Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 42
Wireless Channel Capacity – Water filling
Optimum Power Allocation – Water filling – Part II

I am trying to find out the dates and times for the makeup lectures. We had one missed lecture and 2 lectures that were used for a review I would like to make up those 3. So, the moodle survey is on please do take time 2 indicate we would like to make sure that the times that was chosen are convenient for everyone.

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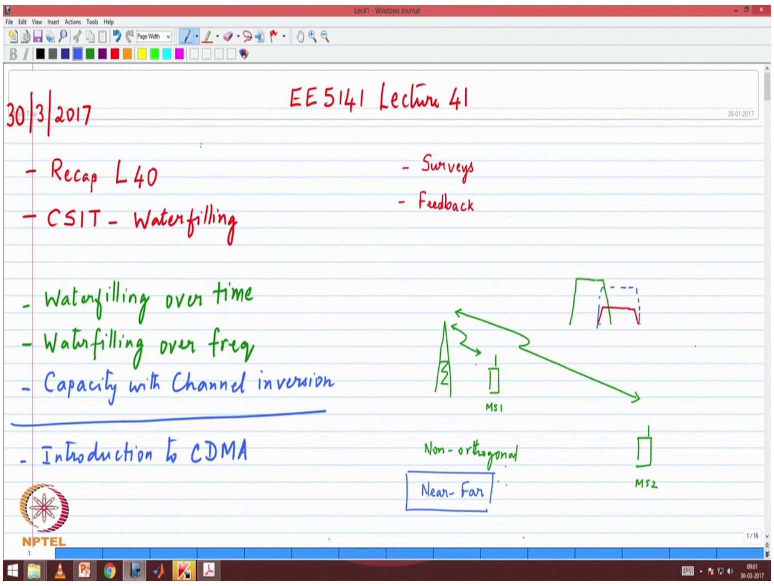

In addition we are also the TAS and I felt that it will be good point for us to get some feedback also. The feedback by the way is anonymous. So, you can feel free to give your comments and we will definitely have a look at that and see how to incorporate going forward.

So, today's discussion is to complete our understanding of water filling. And also look at sub optimal technique which causes us to think very differently from what we have been thinking so far. Because so far what we have been is good channel give more power. Now there is a sort of a counter intuitive method, which says no let us try to equalize the power may be, initially your reaction would be well we already looked at it is not a good way for us to a do to achieve capacity. But let us look at the concept, because this concept more or less leads us into the concept of CDMA.

So, let me just sort of get you a little bit interested even before we talk about CDMA. So, one of the things that you will find in a cellular system is that we have base stations, and we have mobiles. And there are very common scenarios where one mobile, let us call the mobile as MS1 the other one as MS2. Now both are communicating to the base station. And let us say are similar mobiles that is both can output the same transmit power. Now if both are transmitting at maximum power, then what is the situation at the base station? At the base station I am hearing mobile station 1 very loud because it is coming in with full power and the losses are not much. Mobile station 2 is somewhat faint because the mobile station is far away and it there it is suffering significant amount of path loss. So, this is the scenario, mobile station 1 very close to the base station 1 mobile station 2 far away both are transmitting with the full power ok.

Now, under what condition will mobile station 1 interfere with the detection of mobile station 2? Under what condition under? What condition? So, basically with in a cell we assume that the resources used are orthogonal. If mobile station is 1 is on time slot number 1 mobile station 2 is on time slot number 2 it does not matter at all because when I am detecting time slot number 1 I am hearing only mobile station 1 time slot 2. If they are on different frequencies which are completely orthogonal, again no impact; however, if there is a scenario of non orthogonality that loss of orthogonality, one such scenario that can happen is let us say they were on adjacent channels. So, this at the base station mobile station 1 is like this the second signal is much weaker. So, when I try to filter out mobile station 2 I will get a significant portion of mobile station number 1. So, this is the filter that is applied and you can see a significant portion of green is leaking in and therefore, will cause me disturbance.

So, loss of orthogonality very, very important when we talk about different users. And one such situation arises in CDMA, because in CDMA both are using the same frequency. And if for any reason there is a loss of orthogonality then they will interfere with other. So, for this purpose there is a requirement in CDMA which says mobile station all the mobile stations ideally should be received at the same received power; that means, even under loss of orthogonality how much degradation 1 causes to 2, 2 will cause to one it is not that 2 will cause a lot of one will cause a lot of degradation to 2 and therefore, wipe out my ability to detect.

So, this problem is called yeah particularly in the context of is called the near far problem. The near end user can affect the ability to detect the far end user, and you have to you have to take care of this when the users are in some way have lost orthogonality with respect to each other, so very, very important concept. Now how do you how do you do near far, how do you address the near far? You ask the nearby mobile to say reduce your power. I am I have got good channel reduce your power. What do you ask the far the far end user? Your channel is weak, boost your power.

So, what are you trying to do? You are trying to compensate. Good channel guy you say you reduce your power bad channel guy, but that is counterintuitive to capacity because capacity what did you say hey near end guy you transmit with more power because you can get more capacity. So, there is there is a sort of a contradictory, but CDMA requires you to do near far addressing. So, we will take about what are some of the aspects.

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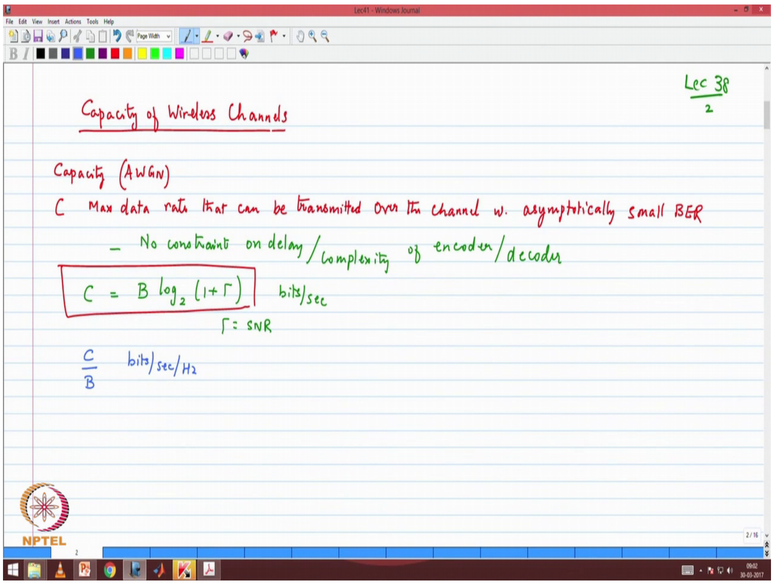

But before that a quick review of the capacity, channel state information at the transmitter instantaneous and perfect.

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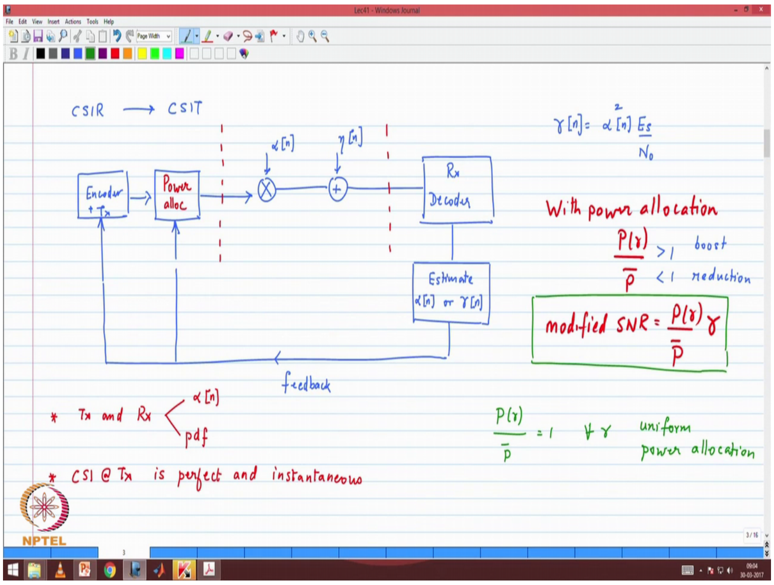

Notice that we now have the ability to do power allocation. So, previously the SNR depended on the average SNR, which is the SNR that you would see in AWGN channel modified by the adding coefficient. But now you do not have to leave with that SNR you can modify it, and the modification is through the power allocation P gamma of over P bar.

Now, P gamma over P bar I want you take a moment to do that. So, this can be greater than 1, if it is greater than 1 you are boosting the power of that particular SNR, boosting of course, it can be equal to 1, less than will equal to 1 you are not modifying anything less than one what you are doing is we are doing a power reduction. So, SNR can be either boosted or reduced artificially because of through power allocation.

So, the modified SNR with the power allocation now the channel does not know I mean when it goes over the channel it does not know whether you need power allocation or not. All it know is a signal was launched and this is what how I am going to receive it. So, the SNR that you are going to receive at the other end is now modified because the instantaneous SNR gamma got modified by the power allocation that you did and. So, there, so you need to keep in mind that you have now artificially modified the SNR the channel the fading channel gave you an instantaneous SNR and you have modified it to for whatever under whatever constraint that you have chosen. So, CSIT environment requires us to keep in mind that we have modified the SNR over the channel ok.

That is a very, very important point. Now what if I use this allocation, P of gamma by P bar equal to 1 for all values of gamma, what does that indicate? I am doing uniform power allocation; I am not changing the SNR at all I am not changing whatever was the original SNR. So, basically this is uniform power allocation uniform power allocation, but that is not the idea the idea is to actually allocate differentiated power to the different SNRs So that you can maximize the capacity ok.

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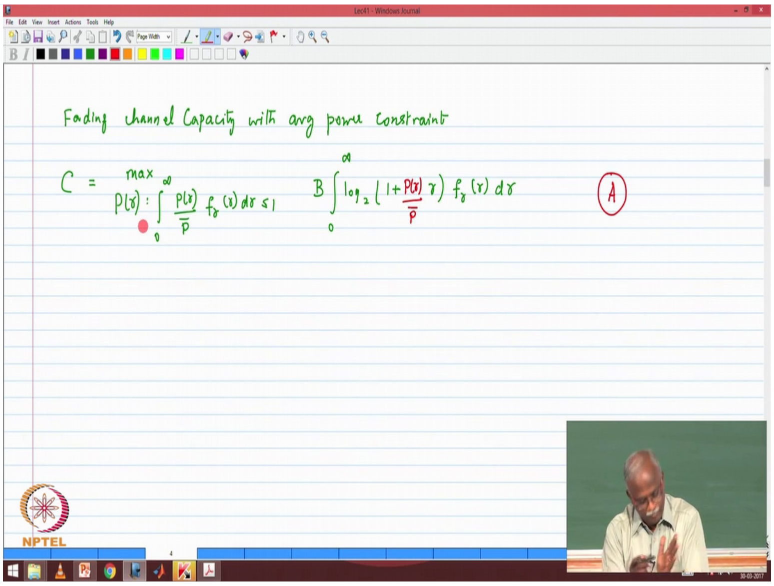

So, the constraint capacity problem as we had written down yesterday, is we want to optimize the capacity with the modified SNR under the constraint that your total power allocation has to be normalized it cannot exceed the average powers. So, this is the average power constraint.

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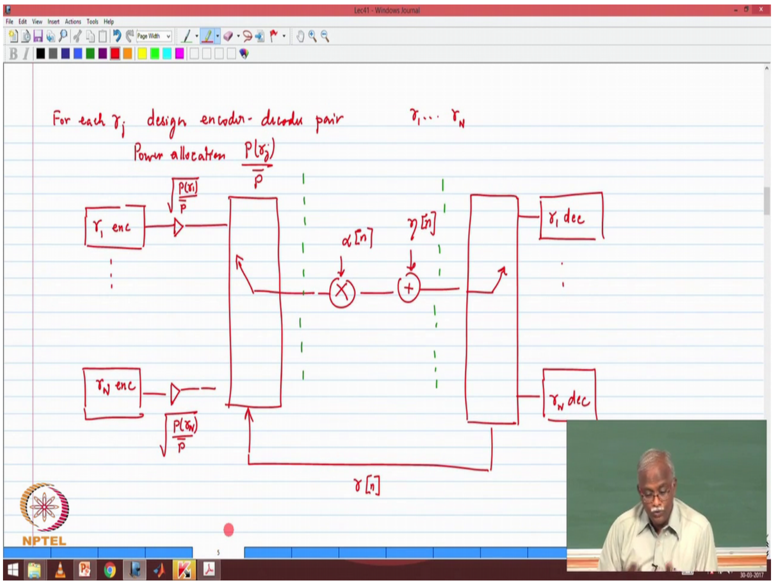

So, this is the statement of the problem, one way to achieve the assuming that the channel has perfect knowledge at the transmitter, we said that the way you would achieve the capacity would be to look at different combinations of encoders decoders which will work for different SNRs, but it is not this SNR that is going to be seen on the channel it is going to be this SNR modified by the power allocation that I am going to do that is going to be seen on the channel.

So, a very important that this is this picture tells us how to achieve capacity once you know the instantaneous SNR and you have design this system of encoders and decoders. But the key question that remained and which we addressed in the last class was how do we find out the power allocation, that is the important question.

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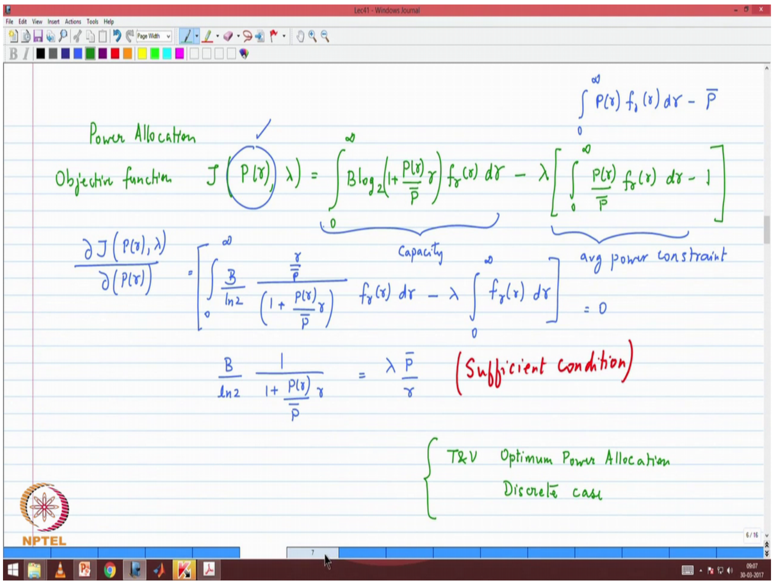

What did we write down we wrote down a objective function, in terms of a lagrangian. And we differentiated and set this equal to 0, very rightly several question arose after the lecture saying that under this condition you cannot say that the integrand has to be equal to 0, because the integrand because you are taking the integral is equal to 0. So, you can have the integrand can take positive and negative values which when integrated can actually go to 0.

So, yes that is a correct statement. So, the correct way to interpret this equation after differentiation is that you have to satisfy this condition equal to 0. And a sufficient condition that will satisfy that is setting the integrand equal to 0. So, the that is that is the one step it is not a necessary condition or at least from here we cannot deduce that it is a necessary condition; however, one of the TS BESOJIT actually pointed that if you look at TSE and Vishwanath, the optimum power allocation has been derived not for the continuous case because what we have what we are doing is for the continuous case, optimum power allocation has been derived for the discrete case, discrete case.

So, that mean you have finite number of SNRs with probability and you are trying to find out what is the optimum power allocation. And I would definitely encourage you to see this where it is shown that the when you under this condition it is not just it is not just sufficient it is necessary and sufficient and it maps to this same condition that we have obtained. But from our succession please be the correct way to interpret this is that there is a sufficient condition which achieves the required setting the derivative equal to 0 and that also will lead us to an optimal solution.

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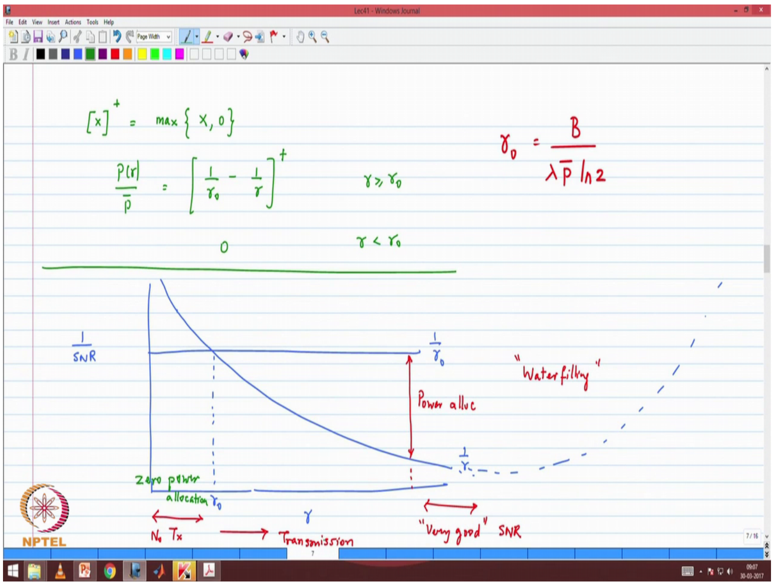

Very important our interpretation of the power allocation we explained it in terms of water filling if you think of this as a bowl the 1 over SNR graph as the as a bowl and the line 1 over gamma naught as a water level, then the depth of water that is the amount of power that you will allocate to that particular channel.

So, the power allocation algorithm is going to be the difference between the water level and the level of or the bottom of the vessel. So, and of course, you will not allocate negative powers. So, this is a quantity that is strictly non negative and if it if the SNR goes below the threshold then. In fact, you do not allocate power that is a same as saying no transmission occurs. So, no transmission for very bad channels and the power allocation done as per the water filling.

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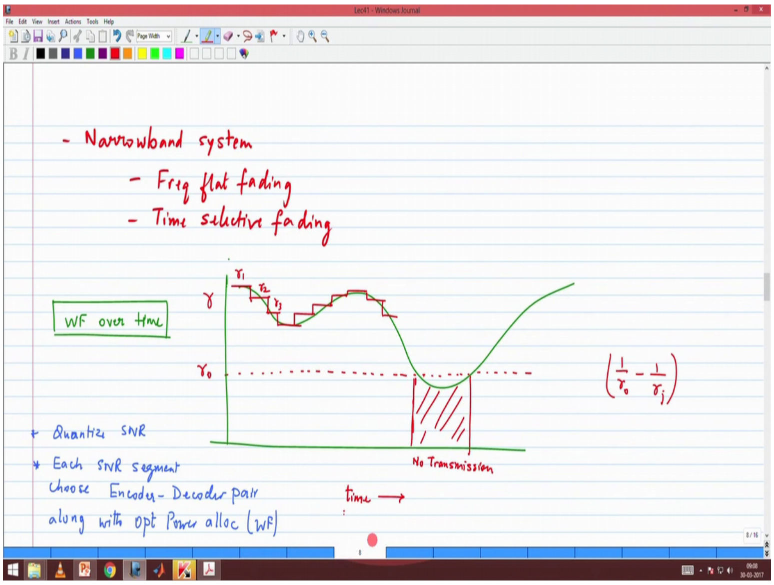

So, the conclusion is that if I have an narrow band system, this narrow band will experience flat fading that any at any given point in time the SNR will go up or down. By goal is to adjust the power transmission power allocation So that I get maximum capacity. So, how do I do that basically look at the SNR and quantize it. And for each of these each of these SNRs what we would what we would do is ah, go back to the a diagram where we saw a family of a encoders decoders, and also be able to allocate power the allocation of power would be based on the water filling algorithm 1 by gamma not by 1 over gamma j and of course, if you go if the SNR goes below the threshold; that means, there is no transmission. So, this would be the way that we would view that.

So, at any given point in time we are transmitting the information with the correct encoding decoding with the correct power level to get maximum out of the capacity. So, over a period of time if you look at a channel with time varying capacity then we have been able to achieve the best possible with the information at the transmitter.

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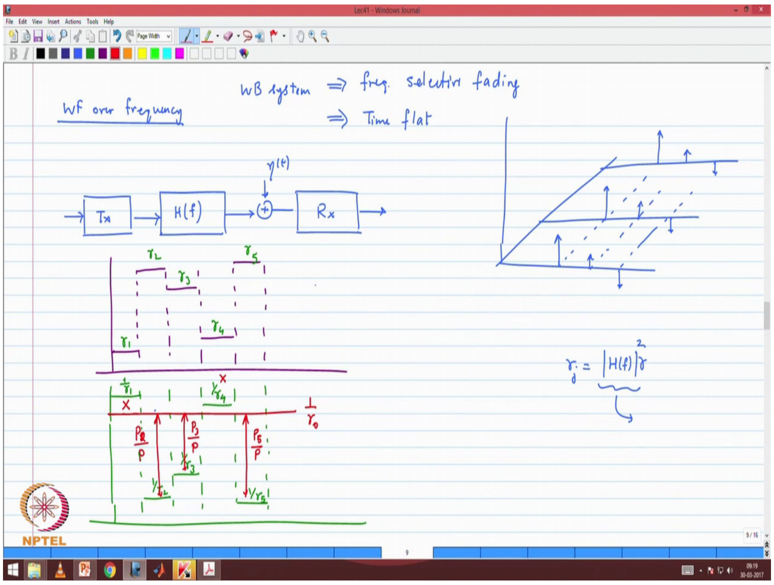

I would like to just spend a few minutes on the extension of this problem. What we are saying is we are going to look at a case where it is a wide band system, which means that frequency selective fading; however, we have made the assumption that it is time flat.

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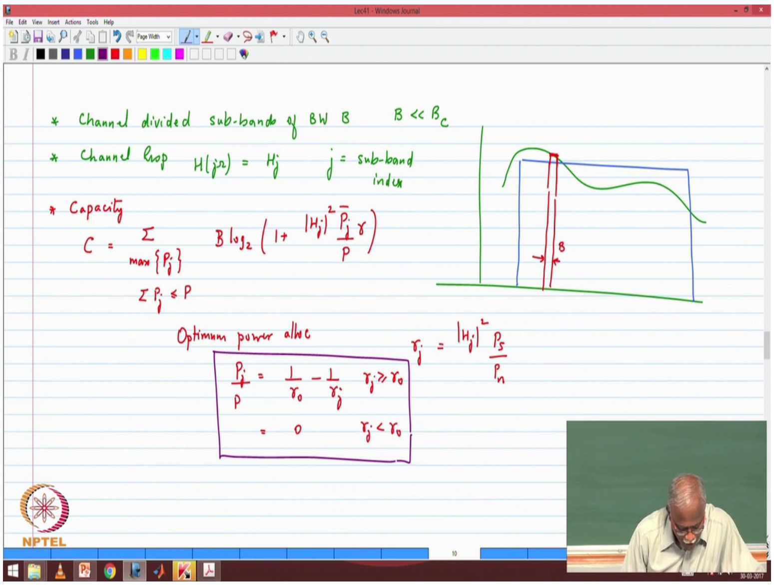

So, the steps that we are we are doing and I would like to just high light that. So, the steps that we are doing are we are going to first divided the channel in to sub bands. Channel is divided in to sub bands of band width B, sub bands of band width B, where B is much less than coherence band width. And that would mean that I can make the safely make the assumption of frequency flat fading for each sub band. The whole signal of course, is seeing frequency selective fading, but I have divided into sub bands which are experiencing flat fading. For each of these sub bands, the corresponding channel response, the channel response H of j omega, I am going to call this as H j, j is the j is the index of the sub band and it is the same for the entire sub band; j is the sub band index.

So, what we are saying we have a wide ban signal, wide ban signal is going to experience some frequency selective fading. So, the wide band signal itself is something like this. But I am going to divide it into small bands of band width B, of bandwidth B, and this particular is band is experiencing frequency flat fading. So, like that I have divided my entire a signal into sub bands.

So, now the capacity for a frequency selective fading channel we (Refer Time: 16:01) down you know without the a full description So, I thought I would make sure make it a point to write it down in today's lecture for completeness. Capacity is equal to summation max over P j. Now is this a probabilistic power allocation, is this a probabilistic power allocation or is a deterministic power allocation. Basically I have to I have to maximize the power, keep in mind that this is a time flat channel so; that means, nothing is changing. So, which means that once I know the frequency response of the channel there is no probabilistic element to it I just need to find out the power allocation for corresponding to each of those channel gains. So, it is maximum over P j summation P j less than or equal to P, it is there is a there is a total power that I have for the transmission of the given signal. And I am allocating it to different sub bands different power to different sub bands.

So, the total power not on an average level I instantaneously also has to be less than or equal to P. So, notice that it is a slightly different formulation then the previous case then it is good for, for us to write it down. So, the capacity itself will be equal to B there are several of these sub channels logarithm base 2 1 plus mod H of j square that is the gain of the channel, I am going to modify it by P j divided by P, P j divided by P that is the scale factor that we are going to apply. And basically the signal to noise ratio or gamma after that, mod H k is already indicating that, let me see mod H k times the gamma is fine. So, the optimum power allocation optimum power allocation as in the earlier case it will be through some method of water approach of water filling. Optimum power allocation P j divided by P is given by 1 by gamma naught by one minus gamma j.

Student: Sir.

Yeah let me just let me just write this as gamma j yeah go head.

Student: If gamma naught in to P j square what it will?

No because the that is the channel gain.

Student: This frequency response comes, because of that multiple power slide right? So.

Wait ok the question is ah, how do you how do you what portion comes from the signal and what portion comes from the channel? Think of it like this I transmit a signal of a certain band width, it is going through a normalized channel with certain gains. So, H j you dissociate from the transmitted signal power. So, basically there is an SNR with which I can transmit the this signal the, that can be written as the signal power by with a certain P s. So, that would be the signal this multiplied by mod H j square is what the channel will produce. So, P s multiplied by mod H z square will be the gain that this signal will produce, now over and above that I may choose to do power allocation ok.

So, the basic signal power multiplied by the channel gain multiplied by any power allocation that I have done, so that will be the resultant SNR. So, P s we can assume is constant for all the sub bands, each of the sub band will see a different gain based on H j, and then I will do power allocation based on that, yeah good. So, always clarify that.

So, basically if I treat. So, how if will gamma j be defined this divided by P n will be gamma j. So, and based on gamma j, I will then decide to either allocate more power or less power and that will be me the resultant signal. So, and of course, this will be equal to 0 this is for gamma j greater than or equal to gamma naught for gamma j less than gamma naught. So, this is the optimum power allocation. And this is how we have achieved the capacity. Let us go back to the previous graph and now see how we want to achieve capacity.

So, supposing the channel has got the channel gains which have the let us say just as an illustrative case, equal all of them must have equal bandwidth. So, I first divide my channel into segments of band width B, bad channel next one is good, next one is medium, another not So good channel and then another good channel. So, basically this is what the channel response looks like. So, I first have to I am going to do a channel in inversion. So, the same boundaries now I want to take 1 over gamma j. So, this corresponds to gamma 1, gamma 2, gamma 3, gamma 4, gamma 5 notice it is the whatever the signal SNR is modified by mod H j square. So, I am going to take the reciprocal. So, this is 1 over gamma 1, 1 over gamma 1, this is 1 over gamma 2, 1 over gamma 2, 1 over gamma 3, 1 over gamma 4 and of course, 1 over gamma 5 is also a good channel.

So, now water filling basically says find that level, which is going to indicate the water filling point. So, that it will satisfy the average power constraint the power the required power constraint. So, this channel no transmission this channel no transmission because they are above the power allocated for this channel will be given by this. So, this will be P 2, P 2 divided by P that is the power allocation here. And this one is going to be P 3 divided by P and P 5 divided by P that is the power allocation, P 5 divided by P. And some channels are not transmitted and then this is how we achieve capacity for a wide band signal. May be one more comment before we move away from this discussion.

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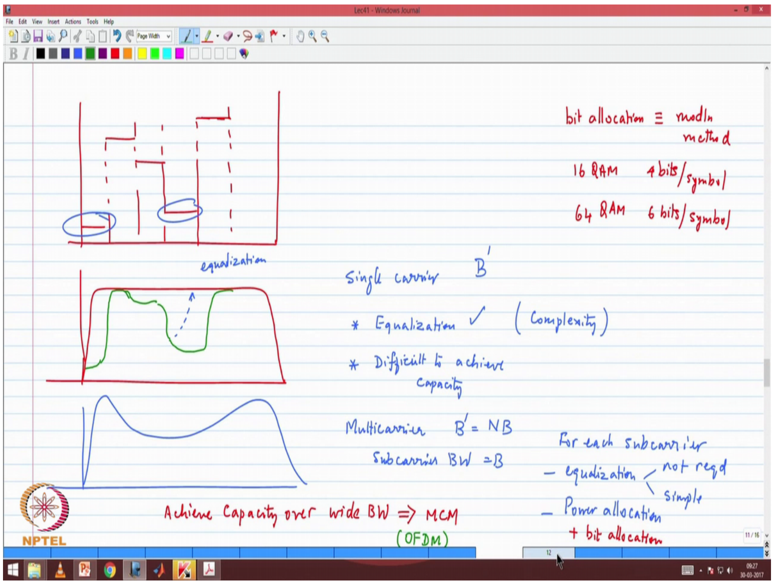

So, supposing you had this channel, supposing you had a channel of this type, where you had very bad channel conditions in for one part, good channel conditions next and then medium poor very good and so on.

Now, what would have happened had I transmitted a single carrier signal through this? If I had transmitted a single carriers system, this is a single carrier system, what would have happened? This portion would have been severely affected distorted. I would see something like this, something like this. So, no you can see that my signal is going to be severely distorted, severely distorted. Now of course, I can recover the original signal by equalization. So, I can go back from here to this point via equalizations. So, my ability to detect the signal a wide band signal which is gone through frequency selective fading is not the issue. But now the question is, how do I achieve capacity? Because what it says is hey in this portion alone I do not transmit any information, because that is that is very bad channel on this part, only transmit in these things and they say I can not do that. Because I cannot create nulls in my transmitted spectrum, because my it is a single carrier system.

So, at most I can do some shaping of my spectrum. So, for example, you can do things like you can shape your spectrum, but I cannot create nulls for you. So, that is a important. So, if I if you tell me that I have to worry work with a single carrier system I am going to say the following. Equalization I can do, I can recover the signal. It may be complex because depends on the complexity may be high, but that is I can recover the signal. But what about capacity? Now it is difficult to achieve capacity, because I cannot change the power allocated to different portions or different segments frequency segments of the spectrum.

So, difficult to achieve capacity. So, what I will have to do either boost the whole signal or reduce the whole signal, and you know and there is no way of matching the transmitted signal perfectly to this information; however, if I could transmit these as individual signals basically multi carrier modulations. So, see this is for a single carrier system single carrier now on the other had we transmitted a multi carrier system multi carrier system. So, this is some band width B let us call it as B prime and this B prime is equal to n times b.

So, basically the multi carrier has got n carriers of band width B. So, each of these sub carriers so, sub carrier band width, sub carrier band width is equal to B. Now for each of the sub carrier for each sub carrier is equalization needed, sub carrier equalization probably is not needed. And even if it is needed it is going to be very simple. So, not either required or it is going to be simple low complexity. So, equalization is not an issue. Now what about capacity? Absolutely no problem because I can do power allocation, I can allot enough amount of power.

Now, what is the use of allotting power for a particular sub channel? That means, that channel is going to see a better SNR. Now why would you do that? If you want to transmit more information on that. So, it is always along with power allocation we do something called bit allocation. What modulations scheme you are going to use for that particular system? So, bit allocation is another way of saying the modulation method, modulation method. So, is it going to be 16 QAM. Then I have 4 bits per symbol. So, you boost the power and say transmit 4 bits on this sub channel on this sub channel just do BP, BPSK 2 one bit per symbol. So, 4 bits per symbol or you could go to 64 QAM which would be 64 QAM will be 6.

So, basically you will always here thee term power allocation and bit allocation. So, please do not get confusion over there is something which I we have not looked at. The only reason you would allocate more power is to send more information, and how would you send more information you will send it my transmitting more bits per symbol. And this is precisely the reason why a multi carrier system can achieve capacity when a single carrier system cannot achieve. Because what can you do with the single carrier system you either choose q 16 QAM for the whole signal or 64 QAM. 64 QAM may be will may end up having a bit errors. Whereas, this particular case when you have multi carrier modulation that particular sub carrier, because it is got sufficient SNR to support 64 QAM you can send 6 bits per symbol. Some other sub carrier has got less SNR you send 4 bits per symbol.

So, that is how you achieve the capacity. So, think along the times of I have to achieve capacity over a wide range of frequencies. How do I achieve it? Achieve it by dividing it up into small chunks it simplifies equalization it helps me do allocation of power through water filling. Once I have that then I can send more information or less information based on the SNR that is currently supported.

So, more or less if you say that you want to achieve capacity for a wide band, achieve a capacity or maximize capacity over wide band width, over band width. It almost forces you to go to multi carrier modulation. And what we referred what we know multi carrier modulation is as OFDM. So, OFDM has comes in with a very natural advantage saying that this is the scheme that will help me achieve capacity this is the scheme that will reduce the complexity of the equalizer. Of course, there must be some penalty that you pay we will look at that as well.

So, already we have already seen the motivation for OFDM, but let us keep that in the background we are going to study OFDM, but before that as a good engineer. We look at this solution and say well you know what this system achieving capacity is actually quite complex. Because first you have to solve the power allocation problem. Then on top of that you have to design this you know bank of encoders and decoders, and you have to keep feeding back ok.

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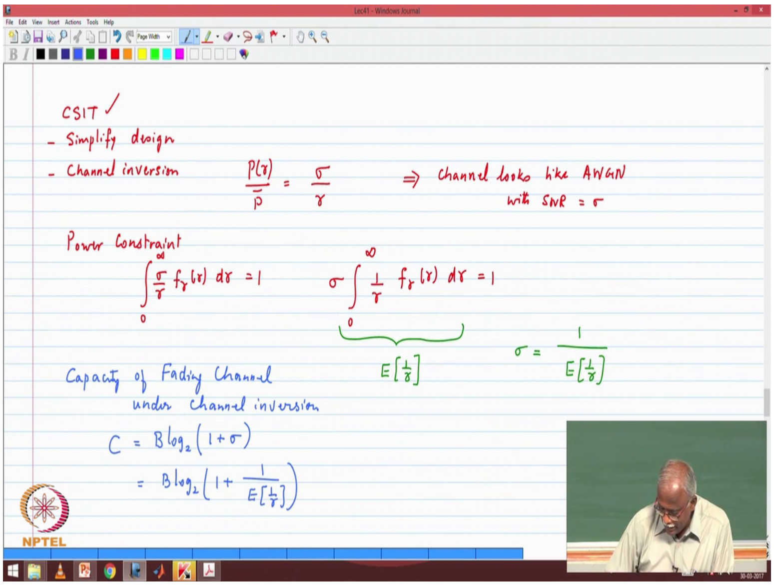

So, the question that you ask as a good engineer saying that, CSIT is a very valuable tool make that assumption of CSIT. But I want to simplify my design, simplify the design of the system. Now you ask the question how much you want to simplify. I say the answer comes back I want to have one encoder one decoder, isn't it? No you have to be realistic your trying to achieve capacity here you have to you have to be you know cannot you just now seen that you know how difficult it was to achieve capacity. So, again the notion that comes about is the following. The notion of channel inversion. So, when the channel is bad boost it, when the channel is good sort of pull it down. So, that you sort of get something which is on average the (Refer Time: 32:15) ok.

So, what is the, how would we represent this. So, channel inversion says the power allocation P of gamma by P bar is going to be such that it is sigma divided by gamma. Sigma is some constant not do not think of it as variance or anything. Just some constant divided by SNR. So, now, multiple the power allocation with the SNR what do you get?

Sigma. So, under all conditions, what is the SNR? Sigma. When you have only one SNR how many encoders do you need decoder do you need you need only one. But the question is this even a system that is going to give any benefit at all. So, let us take a look at it ok.

So, basically you can understand that this is doing perfect channel inversion. Because what it says is by doing power allocation. So, this basically makes it look like mix channel looks like an AWGN channel, channel looks like AWGN channel AWGN with SNR, SNR equal to sigma. It is like a fixed SNR. It is a very interesting concept it is channel inversion. This is what CDMA systems will do to do solve the near far problem, because they want both the signals to be received all signals to be received at the same power.

So, now what is the power constraint? So, the power constrain that we now have to satisfy, there is an average power constraint, because you are going to boost the poor. So, this basically says integral of sigma by gamma, that is P gamma by P bar, f gamma of gamma d gamma equal to 1. Integral 0 to infinity. Now this is a interesting expression because sigma is a constant if I pull that out is integral 0 to infinity 1 over gamma f gamma d gamma this has to be equal to 1. What is this integral? It is expected value of 1 over SNR, very interesting ok.

So, basically we are interested in 1 over SNR, because that is going to. So, and what is your s AWGN level or this constant SNR level? That will be that will be equal to 1 divided by expected value of 1 over gamma. So, this is channel inversion. So, if you now where to ask what is the capacity of a fading channel under channel inversion. Capacity of fading channel, fading channel under channel inversion, basically you are looking for a equivalent competition of ergodic capacity. C is equal to B times logarithm base 2, of what? 1 plus sigma, it looks like AWGN channel right. So, there is no ergodic nothing came into that picture, this is equal to B time's logarithm base 2 of 1 plus 1 over expected value of 1 over SNR and that is a very interesting very interesting expression. Because what it says is if I if you if you can do channel inversion for me then I all I need is one encoder one decoder. And it will achieve the capacity of an AWGN channel which is which has an SNR 1 over expected value of 1 over gamma ok.

So, it is a very interesting scheme, we just want to know how good a how good this method is and whether we can exploit it to our advantage. So, let us take a look and see if we can revisit our example.

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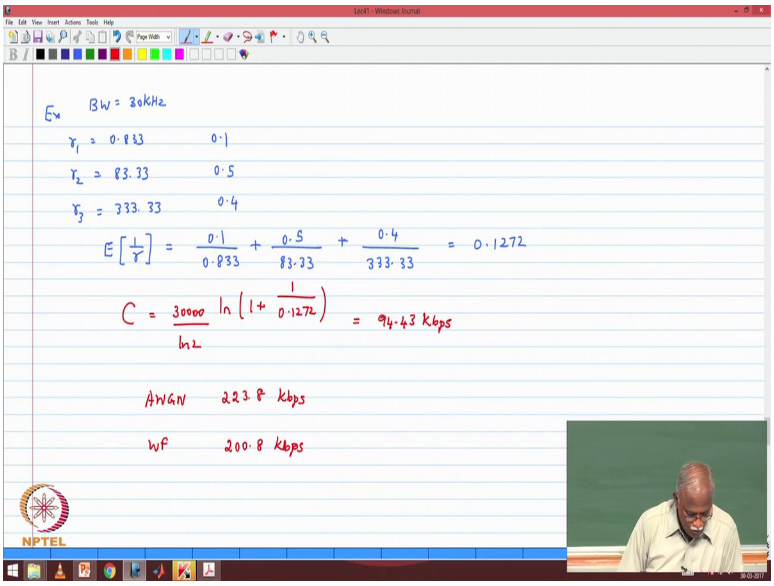

Example The bandwidth is 30 kilo hertz. I have 3 SNRs, gamma 1, gamma 2, gamma 3. This is 0.833, next one is 83.33, this one is 333.3. And the probabilities are 0.1, 0.5 and 0.4 ok.

So, I want to do channel inversion, channel inversion says first find the sigma expected value of 1 over gamma. So, this will be 0.1 divided by 0.833 multiplied by 0.5 divided by 83.33 plus 0.4 divided by 333.3 3. This comes out to be please verify 0.1272. Again I want you to verify that, and now the important thing. What is the capacity under channel inversion? So, this would be 30 kilohertz, log, lan divided by lan 2, natural logarithm of 1 plus 1 over 0.1272. And that comes out to be 94.43 kbps. If you were to put a smiley you were to put a bad smiley, sad smiley, because you know they did not really achieve the you know gave you a disappointing answer you know it is like.

Now, on the other hand had we did capacity of an AWGN channel will the same average SNR. We know that the AWGN, AWGN. AWGN channel with the same average SNR achieve 223.8 kbps. And under water filling with lot of complexity we actually have achieved 200.8 kbps. Now never give up if it does not work the first time, now you say well you know what, what did we try to do you try to invert the channel under all conditions. Say no I think I know from capacity that you know it is not worth spending your resources on very bad channel conditions.

So, we now say that we are going to do something called a modified channel inversion or what is called a truncated channel inversion.

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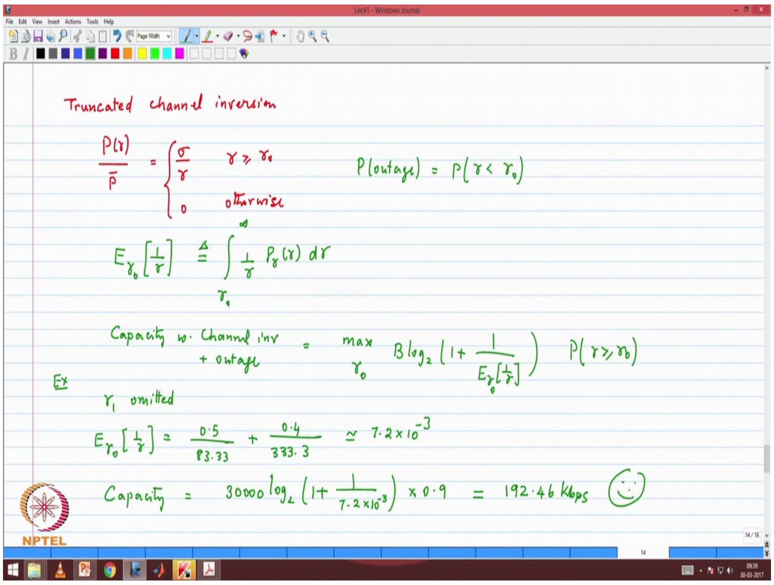

So, I do not want I am not going to invert all the channels. I am going to say when it is very bad ignore. Only the reasonable channels I want to do channel inversion. So, that is called truncated channel inversion, truncated channel inversion because you do not want to mess with the very bad channel conditions.

So, basically what we are saying is, P of gamma divided by P bar is going to be channel inversion sigma by only when it is greater than or equal to some threshold and is equal to 0 otherwise. So, power allocation under bad channel conditions, this is like your trying to take some benefit or intuition from water filling. And of course, this will result in outage. So, probability of outage is when probability that gamma less than gamma naught. So, that that you cannot avoid because you are not going you are going basically not deal with those channel conditions. So now, we now how have to find out what is my sigma level. So, it is no longer expected value of 1 over gamma, but it is a modified expectation with only when it is above gamma naught. So, I am going to write expectation subscript gamma naught to tell you that only over.

So, this is defined as integral gamma naught to infinity 1 over gamma P gamma of gamma d gamma. So, basically it is like a expected value, but I am going to do only over the region of interest. So, the capacity with channel inversion, with channel inversion channel inversion plus outage you are going to understand that, there is there going to be some channels under which we cannot transmit information.

So, this is going to be you maximize over all possible thresholds gamma naught you decide where you want to set the threshold. And beyond that you will do the channel inversion, B times logarithm base 2 1 plus 1 over this modified expectation. And this is basically the channel that you will get, but make sure that we also multiplied by probability that gamma greater than or equal to gamma naught. In other words you have to one minus probability of outage. Because otherwise you know because this is like a constant for all for all time, but you know that certain periods of time there will not be any transmission because you have not done the channel equalization.

So, basically let us assume that gamma 1 is omitted in the example. Example gamma 1 is omitted and all we have is the gamma 2 and gamma 3. So, expected value of gamma naught by 1 by gamma is going to be 0.5 divided by 83.33 plus 0.4 divided by 333.3. This gives us approximately 7.2 into 10 power minus 3. And the capacity, capacity will be 330000, 30000 log base 2 of 1 plus 1 divided by 7.2 into 10 power minus 3 multiplied by 0.91 minus probability of outage. This comes out to be very surprised 192.46 kbps, that is not bad ok.

Ah it actually is pretty close, pretty close to your water filling actually. So, what is it say this channel inversion. So, sometimes you have to be quite willing to look at the options of course, this is for a particular channel with only 3 states. So, you know what happens in a in a, but the good thing is this has much less complexity because all it requires is the ability to have single transmitter single receiver single encoder single decoder and your and your water filling is very straight forward you are trying to achieve something which looks like an AWGN channel. So, there are some practical challenges with you know trying to do channel inversion, because if you have channel that is varying very fast then you know inverting that channel is a little bit difficult, but never the less at least as a concept this is the very powerful method.

So, the key points to take away from here is we have talked about channel capacity. We have talked about the various aspects that effect channel capacity. How the desire to achieve the maximum capacity leads us to multi carrier modulation and to water filling. On the other hand this channel inversion method sort of tells you a do not try to optimize on a sub band level you know treat it you know equalize the channel and you can do quite well. And you know and also be able to work with the reduce complexity ok.

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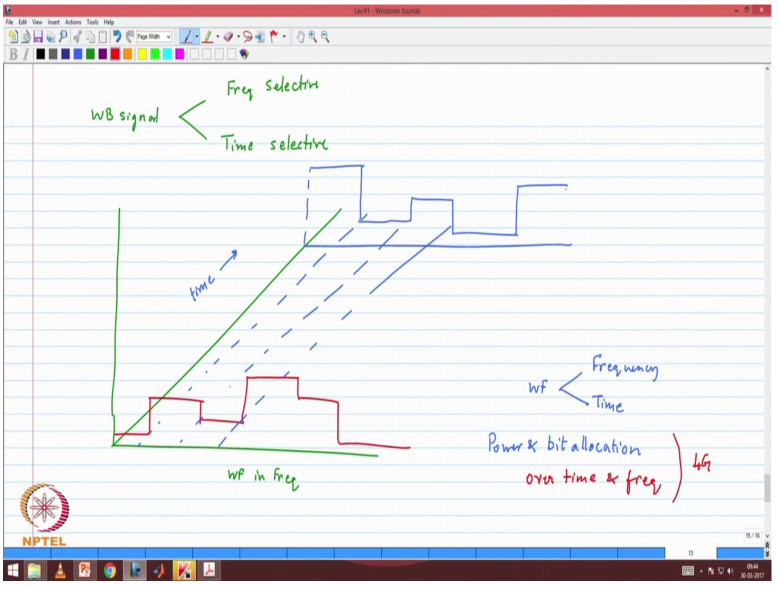

Now, last comment if I now have a wide band signal, which means that I am going to have frequency selective fading. Previously we assume that it was time flat, but now we are going to say it is time selective as well. There is a wide band signal frequency selective fading, which means that at a given point in time I am going to look at water filling in frequency. So, this delay dimension instead of tau now I have moved it over to. So, if I were to do water filling so, this is across time across time. So, at this time instant my water filling is going to be some water filling I am going to do at this particular time instant. But that is not that is not going to be the optimum solution for a forever. So, at some later point in time I may be doing a completely different water filling. So, lot of s signal in the first branch may be the next not So much signal in the third one medium very less.

So, basically you know connect the corresponding points, and you can see that the water filling is not a single water filling. But you are constantly adjusting the water filling as time progresses, so far each times snapshot you are doing the water filling. And you are doing water filling over frequency as well, because it is a wide band signal which is got frequency selective fading. So, if this is the time axis time axis. So, this is water filling over frequency to achieve maximum capacity across the different sub bands. And across time because your frequency response of the channel is changing and therefore, your sub band, your still sub band because sub band is what will achieve capacity. So, basically multi carrier techniques, but you keep adjusting the power level as you go.

So, what are you doing? You are doing power and bit allocation that I am sure you should be able to interpret that power and bit allocation, over time and frequency, over time and frequency is the complete way that you would achieve for a wide band signal that is time varying over passing through a time varying channel you would do the power allocation. And this is exactly what 4 g does. LTI system multi carrier 1024 carriers. Each carrier as a function of time you adjust the power and the bit allocation, and the next time instant you keep changing it.

So, basically this is how we achieve capacity in a in the cellular system. Any questions? If not I would like to just sort of sow the seeds for CDMA systems and then we pick it up from there. CDMA systems actually originally known as spread spectrum systems not as CDMA.

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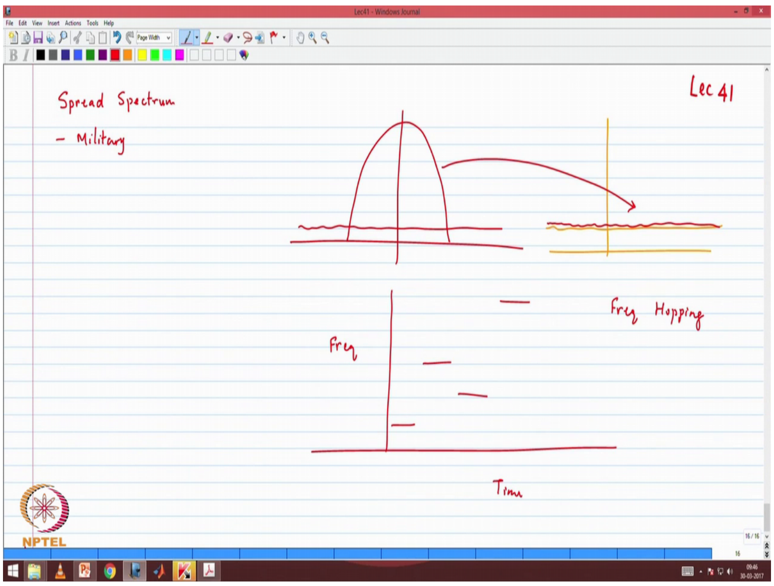

Spread spectrum systems, because CDMA does not necessarily means spread spectrum because you can get spread spectrum in other ways also frequency hopping is also a way to get spread spectrum. CDMA is one way to get spread spectrum. So, CDMA not equal to spread spectrum. So, spread spectrum systems actually originate from the military. So, originate from the military because they have a requirement which says that the enemy should not be able to detect my signal. And it is there is no way you can hide a signal if your noise floor is here and what you have transmitted is sitting up like this, this will this will get detected.

So, the only way that you can expect to you are the enemy not be able to detect your signal 2 ways, one is you keep changing the center frequency. So, if this is the frequency, and no this is frequency and this is time, this is time. So, you transmit at one frequency at this time instant next time you jump to another frequency, a third frequency, basically this is frequency hopping, frequency hopping.

So, unless the enemy knows your hopping pattern or is got a very sophisticated receiver that can track your hopping pattern it will be difficult it is a difficult problem. But what can be eve what how you can make it even more difficult is if you can make your signal look like noise. So, this is noise and your signal is actually looks like noise. So, there is no way the somebody whose receiving the signal. So, how do I make this signal into something that looks like noise? And that is the basic tenet of spread spectrum and ah, but what we are looking at is how does this concept which actually originated for the military to make the spectrum in a way that it is difficult to detect, how is that going to make a difference for a cellular system?

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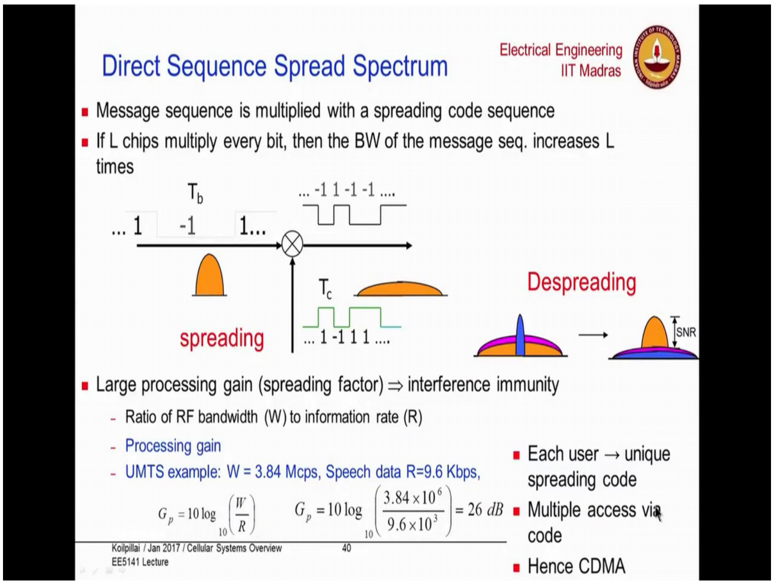

And that is where if you remember 1 of the early slides that we had discussed, says that basically the spread spectrum portion is the part where you take a narrow signal and make it wide band you spread it.

Now, the key point is we have multiple users, it is not one signal that you are trying to hide, you actually have multiple users multiple signals, and that is where you have the orange and the pink and the then eventually you have to recover these signals. So, our goal would be to see what elements of a spread spectrum system are important for us in the context of a cellular context. And how do we spread what are the assumptions that we are making, when does when do this orange and the pink signal loose orthogonality? Basically, if they are perfectly orthogonal remember we said that they will not interfere with each other. But if they loose orthogonality and what is loss of orthogonality in a spread spectrum system, all of those are the elements that we are going to study.

So, as a preparation for that, I would like to request you to whatever spread spectrum you have studied as part of digital communications please review that we will just pick it up from there and quickly move into the cellular system, which is built on the spread spectrum as the underlying modulation method. That will be starting from the next lecture.

Thank you.