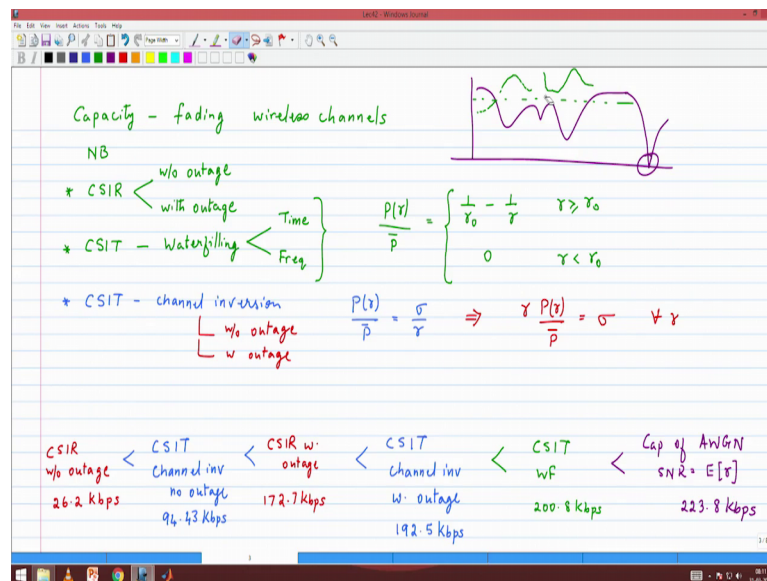


**Introduction to Wireless and Cellular Communication**  
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**Lecture – 43**  
**Wireless Channel Capacity – Water filling**  
**Intro to Direct Sequence Spread Spectrum Communications**

Good morning, we begin lecture 42 we have completed our discussion on the capacity of wireless channels and the various flavors of a water filling and the last class we looked at a very specific suboptimal method called channel inversion which we saw that I gave us fairly good performance. So, with that we now move into a new unit on co division multiple access as we mentioned yesterday this; the origins are from a direct sequence spread spectrum and how do we build on that. So, first let us do a quick run through of the concepts that we have looked at in our discussion of the capacity.

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So, the discussions on capacity we have been talking about fading channels with fading wireless channels and we have been interested in achieving the capacity and we have more or less being looking at narrow band channels where we look at the assumption that there is variation in time, but the frequency response is flat. So, frequency flat response. So, under this assumptions we said that we could look at cases with CSIR that only the receiver knows the channel state information of course, there are theoretical constructs

which tell us that yes even for infinite delay infinite complexity we can build capacity achieving codes, but when we say now let us look at very practical very realizable scenarios then we said that CSIR if I insist on without outage then I am very severely constrained because I do not know what the channel conditions are and I have to assume that it is the worst possible channel conditions. So, basically I am forced to go with a very conservative design; however, we said that if we are allowing outage with outage then we can afford to be a little bit more aggressive in terms of our performance.

So, this we saw in our example the case of CSIR we kept the without outage without outage the 3 SNRs example we saw that if I insist that there is no outage then I go with the design for the lowest rate and the capacity that was achieved was a capacity for the lowest SNR. So, 26.2 kbps on the other hand if we allowed outage, so, basically CSIR with outage with outage we saw that we could do much better of course, there will be periods of time when the data transmission will be lost, but we did achieve 172.7 kbps.

So, there was an advantage with the accepting outage now the next task next version of capacity discussions that we did was the scenario when the information about the channel status available at the transmitter its instantaneous and it is perfect under this condition we said that we could do water fill and that is the optimal method of achieving capacity for a given for a given channel. And as we saw the basic flavor of a water filling when we talk about narrow band channels is water filling over time we also saw that if it was a wide band system then you could have water filling over frequency you divided into small sub bands. And of course, if it is a time varying wide band signal in a time varying system then you can do water filling over time and frequency again that would be the way to achieve it the flavors of water filling are always the following your time to allocate power to the different channel conditions.

So, basically a for a channel condition with SNR  $\gamma$  your allocating power  $P$  of  $\gamma$  divided by  $P_{\text{bar}}$  and the general form of the water filling says that it is with respect to a reference level  $1/\gamma_{\text{naught}}$  and this if  $\gamma$  is greater than or equal to  $\gamma_{\text{naught}}$  equal to 0 if  $\gamma$  less than  $\gamma_{\text{naught}}$ . So, there are times when the there is no transmission and you basically allocate the power else at other times.

Now, in the last lecture we discussed a sub optimal method with CSIT. So, CSIT that transmitter knows the channel, but it is doing a sub optimal scheme CSIT with channel inversion channel inversion channel inversion basically says that I have to allocate power. So,  $P$  of gamma divided by  $\bar{P}$  this is equal to  $\sigma$  by gamma. Probably this is not as easy to interpret as if you rewrite it the new SNR with power allocation is always given by gamma the original SNR times the power boost or the power decrease that is the new SNR this is the new SNR is equal to  $1/\sigma$  for all values of gamma.

So, basically you have made it look like an AWGN oh sorry it is not  $1/\sigma$ . So, you have made it we have made it look like a AWGN channel with the constant SNR and of course, it becomes easier for us to work with this. So, even under a channel inversion we had 2 options that we insist that there is no outage without outage which means that we have to take into account the channel inversion even of the bad channels and then the case where you have with outage.

So, now we have this entire canvas of different methods. So, let us see where each of them fits in. So, CSIR with outage of course, was the most conservative then we found that if I did CSIT with the channel inversion, but I insisted that there was no outage. So, basically CSIT with channel inversion channel inversion, but no outage no outage yesterday we saw that case this came out to be 94.43 kbps. And this of course, is worst then CSIR with outage which achieves 172.7 this in turn is improved upon when you do CSIT channel inversion with outage channel inversion with outage you allow some bad conditions bad channels to be removed we showed that under this assumption the capacity went up to 192.5 kbps and of course, the best that we can do is water filling.

So, this is CSIT water filling and this achieves 200.8 kbps and of course, the ultimate limit for us is the AWGN channel the capacity of AWGN channel with average SNR with SNR equal to expected value of gamma and this came out to be 223.8 kbps. So, what we have done is a taken one illustrative example and we have shown the entire spectrum of the different methods that we can use for you know looking at capacity.

Again it is just gives us a flavor for what is the performance how do each of these schemes perform relative to each other and it is a very instructive example of course, again it is a limited example it is got only 3 states when you apply it in a practical

situation you have to take more factors into consideration. But I believe this is a good way to understand how capacity is achieved how are what are some of the trade off if you do not have information at the transmitter what happens if you allow outage and then of course, always measuring ourselves with respect to the AWGN channel any questions any questions on the various types of capacity yes.

Student: Regarding channel inversion, sir.

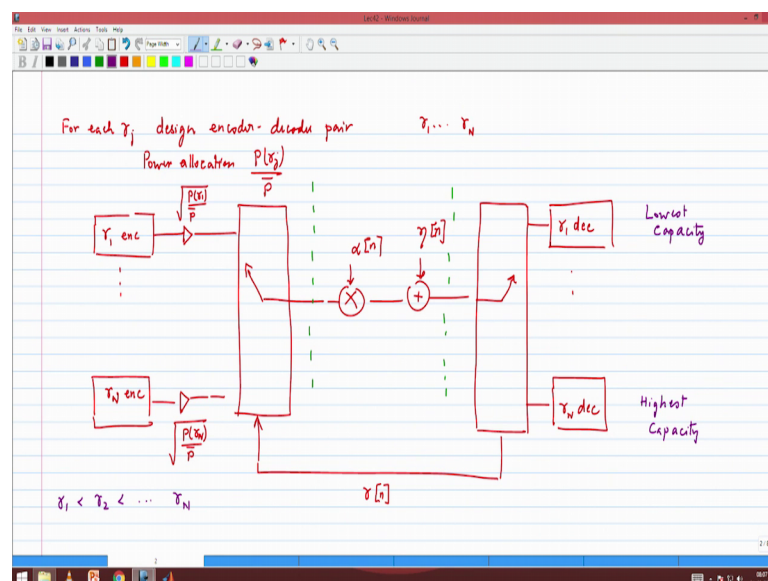
Yes.

Student: Job is bank of encoders and decoders at the without channel inversion.

Student: To which channel inversion we have one encoder one decoder can you just explain once again how come the complexity is reduced with channel inversion.

The question is the way the water filling works we have many encoders many decoders with power allocation, but somehow with the channel inversion technique we do not need many encoders and decoders we all we need is one. So, basically what we have achieved I am glad did not erase that slide what we have achieved with water filling is that we have been able to create channels where the SNR is very different for the different conditions.

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So, because we have also modified the original SNR through power allocation, so, we and if we notice or if you recall our way of doing power allocation was to the good channels give more power that is what water filling told us. So, basically some channels is already good you gave more power and you made it. So, you kind of made it more dispirit. So, because you made your SNRs more dispirit. So, you have channels with the not. So, good SNR and channels with very good SNR because after power allocation.

Now, because you have such large range of SNRs now what you can do is actually achieve better performance for the times when you have the good channel conditions. So, how do you exploit that that is the way of the designing a encoder decoder let us say this is this is for let us make the assumption that  $\gamma_1$  is less than  $\gamma_2$  is less than  $\gamma_n$ . That means, this encoder  $\gamma_n$  encoder decoder pair will achieve highest capacity highest capacity and of course, the on the other end this will be the lowest capacity, but the idea is that you still use the channels whenever it is possible and water filling tells you that go ahead and use it, but you use it in a judicious manner only allocating their amount of power that will achieved.

So, overall when you look at across the entire distribution of SNRs this method achieves the capacity and in order for it to achieve capacity is you have created channels with desperate SNR for each SNR you want to choose the appropriate encoder decoder. So, to achieve capacity now on the other hand very interesting question that you raised what did we do channel inversion we did exactly the opposite we took the channels that were different in SNR and we equalize them made them all look like a channel with the constant SNR. Now is that the good idea we thought it was not such a good idea because that is what water filling told us, but it turns out that this is a way to simplify the problem provided we and take into account that we can we can tolerate some outage.

So, basically channel inversion says make the channel look constant bad channels give more power. So, that it can reach the average level good channels you do not need to give. So, much power again in the bigger scheme of things if you want to maximize capacity water filling is a right way, but this is a sub optimal method which tells you that at least for this example in practice you will find that a channel inversion is actually you will you will have you will pay bigger penalty because inverting very bad channels not a good idea.

So, even if you take willing to take outage you will find the channel inversion. So, water filling is a probably the way to go; however, if the constraints are complexity then maybe you can do outage and the reason you are able to do it with one encoder decoder is because you are resultant channel looks like a channel with constant SNR looks like an AWGN type of channel it is not a the effect of fading has been somehow masked by power allocation and as I mentioned. So, so the method that you would have to do for channel inversion if your signal is varying like this what you are trying to do is make it look like a constant channel right you making it look like a like a constant channel.

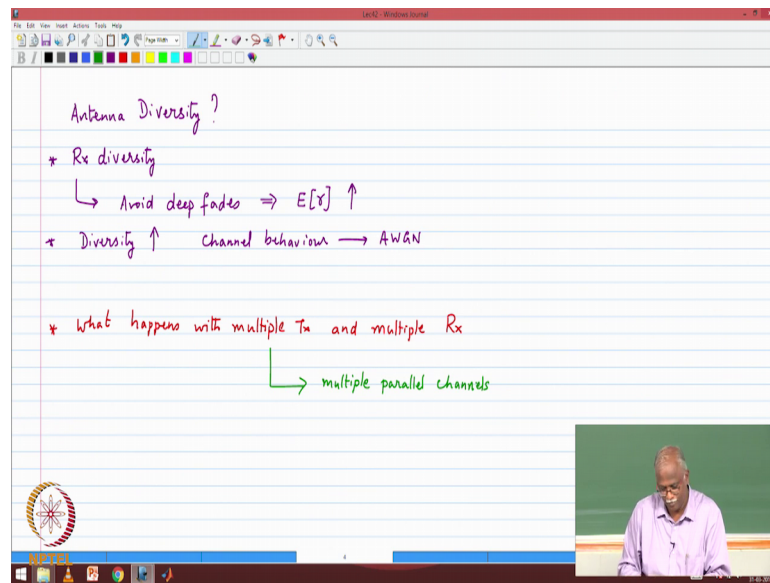
So, your power allocation has to pretty much go the inverse of it because the product of this 2. So, wherever it is low you will have high you have to invert the channel. So, wherever it is; so, at this point it will be high and again it will be high here. So, at the product of those 2 have to equalize. So, the key challenges if you have a if you have a deep fade suddenly if you have a deep fade now this is this is going to be very hard to invert.

So, that is when you will say that that portion I am going to leave as outage, but the problem with the challenge will be some channels are very fast. So, which means that the amount of time for which you get to you get to do the channel inversion is very less and therefore, the channel equalization or the channel inversion is actually a quite a quite a difficult task for us to do in practice because the channel is constantly changing how do you keep inverting it and make him because. Remember this feedback has to go to the transmitter you have to estimate it you have to feed it back to the transmitter and transmitter have to adjust the power.

So, there is a time constant that is associated with all of these algorithm and therefore, again channel inversion with fast fading channels is a challenge, but if you are able to do it low complexity single encoder decoder gives you reasonably good performance good. Let me ask you a few questions how will antenna diversity effect capacity or it will not affect capacity at all antenna diversity will there be any impact on capacity impact on capacity only receive diversity  $R \times$  diversity what is your will it affect how.

Student: your average SNR goes up.

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So, basically what is antenna diversity do it avoids deep fades right fundamentally that is what we are doing avoid deep fades if you had a single antenna the you cannot avoid these portions of time when you are in a deep fade. So, this essentially means that the average SNR improves and the; and capacity is directly tied to the average SNR and therefore, we get the benefit now what happen if I keeps increasing my diversity order 2 3 4 5 6; what will happen. So, diversity order increases. So, diversity in general helps what happens if I keep increasing the diversity order diminishing returns, but is there a benefit in terms of capacity yes because the channel approaches AWGN right because the more and more antennas you have its starts to behave. So, channel behavior approaches AWGN you may say that in terms of the SNR gain not much, but capacity AWGN is the best. So, therefore, there is no this is the performance bound for us.

So, in general we can make the statement that the antenna diversity is good receive diversity will increase the capacity of a fading channel. So, this is still one transmitter many receive antennas. So, only 1 cross m system it is a Simo system single input multiple output; however, what will happen if you had multiple transmitters and multiple receivers what will happen. So, the question very very important question is what happens what happens when you have multiple transmitters what happens with multiple transmitters and multiple receivers multiple T x and multiple R x multiple R x is still going to give you the benefit what about multiple T x multiple T x and multiple R x any

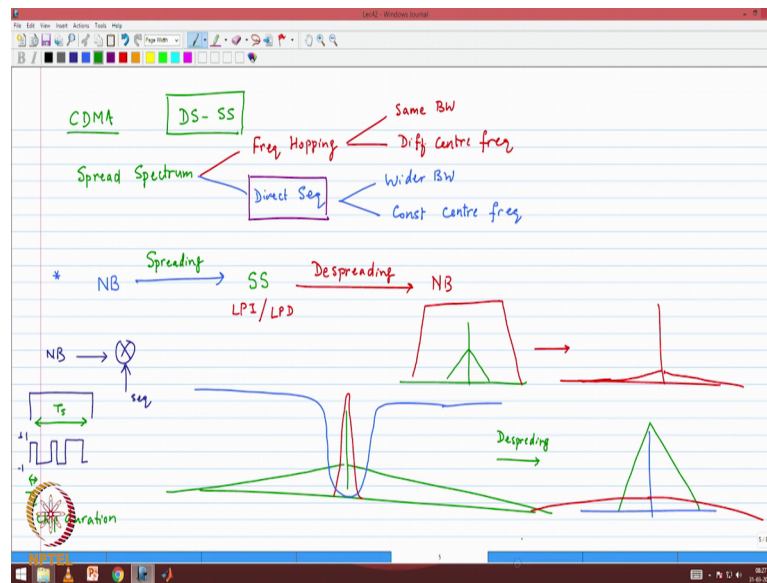
idea any guess no; not much it turns out that if 2 antennas transmit at the same time what happens interference is generated.

So, you have to first deal with the problem of interference. So, it turns out that which we will see when we talk about MIMO systems is that when you have multiple transmitters and multiple receivers and you have more receive antennas than transmit antennas; that means, you can handle the interference then what you can do is you can transmit parallel streams so; that means, what was originally the capacity of one channel now looks like single channel now looks like a number of parallel channels. So, this will lead to a scenario with multiple parallel channels. So, therefore, your capacity is now going to increase linearly with the number of transmitters and so, the relationships are so multiple parallel channels you basically are creating multiple parallel channels same bandwidth  $b$  previously you could transmit some information now it looks like you have you have you know  $4b$  that is 4 channels with bandwidth  $b$  that is the effect of and all of them are getting diversity benefit there also managing the handle the interference and giving you the rate.

So, there is a very interesting study of multiple antennas from the point of view of how much diversity do I get there is also another dimension that we can look at which says how much capacity will I get because you know diversity is good interference suppression is good, but also keep in mind that MIMO systems will give you a capacity increase as well. So, again the notion of capacity of a wireless channel is not a single number it has to do with so many parameters do you have CSIT do you have multiple antennas at the; at the receiver do you have multiple antennas at the transmitter. So, then the minute you say there are multiple antennas at the transmitter then the problem changes it is no longer a single channel its starts to look like a multiple channel a capacity problem that we are dealing with, but it is good that to keep in mind we will we will address it when we come to that section of the course.



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So, the bulk of today's lecture will be for us to get an introduction to CDMA; CDMA is a name that we give to the multiple access. So, realistically speaking that is not that does not describe the modulation or the other things. So, therefore, may be the right name should be direct sequence spread spectrum systems that is what we are using and when we use this direct sequence spread spectrum for multiple access it becomes a CDMA system and as we saw yesterday direct sequence spread spectrum spread spectrum primarily comes from the heritage of military radios and spread spectrum traditionally was a frequency hopped spread spectrum. That was more commonly used frequency hopping frequency hopping is different from direct sequence spread spectrum in the sense that you have the same bandwidth as the signal as without the frequency hopping it is a same signal same bandwidth, but the center frequency keeps changing the different center frequency.

So, basically the center frequency is what is moving around. So, the bandwidth remains a constant on the other hand if you have a direct sequence spread spectrum system direct sequence it has got a very different characteristic the bandwidth is much wider bandwidth and there is no change in the center frequency there is a constant center frequency. So, these are 2 very different ways of generating spread spectrum, but it is important for us to keep those keep that picture in mind and the way we would now our focus is going to be on the direct sequence spread spectrum. So, when we say spread spectrum it is going to mean DS SS not the frequency hopping version.

So, our notation will be that there is a narrow band system which through a process of spreading and we will see in a moment what that process is going to be this is going to give us a spread spectrum system. And the spread spectrum system as we discussed yesterday has got the features of low probability of intercept low probability of detect the some of it is you know depends on how you want look at it is hard to detect the signal because it looks like noise after you have done the spreading when we do the de spreading process in order to at the receiver de spreading de spreading brings back the narrow band signal and of course, your detection happens on the narrow band signal.

So, what is it that we are doing with spread spectrum I have a channel which has got some impairments and I want to get the signal across from point a to point b through this channel in a most reliable manner may be this is a way for us to take advantage of what the military radios use to do and we are going to see how that is going to play a part. And it is interesting for us to note that the narrow band system I convert it into a spread spectrum signal then I transmit I launch it into the channel. So, whatever the channel is going to do it will do it to my spread spectrum signal which I will then take it into the receiver where I will do the de spreading operation and then I will get back what I want to do. So, the spreading operation basically takes the narrow band signal narrow band signal and then it multiplies each symbols let us say one symbol has got some duration it multiplies with a sequence a sequence which looks like this some.

So, basically it has some plus 1 minus 1 combination and you multiply the 2 and what we what we will show in today's class is that if I have a narrow band signal which looks like this when I do this after the spreading process it becomes through the process of spreading it becomes much wider and much lower. So, hard for me to show the; so, basically it is much much wider and the peak levels are much less. So, and the more I spread the more the wider it will become and it the less it will look like noise now in military radios again this is a little bit of interesting side fact it is good to appreciate spread spectrum one of the things that they like to do in military communications is something called jamming let me ask the military people to tell me what jamming is and when do you use jamming.

Student: Jamming is something like if you have a id which is operated remotely.

Ok.

So, like whenever the jammer is the first vehicle which moves in a (Refer Time: 26:23)

Yeah so.

Student: Jam a very neighboring radio station, so that.

So, if you want to move thank you basically that that is jammer is the one that goes into the battle field because you want to disable the communications of the enemy right of one is you want to detect what is what they are doing that is the interception part the other part is you do something to prevent the enemy from communicating. So, now, what happens yeah it is this has got low probability of intercept what happens in the context of a jammer. So, let me just over lay this in with the with the context of a of a jammer basically jammer signal is as follows you it is much higher power and may be a little bit wider than your signal, but by and large it is of the same it is also a narrow band signal, it tries to detect where your signal is or enemy signal is and then creates some random information which is red color. So, at the receiver what the receivers sees enemy receiver will receive this red it has no clue what this red means. So, therefore, their communications is disrupted.

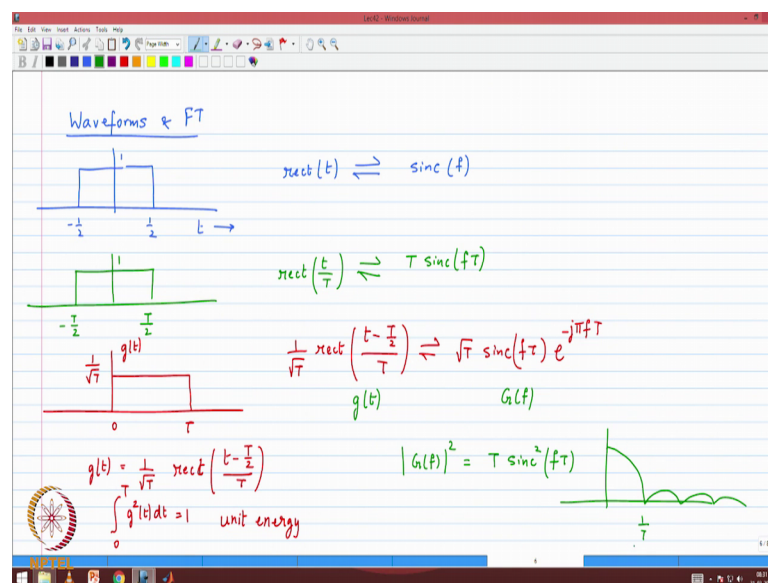
So, now interesting happens what happens in the context of a spread spectrum system. So, if I have already spread my spectrum I am just going just for illustrative purposes my signal has already been spread the jammer is on jammer is like before does this now I have to take this information and pass it to the de spreader right. So, this goes into the de spreader. So, this is de spreading operation is going to happen in my receiver a very interesting thing happens a very interesting phenomenon it happens that your signal the green signal now becomes like this and the process of de spreading actually takes this red one and spreads that.

So, what you thought was actually a jammer actually you know got spread in the process. So, there is a very interesting phenomenon that you can you can understand from here this process of spreading and de spreading is not just a trivial thing that we are doing it actually has a very significant information sometimes the in the military they also knew you do what is called a tone jammer; that means, just a strong tone. So, that is says I am not going to use this type of a jammer I am going to use a very strong tone as my jammer.

So, that that scenario is strong tone I am gone jam with a strong tone if its a if its a spread spectrum signal what it does is it just sort of filters out the tone filters out the tone it loses a part of its spectrum, but it actually can still be detected you can actually throw out portions of the spectrum and still be able to detect. So, this is called a removal of a tone or excision of a tone. So, there are several things that you would have studied in digital communications again the ideas not to repeat, but to just sort of refresh you to the points that a spread spectrum system has got several unique advantages and our question is going to be; what is it that we can do so, that we can benefit in the context of a cellular system.

So, first is some notation and terminology. So, that we are talking the same language in terms of our notation.

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So, let us look at basic way forms and spectrum again this would be a repeat of what you have done in digital communications, but I believe it is very helpful for our for our discussion. So, the pulse shape that we are going to use a by enlarge is going to be a rectangular pulse shape in a lot of the CDMA systems this is what we use. So, I want to sort of build the basic frame work of notation minus one half to one half with the height of one where this is the time access this way form is called rect of t am I correct that is what if the notation that you have used.

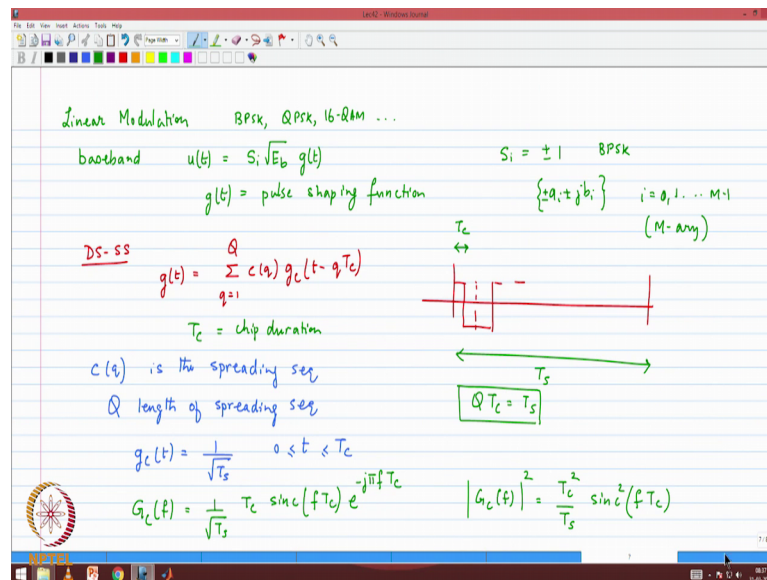
Now, rect of  $t$  has got a Fourier transform which is Sinc function Sinc of  $f$ . So, that is the first basic, but our time periods need to be scaled. So, we will look at the rect function not as minus one half to one half, but as from some symbol  $t$  by 2 to minus  $t$  by 2 to  $t$  by 2 whichever is the symbol period right and the reason I am using it has  $t$  I will explain in a minute because in the context of a CDMA system there are 2 symbol durations that are present this is called the symbol duration  $t_s$  I multiplied with the sequence that has got a duration  $t_c$  this is chip duration versus symbol duration and both of them will use the rectangular way form. So, that is why I am using it as  $t$  I will I will change it over to the notation in a in a minute.

So, minus  $t$  by 2 to  $t$  by 2 to 1 give me the description of this, this is rect of  $t$  by  $t$ ,  $t$  divided by  $t$  scaling in the time access will effect scaling in the frequency Fourier transform I will get  $t$  times Sinc  $f t$  1 step away from the answer the actual way form that I am going to use is not minus  $t$  by 2 to  $t$  by 2 is actually from 0 to  $t$  that is how we always define the way form for a symbol 0 to  $t$  where  $t$  is a symbol period and I am going to use a different scaling  $1$  over  $t$ . Now usually we call the pulse shape  $g$  of  $t$  you may call it  $P$  of  $t$ , but does not matter  $g$  of  $t$  is the notation that we are going to use. So, this give me the description of  $g$  of  $t$ ,  $g$  of  $t$  is  $1$  by square root of  $t$  rect of rect of  $t$  minus  $t$  by 2 divided by  $t$  is that correct is that correct all right. So, basically you can also verify 0 to  $t$   $g$  square of  $t$   $dt$  is equal to 1 basically it is a unit energy that is why we have done this scaling of  $1$  by root  $t$ .

Now,  $1$  by root  $t$  rect of  $t$  minus  $t$  by 2 by  $t$  the Fourier transform will be root  $t$  Sinc  $f t$  delay of minus  $t$  by 2  $e$  power minus  $j \pi f t$  again basic Fourier transforms and the reason that we are interested in is and if this is  $g$  of  $t$   $g$  of  $t$  let me call this as  $g$  of  $f$ . I want to know what is  $g$  of  $f$  magnitude square and  $g$  of  $f$  magnitude square comes out to be  $t$  times Sinc square  $f t$  Sinc square  $f t$  Sinc, Sinc will have both positive and negative going way for Sinc square will basically look like this the first will be  $1$  over  $t$  for 0 crossing.

Now, the key point is how do how do how does this make a difference in our discussion and how do we interpret the signals that we are going to be going to be encountering. So, here is the here is the way we want to look at look at the expressions.

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So, if it was not a spread spectrum system just a regular digital communication system with linear modulation linear modulation. So, linear modulation can be BPSK can be a QPSK 16 QAM all of these are linear modulation methods dot, dot, dot the baseband signal; baseband signal representation will be  $u$  of  $t$  will be the complex signal let me call that as  $s_i$  scaled for energy purposes root  $e_b$  whatever is the  $e_b$  that you want to achieve multiplied by the pulse shape  $g$  of  $t$ .

So,  $g$  of  $t$  is the pulse shaping function pulse shaping function one more step and then we will see why this discussion is beneficial now  $s_i$  are the symbols we are going to be looking primarily at spread spectrum system which are using binary symbols. So, it is usually plus or minus 1 if it is BPSK typically it would be a  $i$  plus or minus  $j b_i$  plus or minus a  $i$  plus or minus  $j b_i$  for and where  $I$  is equal to 1 0 1 all the way through  $m$  minus 1 for a every way form. So, this is the very very broad class of signals that would be characterized by this expression now for a direct sequence spread spectrum system DS SS the way form that I am going to use  $g$  of  $t$  is not just a single rectangular function it is going to be a cascading of.

So, basically over 1 symbol duration there is going to be some plus 1 some minus 1. So, each symbol at each of those sub intervals there is going to be a plus 1 or a minus 1 I am going to denote it in the following manner  $q$  is equal to 1 to  $q$   $c$  of  $q$   $g$   $c$   $t$  minus  $q$   $t$   $c$  let me just fill in all the information that have been written down  $t_c$  is equal to the chip

duration that is duration of the plus 1 minus one sequence which you have multiply that is  $t_c$  this whole duration is  $T_s$  and what we are saying is there are  $q$  chips in one symbol  $P_a$ , so,  $q$  times  $T_c$  equal to  $T_s$  that is the relationship that we have.

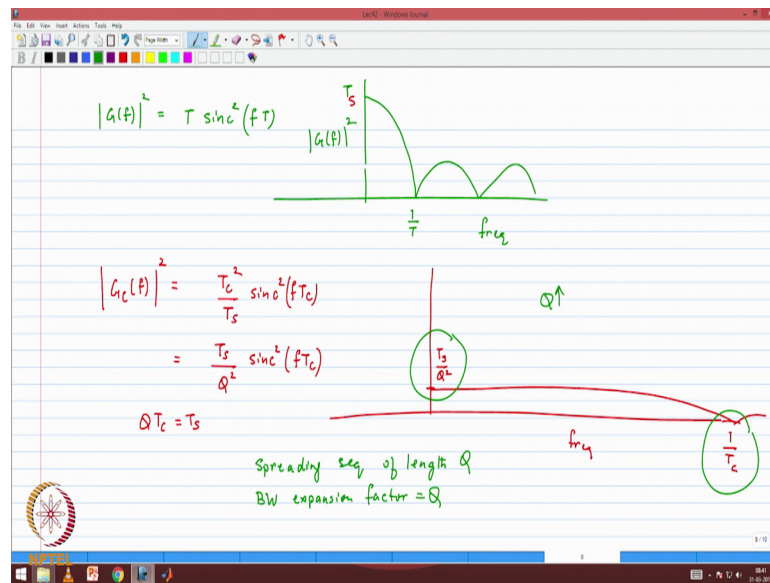
So, one symbol duration that the pulse shaping wave form a last one symbol duration it is going to be made up of  $q$  chips each of this chips has got a rectangular shaping function and that rectangular shaping function will be appropriately be shifted depending upon your talking about a chips chip 1 or chip 2 chip 3 accordingly you will shift the sequence. So,  $T_c$  chip duration  $q$  times  $T_c$  is equal to  $t_s$   $q$  is called the length of the spreading sequence. So,  $c$  is  $c$   $q$  is called the spreading sequence is the spreading sequence this is the one that is going to make the spectrum look wider is the spreading sequence and  $q$  is the length of the spreading sequence that is the basic notation that we will use for the spread spectrum systems length of the spreading sequence.

So, at high level I can think of it just as a linear modulation instead of having one rectangular pulse that that covers the entire symbol duration I now have several some random sequence of plus minus 1s each of which is got a rectangular way form as shown in this figure. So, now, we would like to write down the following expression  $g_c$  of  $t$  that is the chip way form  $g_c$  of  $t$  is equal to  $1$  over root  $t_s$  not  $t_c$   $t_s$  for  $t$  greater than or equal to  $0$  less than or equal to  $t_c$ . So, now, please tell me what is  $g_c$  of  $f$   $g_c$  of  $f$  root take this one over  $t_s$  as a scale factor it is going to be  $t_c$  times  $\text{Sinc}$ ;  $\text{Sinc}$   $f$  times  $t_c$   $e$  power minus  $j \pi f t_c$  everything else remain as if you were only thing wherever there was a  $t$  you replace it to  $t_c$  because that is the chip waveform and this  $1$  over  $t_s$  is a scale factor that comes into the Fourier transform as well.

Now, I want to know what is  $\text{mod } g_c$  of  $f$  whole square this comes out to be  $t_c$  square by  $T_s$ .

$\text{Sinc}^2$   $f t_c$  that is gone to be the spectrum of my going to be that is going to dictate the spectrum of my signal.

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So, now what we have on before us is the following if it was a conventional modulations scheme the spectrum that we achieved or the  $G$  of  $f$  magnitude square we wrote down was  $t$  times Sinc square  $f t$ . So, basically it would be a Sinc function starts at  $t$  this is a 1;  $1$  over  $t$  as the first point at which the; so, this is frequency; this is mod  $G$  of  $f$  square.

Now, if I change it from a single rectangular function to 1 that has got the chip sequence then what we have is mod let me use a different color mod  $G$  of  $f$  magnitude square this is what we have obtained we have written it down as  $t c$  square by  $t s$  Sinc square  $f t c$  just a simple a rewriting of this comes out to be  $t s$  divided by  $q$  square basically the relationship is  $q$  times  $t c$  equal to  $t s$  if you use that you can verify this is  $t s$  by  $q$  square Sinc square  $f t c$ .

So, spectrum starting point is whatever was the this is  $t s$  now the starting point  $t s$  by  $q$  square and where is your first crossing going to be it is going to be at  $1$  over  $t c$ . So, basically  $t c$  is much wider  $1$  over  $t c$   $1$  over  $t c$  is much wider. So, this is the frequency and of course, it is a Sinc square Sinc square function. So, the spreading operation sort of becomes very visible once we look at it in terms of the expression of the pulse shapes and the; I just wanted this relationship to be very clear in your mind the spectrum goes down as  $1$  over  $q$  square. So, as you increase  $q$  what will happen this will level will keep going down more and more and the other thing that will keep getting effected is that 1



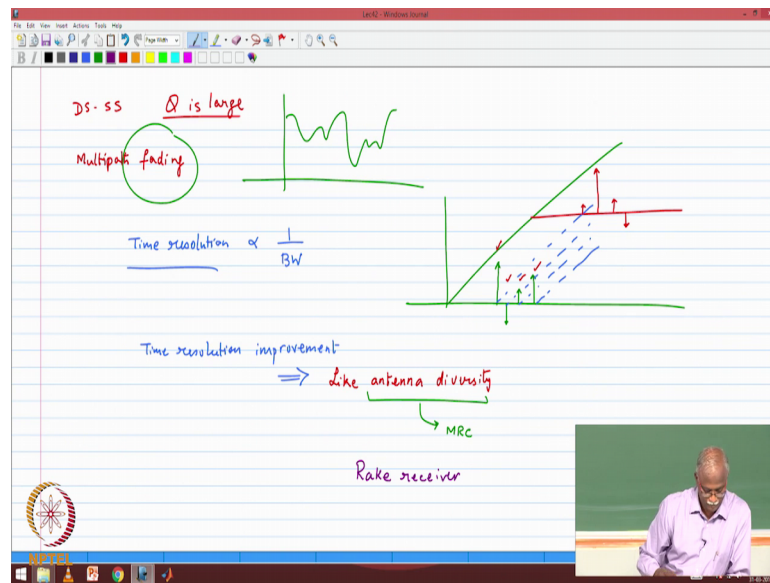
over  $T_c$  because the  $T_s$  is fixed  $T_s$  divided by  $q$  is going to be  $T_c$ . So, that that  $1$  over  $T_c$  is going to go a further.

So, basically it will become wider and wider. So, this is the relationship. So, what we say is we are this spreading sequence spreading sequence of length  $q$  if I use a spreading sequence of length  $q$  then my band width expansion because it went from  $1$  over  $t$  to  $q$  over  $t$ . So, bandwidth expansion is by a factor  $q$  factor equal to  $q$  and the level is going to be effected by  $1$  over  $q$  square. So, again the whole process of spreading a becomes a very important element in our discussion is this is this clear basically I am this must be very familiar to people from the digital communication, but I just thought that its good for us to relate what happens when you have the process of spreading.

Now, maybe it is a good idea to mention even at this point typically in military radios what would you do you would choose  $q$  to be very large 128, 256. So, basically your spectrum looks completely like white noise now in cellular context if you say that I am going to take a 30 kilo hertz signal and spread it by 128 how much bandwidth are you going to ask me for you are going you are going to ask me for now megahertz of bandwidth then you say wait no in cellular this spectrum is very expensive. So, you cannot you know just spread arbitrarily and the other scenario is that we do not have first of all  $Q$ ,  $Q_s$  has to be smaller.

So, then the minute you say  $q$  has to be smaller you must quickly in your minds think now the spectrum starts if you do not use  $Q$  of 128 or 256s let us see use  $q$  of let us say 16 then it still looks like a narrow band spectrum it does not look too much it has its somewhere it is a hybrid between a narrow band system and a and a and spread spectrum system. So, some of those a tradeoffs will start to come into play and then we look at it and, but that is coming in the next lecture you know all the tradeoffs and other things, but for now let us look at it as a spread spectrum system where I have the luxury of spreading it by a large spreading factor.

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So, let us say that it is a direct sequence spread spectrum system and  $q$  is large one of the biggest benefits or one of the main reasons why spread spectrum systems were even considered for a wireless channels was the following multipath fading channels multipath fading channels the problem was this fading right because this is what creates your channel going up and down. Now you must find ways to deal with that now if you remember earlier in our discussions we made a following statement the time resolution of a signal the time resolution capability of a signal in a multipath environment how is it related to band width inversely proportional to the bandwidth.

So, if the bandwidth increases you will then start to see instead of one multi path component you will start to see 3 you will start to see 10. So, the bandwidth increase that we have created by creating a spread spectrum signal from a flat fading signal what you what you now this is the time access I know start to see that there is a pulse here. There is a pulse here there is a third pulse and may be a fourth one what seemed like a single tap to a narrow band system, now you start to see that your it is no longer a single tap. And the important thing to note is that when I look across time these multi path components will change and other instant of time let me take another instant of time the first multi path component has become very small the second one has now become very large and the third one and the fourth one this is the scenario that we are encountering.

Now, one of the most important things when you see this is the following fading means there is one tap and the tap has become small, but what you have told me now is no, no, no, my system sees it not has one tap, but it sees it has a set of taps some are going up some are going down the when you when I treat all of them has one tap it, it even though the total is small or a narrow band system, but for a wide band system it is this scenario. So, my signal is not going to be as badly effected as a narrow band system because there is still a strong tap that is present and I can capture that in information.

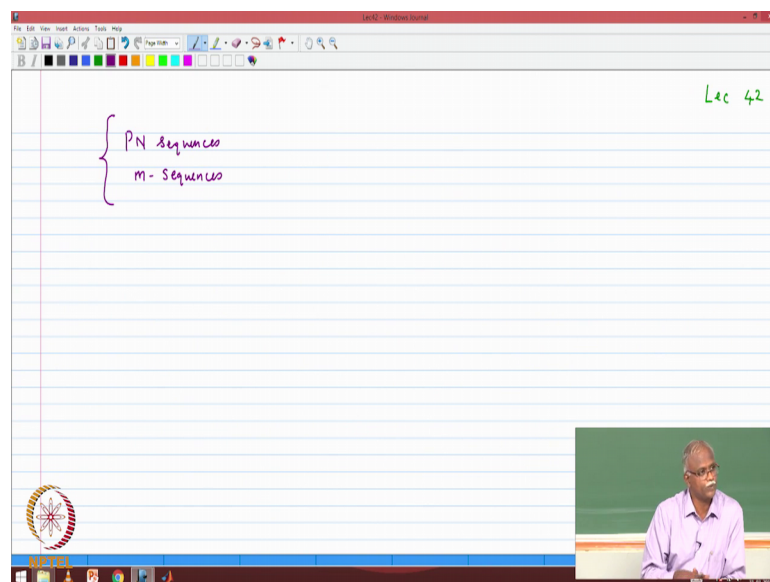
So, if I can capture the information that is there in this tap in this tap in this tap and in this tap I can receive that information from each of those multi path components then this looks like 4 branch diversity what should I do when I have diversity I should do maximal ratio combining. If I do maximal ratio combining with 4 branches what will you tell me the performance is going to be much better than a single tap Rayleigh fading and I got it for free I use more bandwidth, but at the end of the day it was the same channel before which I would have been suffering with the fading all of a sudden says hey no problem I am going to do some form of MRC because each of those and when keep in mind that you know these taps fade independently. So, as long as there are a few strong taps you are going to be better off than a narrow band system. So, this is one of the main reasons why in a cellular system we like spread spectrum because you can now differentiate the multipath components and then you can get the benefits in the form of diversity.

So, this is like this time resolution the time resolution getting better time resolution improvement; that means, you are going to see more multi path components distinct multi path components this is actually going to give you benefit like antenna diversity you are going to get multiple copies of the signal it is like antenna diversity, but do you have to use different antennas no do you have to use multiple receive chains no you got it for free because you got it with one receiver one antenna one receiver you got 4 copies of the signal which is really good. So, like antenna diversity and the best thing that we can do with antenna diversity the optimum way of combining is called maximal ratio combine and if you recall from your digital communications course you the receiver that you use for a spread spectrum system was called a rake receiver and a rake receiver is nothing, but MRC combining of the different multi path components. So, that is why the spread spectrum system works so well.

So, that is the background of CDMA systems that is the spreading part what is the benefits of spreading again we are not so much looking at anti jam or low probability of interception, but we are going to be utilizing the diversity component of it and the ability for a spread spectrum signal to coexist with interference because in a cellular system we are primarily limited by interference. So, basically the ability to coexist with interference is a very powerful property that we would like to exploit in our cellular system.

So, this is the background if you can you know I would encourage to read up whatever you can on the direct sequence spread spectrum very quickly we will go into the properties that we need for a cellular system. So, the things that I would like to like you to read about are the following.

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So, in general P n sequences these are the sequences that we will use extensively in spread spectrum systems there are a set of a class a family of P n sequences called m sequences those also we would be using because that is a basis on which we will build our system.

So, if you can review or refresh your memory about random sequences or P n or pseudo random sequences and also very specifically the family of m sequences which are linear shift register based P n sequences those are very useful for us in our discussion and we will use that to build our understanding of how the CDMA system has been designed how the wide band CDMA system has been designed very very briefly and then

summarize saying that these are the benefits. But at the end of the day if I want to transmit ten megabits per second I cannot use spread spectrum because you will ask to spread the spectrum its already ten megahertz wide the signal if I multiplied it by a factor of 64 I am going to ask for 640 megahertz of spectrum, so, not there.

So, the limitation is that you can do very nicely spread spectrum systems for narrow band signals, but when you are stop talking about a 10 megabits per second or hundred megabits per second P n sequences are this is not the right way to do it, but many of or applications require us to work with many narrow band systems and P n sequences are a good give me couple of examples of direct sequence spread spectrum systems that you use every day.

Student: GPS (Refer Time: 54:38).

Yes, GPS if you any time use your phone and you look at your position that is a spread spectrum system Wi-Fi that is a spread spectrum system it is a direct sequence spread spectrum system (Refer Time: 54:50) 2 dot 11 b Bluetooth frequency hot that is a that is a spread spectrum system as well. So, you know all around you there are spread spectrum systems and again we have used them for cellular systems as well in the third generation after that we will move to the fourth and fifth generation which will be based on OFDM.

Thank you.