T c; again I have written it down for the first branch basically these will be the type of noise this is for branch one for branch one.

Now, one of the important elements that we will need to analyze this very accurately would b 2 estimate what is expected value of the noise that is item number one and that you can verify it is a combination of the noise term plus something which depends on b expected value if you take the noise term will give you 0 the b has a plus minus 1. So, that also; so, basically this is a 0 mean. This impairment is 0 mean. The second component that we need to be careful about is that expected value of n L magnitude square the variance. So, the variance expression basically involves us to work with this work with this term. So, basically it will be expected value of eta L squared plus there will be cross terms the cross terms will be have a product of eta L when you take expression of eta L the cross terms will go away and then you will get the square term that is coming in from here.

So, you will get e b times expected value of summation L equal to 0 to L minus 1 h of L squared R g g of I am just going to write L minus L the T c. I am deliberately ignoring just for that it does not. So, this is the first term and you have to multiply it by its conjugate. So, I am changing the variable of summation m is equal to 0 to L minus one mod wait a minute I think I have written it wrong this should be only hl this is the second term will be h m R g g L l minus L upper case L minus L with conjugate signs and so, when we when we look at this expression basically you will get the square terms which will be e b times expected value where L and m are the same L is equal to 0 through L minus one nod h of L squared times R g g L minus L magnitude squared plus you will get those cross terms cross terms are also present.

Now, here is where the basically I am trying to get a field for what is this impairment we initially said we are going to ignore this; this multipath component right we just said we are going to ignore it. So, what we are trying to do now is sort of get a little bit more insight into what that impairment term is and what is its mean is not a problem we had looking at the variance now of course, there is there are these cross terms where you will have h and where L and m are not equal you need to take those terms into account now if you if you sort of take the fact that expected value of h L h m star if these are there is a they are different tabs of the channel model. So, which means that they are not correlated to each other.

And if each of those channel tabs if this; this was a Rayleigh fading channel and you had modeled it the each of these coefficients will look like they are Rayleigh coefficients and so this one you know; we are saying this is not going to contribute a significant portion. So, the path that I need to worry about is only where you get magnitude squared. So, then I take the expectation inside I know that expected value of R g g of L minus L magnitude squared this should be approximately one over Q for my if it was a random sequence or a p n sequence and most of the time this is the type of cross correlation that we will achieve sorry self orthogonality that we will achieve. So, if i you have achieved that level of self orthogonality then comes the then comes the expression that expected value of mod n L squared is the same as n naught which is the noise variance plus e b divided by Q summation L is equal to 0 through upper case L minus one mod h L squared.

So, each of those noise terms in addition to n naught has got this quantity. So, what we have done is we said initially we said we just said we are going to ignore the multipath components we are going to take only n naught and we showed that it is close to MRC that is correct that gives us a certain level of insight, but we now are saying let us go in and look at the impairment a little bit more closely we are saying that the major term is going to come from major part is going to come from the this square terms the square term basically gives us a contribution of this type.

The most important step comes now if Q is sufficiently large if Q is sufficiently large then everything hangs together because if Q is sufficiently large compared to L which is the assumption that we have made right at the beginning then this whole term kind of drops out and I can still go back and say well you know what it does look like MRC anyway my spreading sequence give me the benefit of our suppressing the unwanted multipath related components and therefore, we are actually doing as good as the system as an MRC now it is not the same as MRC because these terms have been neglected right please keep in mind if this is the penalty that you have paid which you have sort of ignored. So, it will not achieve the same as full ml plus one branch MRC, but it will be slightly worst, but it is it is done a fairly good job.

2 more very interesting observations which after that we will conclude this discussion first one is this a linear system if you look filtering, it is a linear system right this is a linear system nothing no squaring term nothing which is non-linear. So, if I am permitted to move the linear parts; I am going to draw a dotted line around what we consider as a linear operation; we do it with the blue line; the these are different for each other branches the part that I want to move is the do I move the yes this whole branch is common for h of the branches. So, I can move it to after the summation. So, basically I am going to do the multiplication by g star of t minus L T c followed by the integration and; that means, it is been removed from that that part. So, this will be L T c to L T c plus t integral d t. So, invoking the property of an LTI system; I have moved this operation which is a basically this is a filtering operation right when you multiply basically it is like a multiply or convolving with a matched filter.

So, that you have moved to after the summation because if the filtering is a linear operation addition is a linear operation. So, you can move to the other side. So, then what happens? It is the delay line follow along delay line multiplied by h L star h L minus one star h 0 star how would you describe that how do you describe that it is a matched filter to what.

To your channel matched filter to your channel the channel was an L tab L plus 1 tab delayed. So, what you did was basically you are applying the matched filter to your channel model very important observation. So, then it this operation this structure is actually doing the following when you have moved the filtering operation outside you can say that you are doing a matched filter to your channel model time reversed and conjugated is the structure that you are applying.

But match filter requires you to have matched filtering to the entire wave form which means that it is a combination of the chip waveform and the channel, but notice that the match filter to the chip is also present. So, in effect you have done a match filter to the entire the chip wave form plus the channel you have constructed a match filter. So, basically your; the structure is implementing. So, the structure is implementing a matched filter to the channel model the correlation plus the integrator is the correlation based receiver correlation based receiver is doing the matched filter to the chip spaced the chip waveform h m correlation based receiver is doing a match filter to the chip wave form.

So, you have a very conventional optimal receiver and this would be your optimal receiver in AWGN of course, it is not strictly AWGN; what you have done is you have neglected some terms, but very important that you know this is fairly close to what you would have done in terms of optimal receiver's last observation. So, this is a one of the thing I said there were 2 things if you look at this structure what is it look like? It has got lots of parallel branches which then joined together.

Now, if you have done gardening and you want to gather all the leaves there is a tool which you use which is got a wooden stick and then it has got certain branches like that right that is what you use to gather the leaves dried leaves and that is called a rake and this one actually looks like a rake; that is why the name comes from? It is not there is no person called the mister rake who would invented the system because just the structure looks like a garden rake and therefore, it is called a rake receiver. So, in case you wondering where the rake name comes from that is where it comes from. So, that is multipath how CDMA systems work in the presence of multipath.

what I would like to do is introduce you to the multiuser environment again I was hoping to go a little bit further than today, but again more important that you are comfortable with the material that is being covered and we are able to you know cover it at our own space. So, the first thing that we would like to do is look at the; what is the difference between a multipath environment and a multiuser environment. So, multiuser environment is what we would like to develop now.

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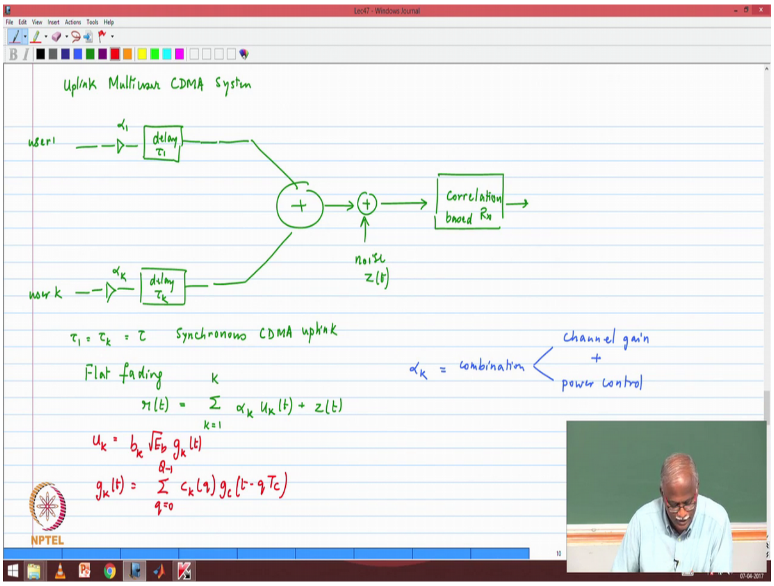

So, multi user environment is going to have a large number of users user one to user K; each of them have spread their signals we will assume that they have different gain terms we will we will we will explain this in a moment what these what these terms are.

Now, each of these can go through different delays. So, let us call them as tau one through delay tau K and all of these will then get combined into as we are looking at the uplink of a multiuser; multiuser CDMA system CDMA system. Now each of each user does not care about the others he says I have got my code I have to; I have mis spread my signal I have transmitted it. So, in the channel that is in the wireless channel all of them are transmitting the same frequency. So, therefore, the channel adds all of them together. So, that is why you get the presence of the multiple users.

Now, in addition to this; there will be noise that gets added that is the noise term and then at the receiver I must build the receiver that can detect each of these signals I should be able to detect because the base station should detect all of them. So, it is a correlation based receiver because if I want to detect a user one I need to use a correlation based receiver for ah. So, the channel model that we would like to do is first of all we will make the assumption that all the delays have been controlled. So, that they are all the same. So, this is called a synchronous CDMA system synchronous. It is a little bit easier work with than a asynchronous system, but because the base station controls all the users you can actually control the time they actually arrive at the same time. So, synchronous CDMA uplink that is what we are studying.

Now, in the presence of flat fading flat fading a channel; that means, there is no multipath the combined received signal r of t is equal to summation K equal to one to uppercase K alpha K u K of t u K is baseband signal representation of each user signal plus the noise term z of t.

Now, what is alpha K I just want you to take a little bit of time to think about it alpha K is a combination of 2 things. One is if you did not do any power control; it will be only the channel gain which will be a Rayleigh coefficient. If it is a Rayleigh fading channel, but if you have done power control which is the; what you would do in a CDMA system. So, this would be the channel gain plus the power control we have represented it by a single alpha which is fine because if this is a flat fading channel; anyway the channel gain would have been a single number. So, this is. So, each of this u Ks again you will see a lot of similarities with the previous u Ks can be written as b K times root eb that is a bit transmitted by each of the users each of the users waveform g K of t and g K of t is the spreading sequence with Q chips Q is equal to 0 to Q minus 1 c subscript K to denote the spreading sequence of user K times Q g c of t minus Q T c.

Actually the multiuser system is very simple, very elegant. We will write it down and then conclude today’s lecture. So, if I want to pick up user one signal I do a correlation with respect to user one.

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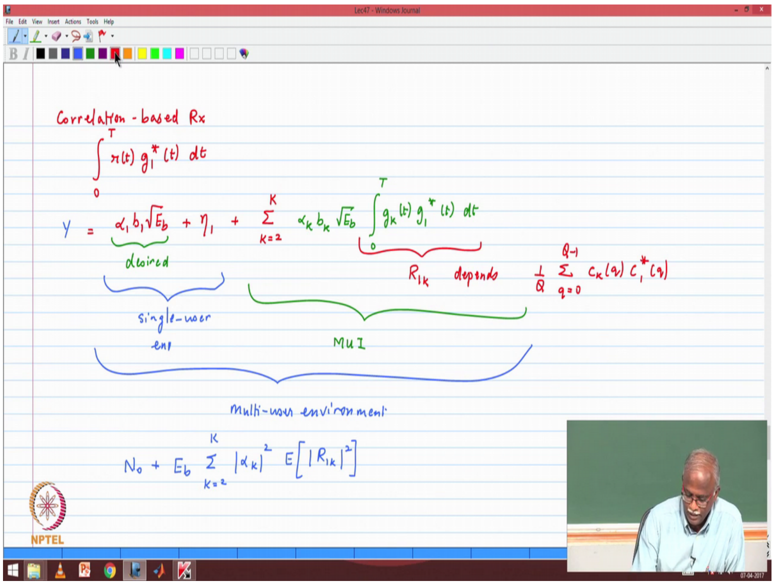

So, a correlation based receiver; correlation based receiver for user one would take the signal r of t and multiply it with g 1 star of t g 1 because it is used ones waveform times d t if you now write down the expression what you will find is this will be alpha 1 b 1 root e b plus the noise z 1 which is passing through this correlation filter. We will call it as eta 1 and you will have the different other users signals. So, the other users signals are K equal to 2 to uppercase K those are the user 1 is our desired user you will get alpha K b K root e b and you will get a term which depends on the cross correlation between the different users it will be 0 to t g K of t g one star of t d t; this is mutual orthogonality.

So, basically the strength of your multi user performance is going to depend on. So, we will denote this as r one subscript k; that means, the cross correlation between user K with user ones waveform. So, the first term is the desired user and the others are. So, basically this depends on the cross correlation which in turn depends on 1 over Q; Q equal to 0 through Q minus 1 c K of Q c 1 star of Q that is the chip sequences. So, this is what we referred to as the multiuser interference that is the term that comes from or when I do my decision statistic there is a term which desired signal this is the desired signal. If there was no one else present in the system; it will be a single user system; that means, it will be my desired signal plus the noise term. This is what will be if it was a single user environment.

If it is a multi user environment, you have this multi user environment. So, this essentially what we have. So, what we now say is multiuser not a problem that this; all the users are transmitting I will build a receiver that detects user 1 and we then find them. So, let us call this as the decision statistic Y.

Now, we would like to understand what are the statistics of the signal component in Y and the impairment component in Y and then basically an analysis which is very very similar to what we have done in the in the in the previous case and we would like to know what is the when we look at the impairment want to know the mean and variance of the impairment. So, then we can statistically characterize. So, again it just requires us to write a couple of steps, but a good place to stops that you can then go back and see what are some of the similarity between the wave.

We have handled multipath interference and now we are going to do multi user interference. In fact, even without my input I am sure you may be able to write down what is the impairment component its mean and variance. So, maybe just let me write the answers for you can; then you can you can verify that this is indeed the case. So, if I were to look at the noise the variance of the impairment it will come out to be n naught plus e b times summation K equal to 2 to uppercase K mod alpha K squared expected value of r one K magnitude square.

And since we know that this for the case of the p n sequence a p n codes will be approximately one by Q. It starts to look very similar to what you encountered in the multipath case and basically; we are we will follow the very similar step if you can try this, but I will pick it up from this expression for Y and then derive the impairment variance.

Thank you.