Introduction to Wireless and Cellular Communication
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Lecture - 44
Introduction to CDMA
Properties of Spreading Sequences

Welcome lecture 43, we have seen an introduction to CDMA very interesting topic very lot of in insights into what used to be advantages for military communications, and how it has played a very important role in cellular systems.

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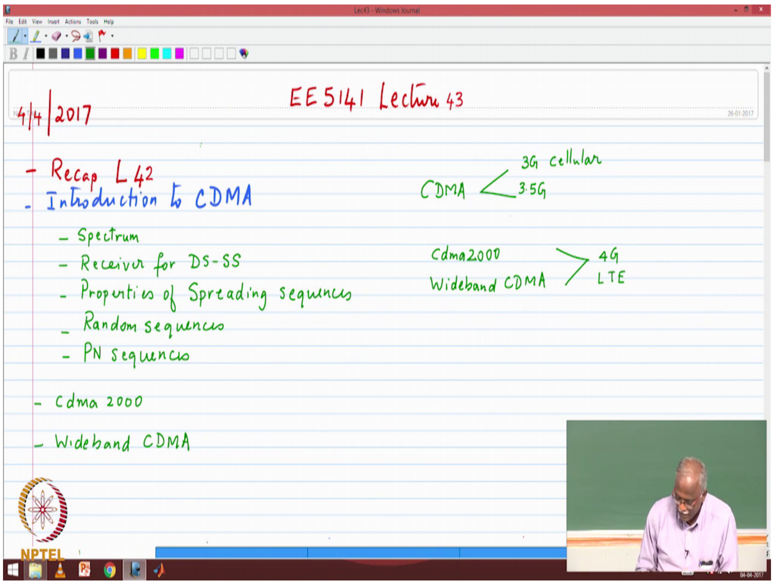

So, basically what we are saying was CDMA has formed the backbone of what we referred to as the third generation cellular systems, and third generation cellular systems is not as we saw in the introduction is not a single standard, but it has evolved over a period of time. So, there are variance that are called 3.5 G also. So, that is also been based on CDMA, and there are 2 dominant flavors 1 is what we referred to as CDMA 2000, it was introduced into the market as a competitor to GSM, and GSM itself evolved to a system called wideband CDMA.

So, if you go by history they should have been competitors and they were and but eventually in the fourth generation they became a single system called LTE, but in the third generation they were both CDMA base systems both had some unique characteristics we will study that, and you will also see that the design of a cellular system based on CDMA is something that is far from non trivial far from trivial, it is highly non trivial because the complexities of how to design basically you have taken something that had very different application in military communications, and you are using it for a cellular system and we like to see what are some of those benefits.

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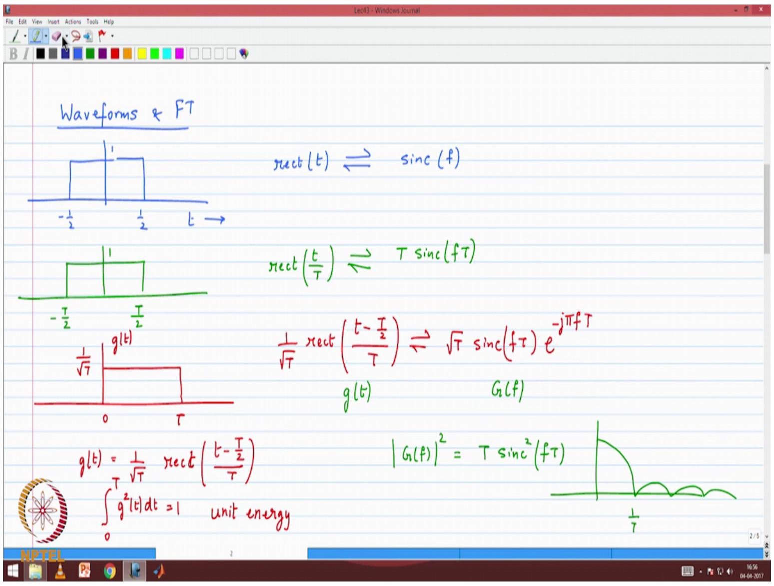

So, but for now the first thing that wanted to just refresh your memory is the understanding of what is the basic pulse shape. So, rectangular pulse shape, and what we will actually be using is a shifted rectangular pulse shape with the scaling to make it unit energy, and we showed that the corresponding spectrum looks like this. Now there is a difference between the terminologies that we will use and we will I may be just good point to highlight that is the difference between symbols and chips.

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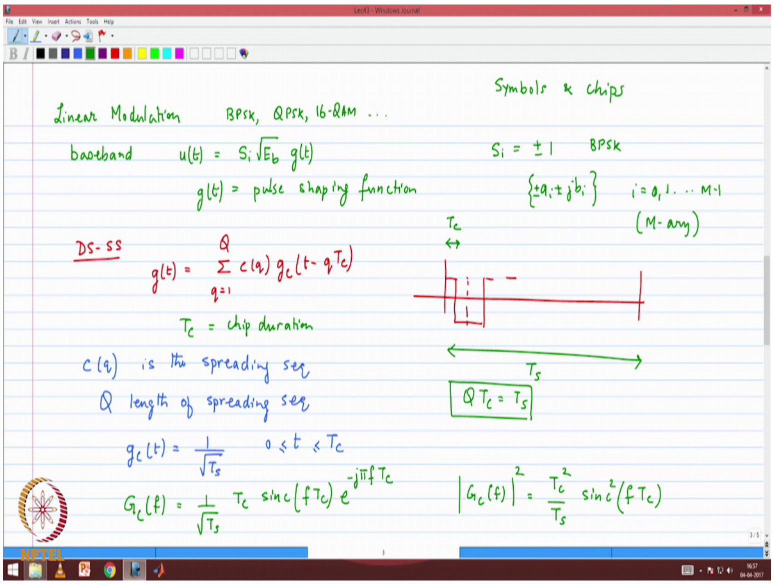

So, the never have a confusion between symbols and chips; whenever I say whenever we referred to chips; that means, we are referring to a this signal which has already been spread. Symbols there are symbols which are being transmitted in the direct seq in the spread spectrum system, but there could all it could also be before or after the spreading so therefore, we have to interpret them carefully.

So, in general the convention is symbols means it is the narrow band system, chips means that it has been spread. So, the information is still there, but the fundamental waveforms are different ok.

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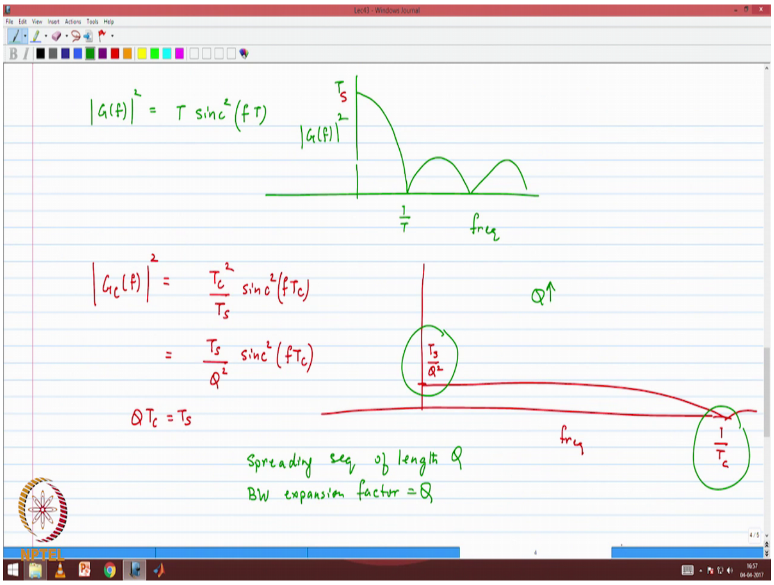

And we had looked at the spectrum how the once you spread it these the bandwidth of the signal expanse by a factor of q. So, we pick it up from there and build up our development of today's lecture ok.

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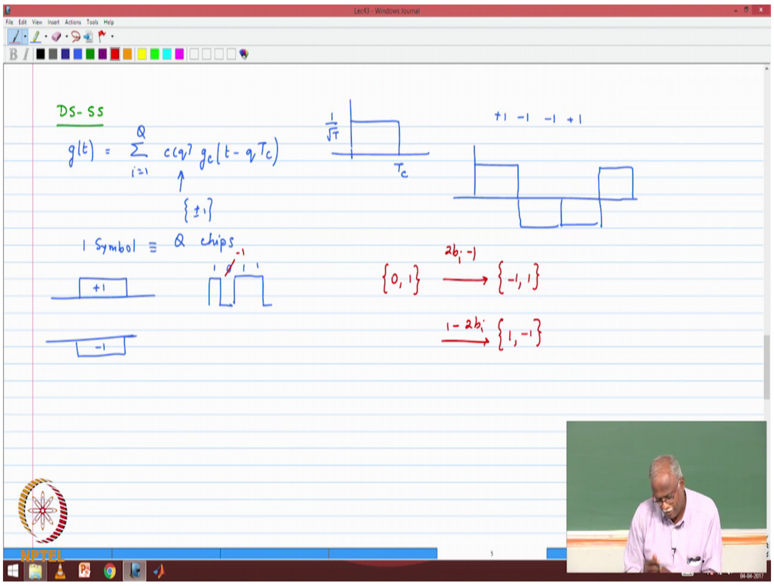

So, the pulse shape or the waveform that we are using for direct sequence spread spectrum, we have not yet introduced it as a CDMA system still at in the at the basic level, We say that the pulse shape or the shape that will be used for representing a symbol is not a single pulse, but it is a combination of Q pulses.

So, i is equal to 1 through upper case Q, c subscript of Q g c t minus Q t c where the g c has this waveform 1 over root T not T, T c this is T c and this is the basic pulse shape and these c qs c qs are belonging to plus minus 1. So, basically it is a sequence of plus minus ones which are then modulated now I hope you are comfortable with this notation g c t minus q T c; that means, for the if the first symbol first chip was a plus 1, second chip minus 1 third chip minus 1 plus 1, the way form that we are talking about is up to t c is plus 1 minus t c is minus 1 minus 1 plus 1 this would be the way from corresponding to that and that is what is represented and this is. So, 1 symbol is what we are trying to represent of the narrow band system, this corresponds to Q chips and always make sure that you are comfortable with that and. So, typically if we keep it very simple if the symbols let us say can be either a plus 1 or a minus 1.

So, this represents a plus 1 minus 1 the symbol, the corresponding chip way form let us say is 1 0 1 0 1 1 again these this may be material that you have very familiar with, but I just want to avoid any confusion. So, 1 0 1 1 or 1 minus 1 may be 1 minus 1 11. So, if I want to send a waveform of the other polarity basically it will be a flipped version. So, again the basic operation of spreading I am assuming you are familiar with. Now very often in the context of the spread spectrum systems again I just want to say that at the beginning itself we are talking about waveform that is binary for example, Walsh Hadamard code the spreading code we will talk about it in terms of ones and zeros, but when it actually gets transmitted it is not zero, but it actually is a plus 1 minus 1. So, there is a mapping that will map it to 0 to minus 1 and a plus 1. So, that mapping is 2 b i minus 1 there is another mapping which is 1 minus 2 b i and that will a map is 0 2 a plus 1 and the minus 1, and again it does not make a difference, but just be aware that we will always when it actually comes to the waveform implementation on the channel it will be a plus 1 and minus 1, but very often you will find the discussions regarding these spreading codes we will be in the context of zeros and one.

So, again always make sure that you have to quickly translate it into away from that. So, with this basic introduction now to move it from the just a single user system into a multi user system and how to build on this particular effort, but before that lets answer or let us get a feel for the communication system as the whole ok.

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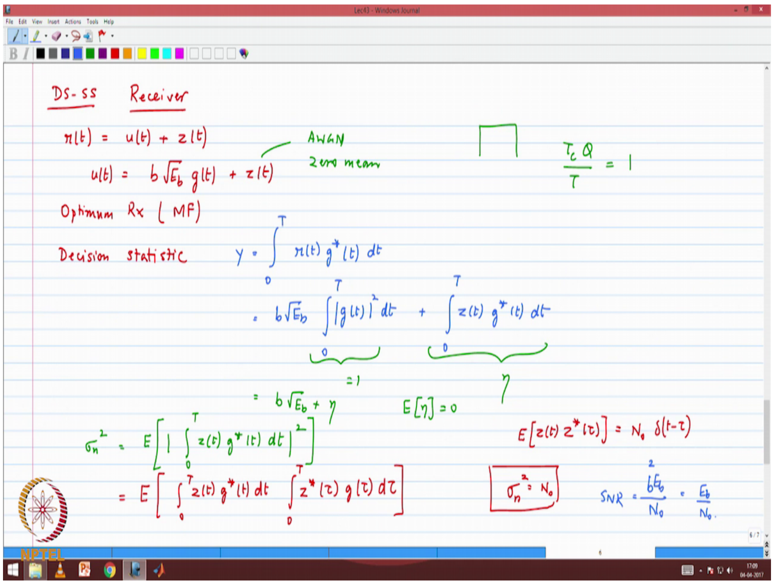

So, I would like to look at a direct sequence spread spectrum system, and what sort of receiver we would have to use for that. And this is a good starting point for us to get a feel for the spread spectrum systems and then how do we expand them into a multi user system.

So, typically the received signal r of t is the transmitted signal plus some impairment and the u of t the transmitted signal is b some value plus minus 1, square root of Eb that is the energy of the bit, g of t that is the waveform that we have used for the symbol. Again at this point we are talking about spread spectrum systems. So, g of t is these spread waveform plus z of t, and assume that z of t is AWGN the optimum filter in AWGN is.

Student: Match filter.

Is a match filter. So, basically you must apply filter that is time reversed and conjugated and then sampled at the end of one symbol duration. Now match filter can also be implemented as a correlation receiver correlation base receiver. So, it is very intuitive and helpful for a still look at it in the form of a correlation base receiver.

So, my optimum receiver will be a match filter implemented using a correlated. So, my decision statistic will be of the following form again this is material I am assuming is very familiar, but good for us to have the foundation. So, the decision statistic will be you multiply the received signal with a conjugated version of the trans of the pulse shape g of t integrated over 0 to T a symbol duration. So, now, go back and substitute this will come out be b times root Eb integral 0 to T, g of t magnitudes squared d t plus integral of 0 toT z of t, g star of t d t ok.

Now, we will call this as a noise term eta and basically we have made the assumption that this is a AWGN 0 mean some impairment that is happened. Now is this if I say that this is equal to 1 would you agree it is 1 over root t, but there are Q of them when I add all of them basically I will get each of them contributes T c right each of these pulses we will contribute T c there are Q of them. So, it will become Q T c divided by T which is equal to 1 that is the relationship. So, this is equal to one. So, what we get at the output is b times root Eb plus eta ok.

Expected value of eta is 0 basically it is a it is y it is a 0 mean gauss is Gaussian signal that it being filtered. So, therefore, it does not affect the me. Where interested to know what is sigma n squared. So, that will be expected value of magnitude of that of eta squared 0 to T z of t g star of t d t magnitude squared that is the variance. So, I just need to want you to be comfortable with this step because we will see many times the this operation and again this is something not new t u, but I just wanted to make sure that we are comfortable with this. So, I am going to write this magnitude squared as 2 integrals, 1 there is integral and then it is conjugate. So, this expected value of integral 0 to T, z of t g star of t d t, then integral 0 to T I will change the variable of integration I have to conjugate everything z star of tau g of tau deta ok.

Now, group the terms take the expectation inside this is z of t is white noise. So, expected value of z of t, z star of tau is equal to correlation. So, if it is not aligned it will give 0 such a delta function N naught t minus tau. So, only when both are aligned you get the power of the noise otherwise they expected value goes to 0. So, and of course, once this constant comes out t and tau become the same you get integral 0 to T mod g of t whole squared that we will be give you equal to 1. So, sigma n squared equal to N naught please verify that. So, the last step again very important for you to note SNR of my decision statistic right that is the you know what is what is going to determine whether I will make a correct decision or not will be equal to b times Eb that is the signal power by the noise component Eb by N naught sorry b squared it will come right b squared yeah and b is plus or minus 1.

So, this is comes out to be Eb by N naught. Now at this point you wonder you know even through all this process to tell me that you know you still have Eb you have not done anything to improve the signal to noise ratio the answer is yes not done anything to improve the signal to noise ratio and that is one of the fundamental things that you need to learn, that if you are impairment is only white noise spreading does not help at all. You just expand the bandwidth you contact the bandwidth in both operations the noise does not play a role and therefore, you end up having the original noise spectral density as before. So, there is no advantage for spread spectrum in pure AWGN, but multipath oh huge advantage interference huge advantage.

So, in a cellular system it is both multipart and in interference. So, therefore, the advantages are coming. So, maybe just a footnote saying that n AWGN it is not going to give me advantage at all that is in important. So, now, given that you already know starting to think about the receivers.

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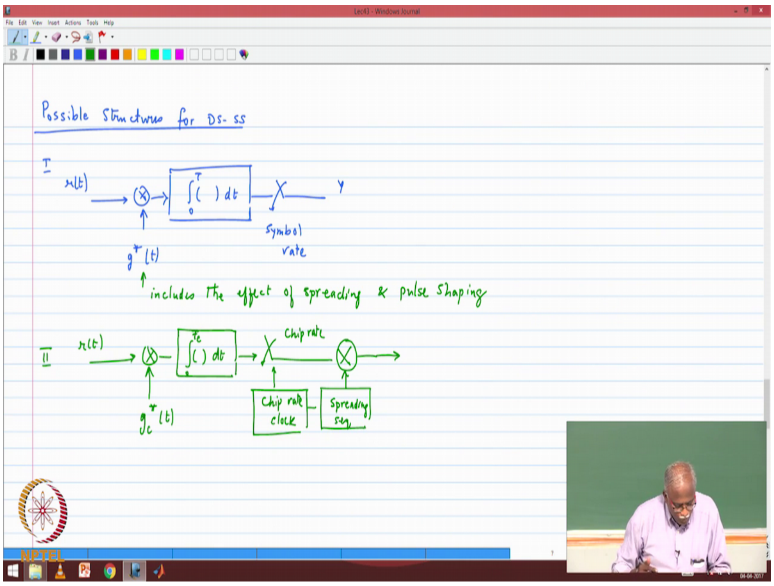

Maybe the next step would be to write down some possible receiver structures, possible structures for spread spectrum receivers. You may say already deigned the correlation based receiver, I just want to take you 1 step beyond that initial perspective to the wave you would actually like to implemented. Possible structures for direct sequence spread spectrum systems. Now of course, we said that the match filter approach is the way we going to do it. So, take the received signal multiplied with g star of t, this is basically 1 symbol long after that it will be followed by an integrator from 0 to T again one symbol duration whatever is the output of this multiplication process you integrated over one symbol period, and then once a symbol duration that is at symbol rate you sample and that will be your decision statistic y.

This is the structure that we have just now implemented. So, this is the match filter based approach. So, this g star of t. So, this match filter approach is for anything if you whether spread spectrum or not. So, this waveform when it is a spread spectrum system includes the effect of spreading includes the effect of spreading, which means it has Q chips and then includes the effect of spreading and the and the chip waveform effect of spreading and the pulse shape; pulse shape of each chip, pulse shaping of each chip it has includes both of those. Now there is a another structure very intuitive which more or less emerges from here, but it helps you to start to think in the way that we want to approach the spread spectrum systems, is now that the when we start to have multiple users one of the things that we will have to allocate to them is different users we will get different spreading waveforms that is how we are going to preserve orthogonality.

So, which means that every time something changes you have matched filter has to change. So, now, if you say I do not want to keep modifying my match filter, I want to have a structure that is a little bit not so much dependent on the current spreading sequence, there is a second structure which is very very interesting and helpful. So, r of t multiplied by g c star of t, now regardless of what is your spreading sequences g c is g c does not change it is still that rectangular waveform for 1 chip and this one also it is a match filter. So, we will now integrate from 0 to T c integral d t, integral 0 to T c and the output I am going to sample at chip rate at chip rate. So, basically there is a chip rate clock just like in the previous case you would have a symbol rate clock chip rate clock which is controlling this sampling device.

But you are not done yet, we still need to de spread what you are done is your achieve the best detection possible for each chip, but now you need to remove the spreading. So, whatever was your speeding sequence assuming it was some sequence of plus minus ones, if you multiplied again by the same spreading sequence what happens you will remove the spreading. So, basically the spreading sequence you same spreading sequence if you multiply then what you get is a sequence of chips, that which we can then used to do the subsequent detection mechanisms. So, basically there are 2 ways of looking at the direct sequence spread spectrum system, either at a symbol level it is good as a starting point to do that, but very quickly move into start thinking in terms of chips and how we put how we work with the chips, and that is that is very helpful for us ok.

So, now would like to move into guiding you leading us into the multi user environment. So, we talk about a spreading sequence, we will use the notation c subscript q stands for the chip index.

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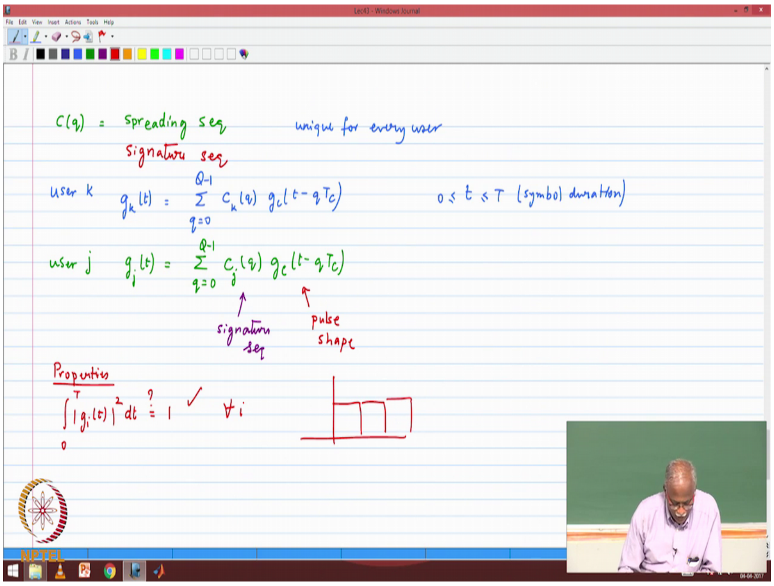

So, this we will call this as a spreading sequence it is also called a signature sequence. So, basically something that uniquely identifies a user. So, removing into the multi user environment it may be the signature sequences a is a very appropriate term. So, and this is unique for every user; unique that is only way we need to be we can separate them unique for every user that is an important element.

So, let us look at user k, let us just look at 2 users k and j and start to make some observations about their spreading sequences. User k the waveform for 1 symbol we will be a combination of the way forms of the different chips, the chip waveform is the same, but the chip values are different. So, that is captured in the following form Q is equal to you can do it 0 through Q minus 1 or 1 to Q, apologies if I have mixture, but it is basically a total of Q chips C k subscript q, g c of t minus q t c. I yeah in order to write this 1 you must have starting from Q because the first chip waveform should be at the origin. So, it should start from Q equal to 0 the for the user next user that is call the user as user j you must have a different waveform and we will now quickly and. So, the duration of this is t greater than or equal to 0 less than or equal to t that is the symbol period.

So, the waveform that is the spreading waveform last duration of one symbol of the narrow band system symbol duration, it is made up of q chips and each of them each of the chips have got it is own unique same waveform. User j g subscript j of t to denote the symbol waveform of user j, we wanted to be different unique. So, Q is equal to 0 to Q minus 1, C subscript j of q g c t minus q T c notice the g c is the same. So, regardless of which speeding sequence you are given the match filter we will remain the same, the processing after the match filter is what will change. So, the 2 components here are this is the signature sequence that is the signature sequence component and this is the pulse shape component chip level pulse shape. The combination of the 2 give you the symbol waveform and then that is a ok.

Now, we already wrote down this property, but I just after we wrote down in the form of chips will this property hold properties integral 0 to T mod g i of t whole squared d t is equal to 1 is it correct, would you agree yes because what is this it is all these type of waveforms right, if I take the magnitude squared and add it way it will come out to be the right value. So, yeah. So, this is correct. So, this is one of the basic property. So, unit energy property is preserved even after we do this. So, this is true for all of the spreading sequences as long as your sequences are plus minus ones ok.

Now, what we would like to add in addition to this are the following, and now here is where it becomes very interesting and request you to pay attention and if there are any doubts we will discuss them.

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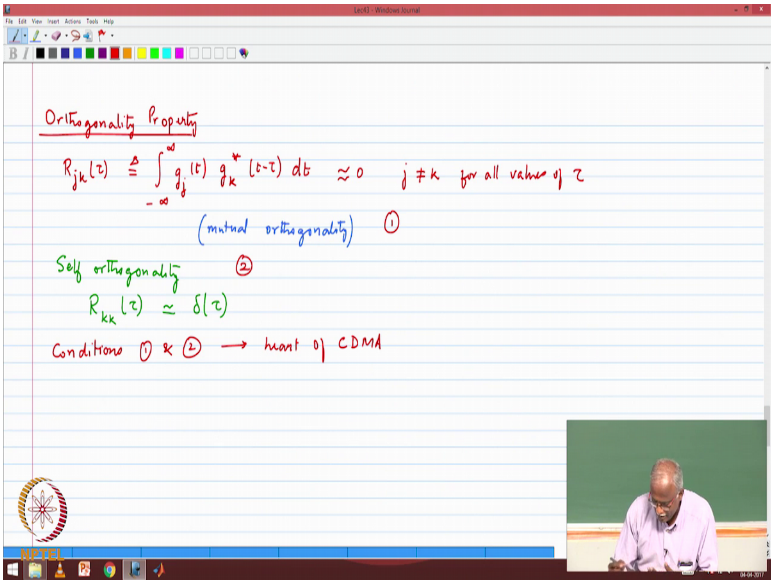

So, I am going to first write down and orthogonality property, that both these waveforms must be orthogonal only then we should we will be able to separate them orthogonality property. The orthogonality property basically says that if I take R j k of tau; that means, the spreading waveforms of the pulse shapes of the j th user and the k th user define this as the integral basically these waveforms are defined over a period of time, but because I am shifting them the most in most general form is minus infinity to infinity, g j of t, g k star of t minus tau d t. So, in other words basically I am correlating the waveform of user j with user k tau can be 0, tau can have shifts as well what we would like to have is that this property is approximately or as close to 0 whenever j is not equal to k for all values of tau.

So, this is a very very important property, this is the property of mutual orthogonality right. So, basically it is orthogonality, but it is between users mutual orthogonality. Now there is another component that is very important for us, that is the next property which is self orthogonality. Self orthogonality is also a part that this very important basically says that if I take autocorrelation of users way form with itself at a delay of tau this must be as close to delta function. If I gave you these 2 conditions and say give me some candidate schemes for the spreading waveforms; what are your what are some suggestions you could give what are things that you if you take white noise and take one segment for user k, self orthogonality is guaranteed because white noise if you only at 0 lag you will get any correlation and take a completely different segments for another user and now these 2 are independent segments of white noise. So, correlation between the 2 does not exist and therefore.

So, start to think in terms of trying to make each uses waveform look white basically that is that is ultimately what we are trying to do. So, that it is not correlated with other sequences and it has only the correlation at 0 lag, at all other lags it does not have correlation this is the very useful property, so 1 and 2 one being mutual orthogonality, self orthogonality being number 2. So, conditions or properties 1 and 2 are pretty much the heart of CDMA systems. The minute we get these 2 pictures in mind because we will be we will have and let me substantiate the statement with the following ok.

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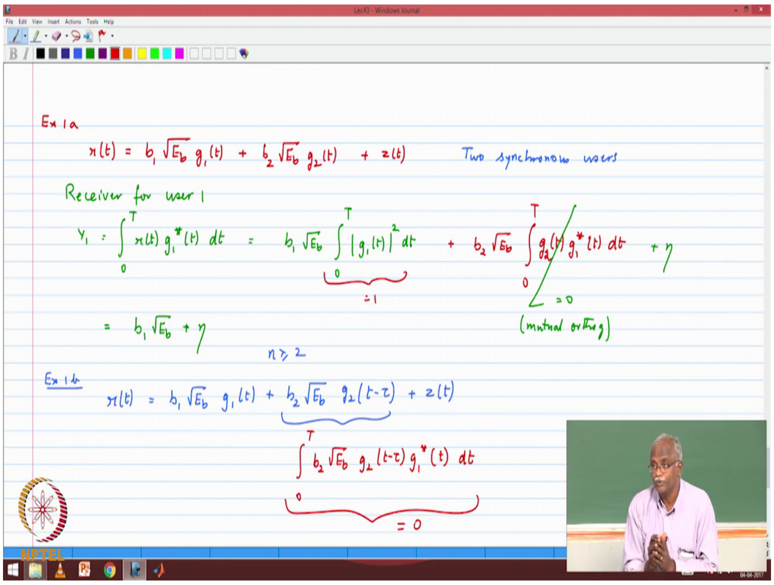

So, I will going to have an example with 3 parts. So, example 1 a, so if I now have a signal r of t which is b 1 root Eb times g 1 of t that is user ones waveform, but what is happening is there is a second user also in the using the channel.

So, in CDMA systems both are using the same frequency at the same time. So, they will interfere with each other. So, the you interfering signal is b 2 root E b, g 2 of t and of course, as always there is noise present in the system z of t. So, this is the case where what you would call as 2 users who are synchronous because both of them are perfectly aligned in terms of their symbol boundaries, 2 synchronous users synchronous users. Now very important that it could be any number we have just taken a simple case of mo more than 1 and now (Refer Time: 29:25) very close attention to. So, what is the optimum receiver for user one. Receiver for user 1 we say that match filter is that is all I know maybe that is that is that right thing to do in the context of the of this environment let us take a look and see.

So, the decision statistic if I do 0 to T r of t g 1 star of t d t. If I do that and if I have satisfy the properties of mutual orthogonality and self orthogonality in this case mutual orthogonality, what will happen the second term when I do the integration goes to 0 right because this will be equal to b 1 root Eb integral 0 to T mod g 1 of t magnitude squared d t now that is equal to one. So, we have gotten the desired signal the second one is b 2 root Eb integral 0 to T g 1 of t sorry g 2 of t g 1 star of t d t mutual orthogonality says this the term will go to 0 if mutual orthogonality due to mutual orthogonality and of course, the noise is a filtered form of the noise basically let me call that as eta. So, that is my decision statistic.

So, this is nothing, but b 1 times root Eb plus eta this. So, what did it do it took you from a multi user environment to a single user AWGN like environment, all because you could satisfy mutual orthogonality. Now you could; obviously, this is applicable for n greater than equal to 2 also. So, we can have, but the key thing is you must have waveforms that are orthogonal and the uses have to be synchronous. So, therefore, it is an important element. Second now what if the users are a synchronous; so, example 1 b a synchronous users you will capture in the following form b 1 root Eb, g 1 of t please tell me how to write the a synchronous user b 2 root Eb g 2 with some shift t minus tau called the plus z of t, apply the same state decision state I mean the receiver mechanism user the date of user 1 comes out, the second term when I do the integral 0 to T b 2 root Eb, g 2 t minus tau g 1 star of t d t ok.

Now, of course, b 2 root t b you can take out the mutual orthogonality is not is valid for all tau. So, therefore, this term also goes to 0 or very close to 0 and therefore, you are once again. So, whether it is synchronous users or asynchronous users you find that you are able to because of the assumptions of. Now how difficult is it to get mutual and orthogonality that is the challenge, but if you have that then you have the very powerful system the last of the last part of the example.

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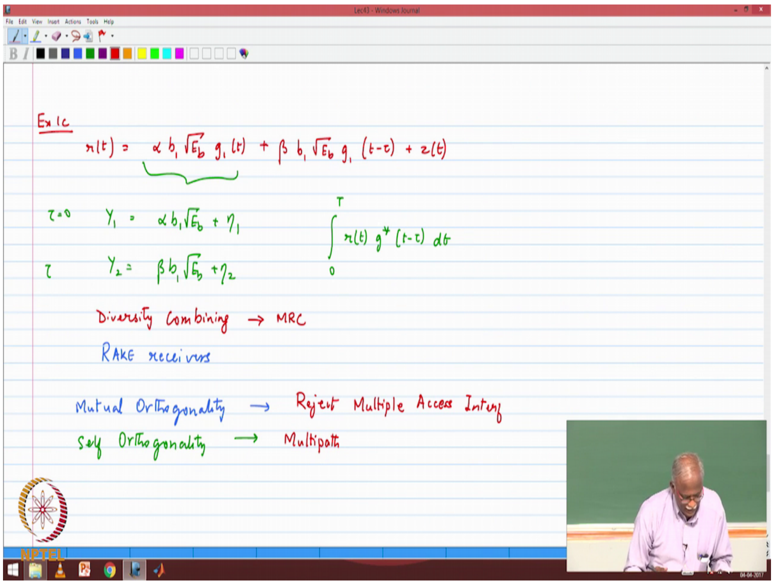

Example one c now look at this example and describe this signal for me, r of t is alpha b root b 1 root Eb g 1 of t plus beta times b 1 root Eb g 1 of t minus tau plus z of t; what type of channel is this? Multi path channel and it is got different multi path gains alpha and beta.

So, now what I do I apply the same type of receiver and what comes out is this 1 comes out as y 1 and I get alpha times b 1 root Eb plus eta 1 what happened to the second term because of self orthogonality right when you are not aligned perfectly at tau equal to 0 that term goes to 0. Now, but the good thing is if you now shift your correlation to align with the second term you get a second output which will knock off the first term, but we will preserve the second term beta times root b 1 root Eb plus eta 2. So, this comes out when you do integral 0 to T, r of t g star of t minus tau d t. So, basically when if you slide your correlation instead of just aligning it first time y 1 you got with tau equal to 0 right with this shift of the correlating waveform this one value of tau.

So, basically you got these two. So, you got now 2 copies of the data that you are trying to detect of course, this looks like diversity and what should you do maximal ratio combining or something that is as close to as best as you can. So, basically you would do some form of diversity combining to get the best performance and of course, this was going to give you a benefit in the fading waveform because alpha and beta could be independently feeding. So, therefore, you will get the diversity benefit and. So, you would try to do something like MRC if you can and CDMA systems, actually use a technique call the rake receiver which is a form of diversity combining has we have already mentioned. So, the 2 properties are going to play a very key role, mutual orthogonality is a key element or key feature that will help us deal with multiple access interference. It rejects all other user signals this one rejects or suppresses multiple access interference that is the interference cause by other users were using the channel at the same time multiple access interference.

The other component is the shelf orthogonality and that is the 1 that helps us capture the multi path, this is the one that helps capture the multi path and these are the 2 impairments in a cellular system and these are though and the spreading waveform and the receiver for the co corresponding spreading waveform is going to play a huge role in helping us work with these, any doubts or questions.

Now we move into an interesting discussion and hopefully this will start to build the intuition regarding the sequences themselves. So, the question that we know ask is now where do we find such sequences, you know you this all nice to write down mutual orthogonality self orthogonality, but I need to find the sequences. So, again you are going back to our understanding that we need to be looking at the random sequences or random types of sequences.

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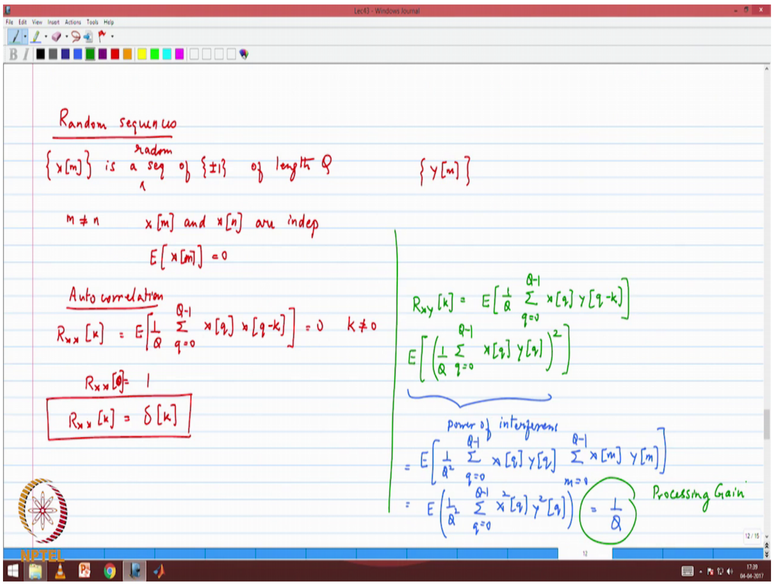

So, we will study a basic properties of random sequences and the best that we can do in practice are pseudo random sequences. So, we will we will basically so, but first let us look at random sequences. So, random sequences are defined as a sequence of plus minus ones, is sequence random sequence of plus minus ones of length Q.

Basically you can think of it as a segment of length Q of a much larger sequence if that we will help because you know random sequences are not necessarily defined for a specific length. Now these sequences have the following properties that if I look at 2 different indices m and n they are not the same, then x of m and x of n are independent and that is a basic property of a random sequence, they are independent and because these are bipolar plus minus ones the expected value is 0 expected value of m is 0, and now if I have another sequence Y of m. So, now, I can think of it as now the waveforms that I need for 2 users, and now we want to look at the properties of this type of a system.

So, first thing that I would like to do is autocorrelation; autocorrelation property and how it manifest itself and cross correlation property and how that would manifest itself. So, R x x of k this would be expected value of 1 by Q because I am doing a summation over k summation Q equal to 0 through Q minus 1, x of q, x of q minus k, x of Q minus k. So, as long as k is non 0 this term is going to go to 0 because of the independence property if k is not equal to and if k is equal to 0 then what we get is R x x of 0 will be equal to 1. So, basically we can write R x x of k to be equal to delta of k, that is good property of the random sequences. Now move over and start to think in terms of multiple users, I need to worry about what will be the cross correlation between these waveforms this would be expected value of 1 by Q summation Q equal to 0 through Q minus 1, x of q, y of q minus k.

Notice I have drop the conjugate because I am basically dealing with the real sequences. Now this is a very important quantity for the following reason. When we try to look at the way the receiver for user 1 and I apply the way the correlation based technique, what is going to happen is the second term we will contribute something. So, basically there is a user 1 signal user 2 signal and then there is noise. So, the user 2 signal is going to contribute a term that is going to be affected by the cross correlation. So, ideally you would like this to be 0, but if it is not 0 what is the value. So, that is what we are trying to understand ok.

So, what we would like to do is look at the mean and the and the variance mean you can easily shows equal to 0, but now I want to look at expected value of the magnitude square of this because that is going to tell me how much is the interference power because this is the correlation value this is what is going to look into my decision statistic, the magnitude square is going to be my interference power. So, I want to look at 1 by Q summation q equal to 0 to Q minus 1, x of q y of q whole square. So, basically I have taken Q equal to k equal zero, but the again this is a useful form to say that this is the type of interference power. So, this is going to be the power of the interference it represents the interference power of the interference component in my decision statistic power of the interference and. So, when I write down this form I will again go back to the form that we had written down earlier, becomes 1 by Q squared summation q equal to 0 to Q minus 1, x of q y of qsummation m equal to 0 through Q minus 1 different index x of m y of m.

Basically if Q and m are not the same the this expression we will go to 0, and if they are the same what you will get is basically you will get expected value of 1 by Q squared summation q equal to 0 through Q minus 1, x squared of Q y squared of Q x squared of Q become plus 1 y squared of Q becomes plus 1. So, this value becomes 1 by Q what is the takeaway from this discussion? This said you would you took this random sequences and looked at the. So, when I apply the receiver for user 1 some signal is going to leak in from user 2 the power of the interference that is cause by this leakage has got a 1 over Q factor when I if I use random sequences, Q is my spreading factor.

So, the larger my spreading factor the multi user interference is going to go down and that is the fundamental property that of that we have taken from spread spectrum systems. Because if you remember I told you jammer tone jammer if I have sufficiently large spreading factor what will happen it will suppress the interference by a factor of Q, and the which means that if I have a sufficiently large spreading system a spreading factor it will pretty much surprise the interference and you can see that this suppression is a very powerful one, and this is what we referred to as processing gain. Processing gain is what you get because of your spreading factor and this is the reason why we do spread spectrum not for AWGN because an AWGN we will not get any benefit. So, what have covered so for? What we said was I would like to have multi user scenario, the best scenario is if I can set my user multi user waveforms we will satisfy mutual orthogonality and self orthogonality, again I do not know how to find such sequences.

So, you know in the quest for the sequences we come upon a random sequences look like what we are look what we want you know if it take Q long enough autocorrelation the self orthogonality is guaranteed mutual orthogonality is not 0, but it is 1 by Q which is quite good we will get the advantage of these of the spreading factor. So, this is the pointed which we did not say now where do you find random sequences and we said the random, sequences do not a exist.

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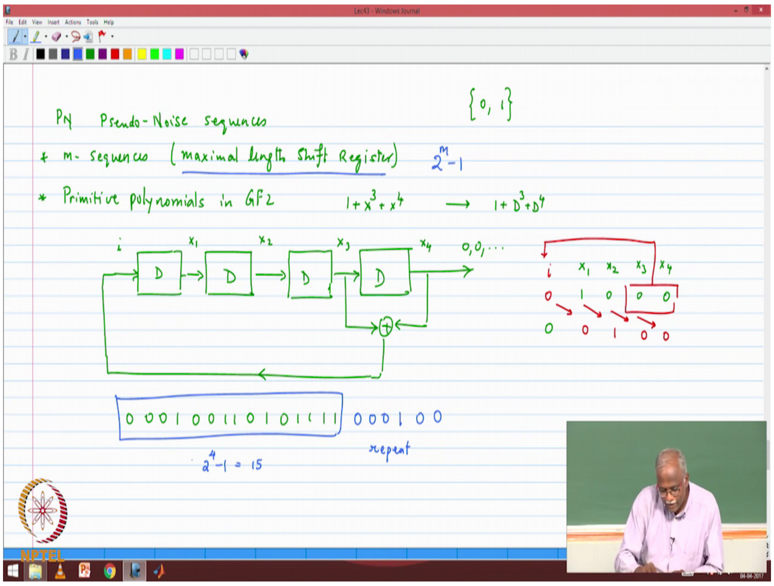

So, therefore, we have to go with PN sequences pseudo noise sequences or pseudo random sequences. So, pseudo noise sequences, can I assume everybody knows the about PN sequences that something I can assume or do we need to spend a few minutes just to clarify the concepts. PN sequences studied in digital communications yes I am not getting after machine.

So, let maybe we should spend a few minutes. There is a class of PN sequences which are called m sequences which are very very widely used in CDMA systems, and these are also called maximal length shift register sequences I would definitely would like my students to be familiar with m sequences and their properties because these are so, widely used they are also referred to as a linear shift register sequences. So, again we will the name becomes very very clear when we look at the properties.

So, what you do is you look for primitive polynomials, primitive polynomials in GF2 polynomials. Once you have the in GF2 and once you have these primitive polynomials and I am sure you would have you know. So, basically for example, this is a primitive polynomial 1 plus x 3 plus x 4 is a primitive polynomial in GF2. Now such primitive polynomials we will take and constructed into a interconnection of shift registers basically the way we would do this is we would write this in terms of delays. So, this would become 1 plus D cubed plus D 4 and the linear shift register sequences feed forward shift register sequences would be a delay 4 delays is what I will need always moving to the right D and D.

So, the output of the third delay is the coefficient of D cubed and output of the fourth delay is the coefficient of D power 4. So, basically I have to add these 2 components. So, that will be 1 plus sorry D cube plus D 4 that is basically this is a cube plus D 4 now because of the binary property, which says that if I add that to itself that is what it mean say you can write it as you know take 1 to the other side. So, one is equal to the input is equal to D cube plus D 4. So, basically the shift register sequence now what I do. So, this is the implementation of the this 1 plus D cube plus D 4 in a binary context. So, basically now think in terms of zeros and ones not as plus ones and minus ones we finish generating it then you can go back and think about.

So, basically let us call these variables in the following form, let us call this variable as x 1 x 2 x 3 x 4 and 5. So, basically if I initialize x 1, x 2, x 3, x 4 with 1 0 0 0 that is my starting state you can start with except the nonzero all 0 state you can start with any of the other states basically you initialize x 1 x 2 x 3 x 4 the operation says I do exclusive or of x 3 and x 4 and that has to go back as my input and that I will call that as I and that is 0. So, at their first time instant I have initialize x 1 x 2 x 3 x 4 that we will decide what the input i s. So, now, we have all these at the next clock cycle what happens all these guys we will move to the right every one of these we will move to the right. So, 0 moves to this position 1 moves to this position, I have 0 and 0. So, 0 0 the exclusive or I have comes here.

So, basically now you start to see the sequence what is my output this sequence. So, the first time instant this was 0, next time instant it was 0 and then you can fill up. So, just, so that we would not spend too much time on this, please when you get a chance go through and write down this sequence and what you will find is the following. The sequence that you will get at the output we will be of the form 0 0 0 1 0 0 1 0 0 1 1 0 1 0 1 1 1 and then you will find that the sequence repeats itself then you will get 0 0 0 1 0 0 dot dot dot. So, basically this is 1 full copy of the waveform of the sequence and then this is a repeat. You can verify that this has the length 15. So, this is nothing, but 2 to the power minus 1 equal to 15 and this is the basic property of maximal length shift registers.

So, if you get a sequence that repeats only after 2 to the power of m minus 1, then you get a maximal length shift register sequence, it is called a PN sequence for a very specific property of the randomness of the form it is deterministic and say you know a because if you initialized it correctly and you know what the feedback types are the or the in interconnection types are, you can actually predict the sequence. So, it is not random in that since, but the occurrence of the zeros and ones because of the GF 2 property is the primitive polynomial all of that you will find that it has a certain pattern that is very helpful. So, let us stop we will continue on this.

Thank you.