Introduction to Wireless and Cellular Communication
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Lecture - 40
Wireless Channel Capacity - Water filling
Optimum Power Allocation - Water filling

Good morning, we begin lecture 40. To avoid confusion they have asked us to keep the numbering sequential. So, lecture 39 was actually a review lecture which will be made up with another you know content lecture, but for now lecture 38 and then lecture 40 the continuation is from lecture 38.

And since it is been almost a week since the last lecture will go through little bit extended review, because we need to cover build on what we have already covered. Today we will probably cover the 2 most interesting concepts in channel capacity, which is a water filing and then 2 variance of that water filling over time water filling over frequency. Once you understand this is the entire concept of capacity, how do you achieve capacity and all of those answers starts to fall into place?

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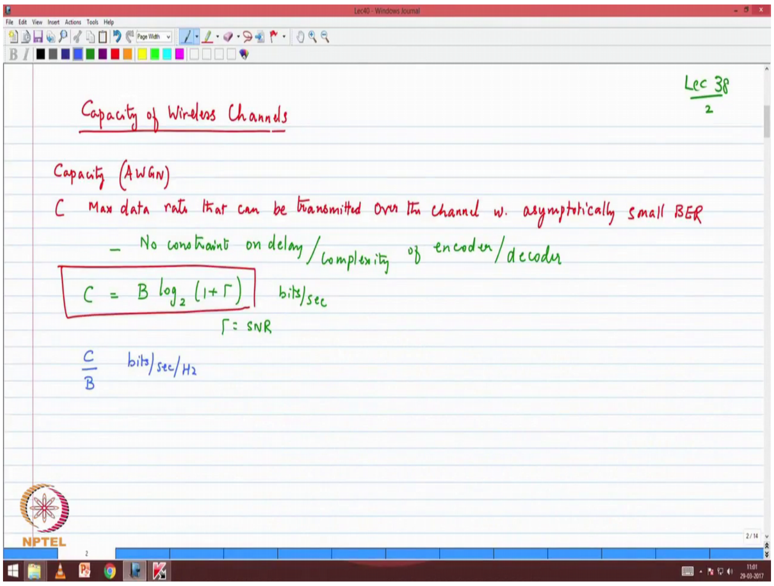

So, hopefully this will be a interesting lecture. A very quick run through we are having as the foundation Shannon's definition of the capacity of a AWGN channel B times logarithm base 2 1 plus gamma is the SNR and the units are bit is per second. So, this is a capacity that is scales logarithmically with respect to SNR, linearly with respect to bandwidth. And that is the reason why very often we look at a normalized capacity which is C over B per unit bandwidth which is bit is per second per hertz.

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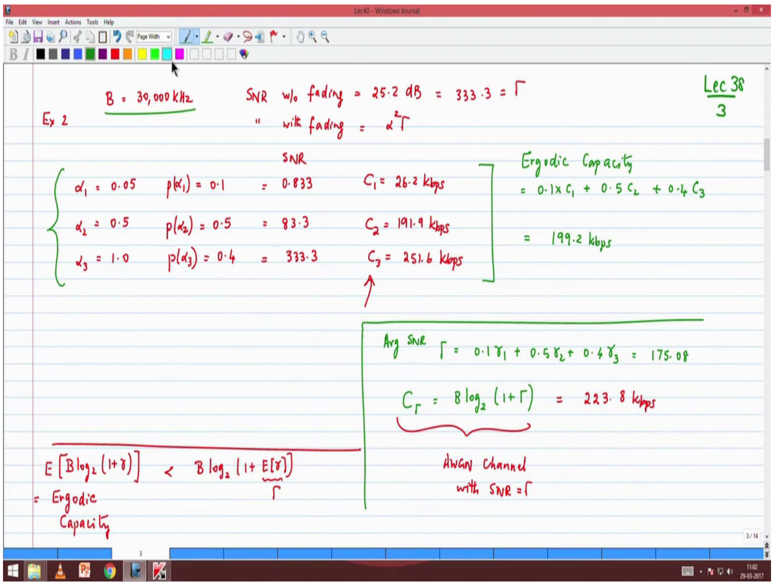

A classic example where we saw a channel with bandwidth thirty kilohertz; 3 different SNRs caused by the 3 different fading coefficients each of them has got a certain probability. Now how would you describe this channel? Is it a very bad channel or is it a channel? It is a reasonably good channel, a mild channel? What would your description be? The bad state is occurs fairly infrequently.

So, it is a it is a reasonably channel. So, keeping in mind that capacity is not about channels with good SNR, because that you know how to get the get the benefit of that the.

Challenge is how do you get the maximum out of channels which are which are not so good in terms of SNR, that is what we are going to be addressing today. But the example that we looked at had 3 states 3 different probabilities and that gave us a competition of 2 important quantities. One is if I calculated the average SNR based on the probabilities of the different SNRs and then calculate a capacity that would be the capacity of AWGN channel with the same average SNR. That came out to be 223 kilobit is almost 224 kilobit is per second.

On the other hand recognizing that I am actually the issue is that SNR directly affects the capacity. So, we now then look at an ergodic capacity where it is a capacity multiplied by the probability. When we did that calculation it came out to be 199 kilobit is per second substantially less than the corresponding awn channel capacity, but never the less even this 199 kilobit is per second is going to be a challenge to achieve under different SNR condition notice.

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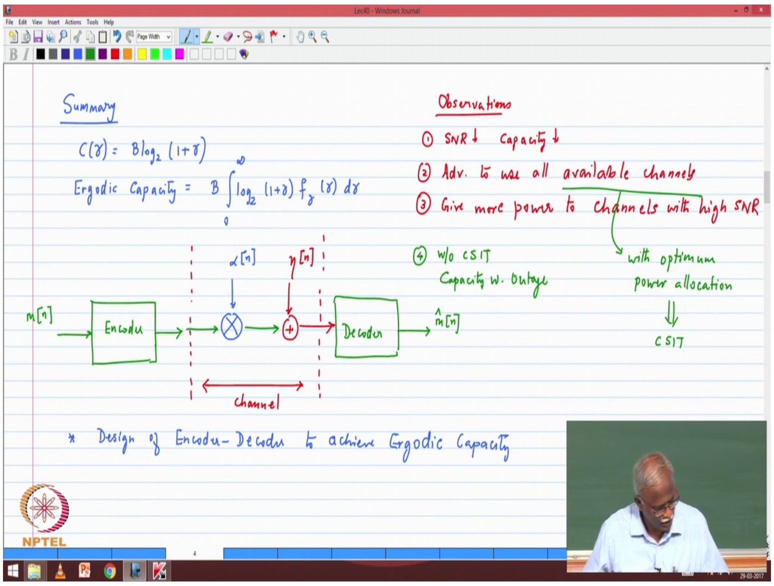

That this capacity is for these specific scenarios with in terms of SNRs and the probabilities of the different SNRs, and using the Jensen inequality we showed that the ergodic capacity will always be upper bounded by the corresponding AWGN capacity ok.

So, the summary is that we have a channel, typically would have been indicated by an AWGN source. Now we also have a fading coefficient. So, which means that this is our fading channel, for this fading channel I need to come up with a suitable encoder decoder or a set of encoder decoders that will achieve capacity. So, basically that is the underlying problem statement. Some observation that we have made already whenever SNR goes down, capacity is very adversely effected. As much as possible, we would like to use all the available channels. May be now that we have a little bit better knowledge we would say available channels with optimum power allocation, optimum power allocation would you agree, because it is not just about using all the channels it is about using them with allocating the powers the good channels would have.

So, with optimum power allocation, but power allocation requires you to have what in order to do optimum power allocation CSIT. So, this requires CSIT. If you have CSIT then you can use more channels and you can also use the optimum power allocation. And of course, the intuition is that gives more power to the channels with the higher SNR, and therefore gets the maximum out of it. Now if you did not have without CSIT, what is the best that we could do? What is the best that we could do? It would be if you insist on no outage then I will be severely constrained. So, the best that we can do is channels capacity with outage, right? These are the scenarios where you say some percentage of the time data will not go through.

So, capacity with outage; so these are the concepts that we have covered and this is what we will build up on. So, the notions of CSIT and CSIR very important, and how they will impact our capacities are visible right at this at this top level ok.

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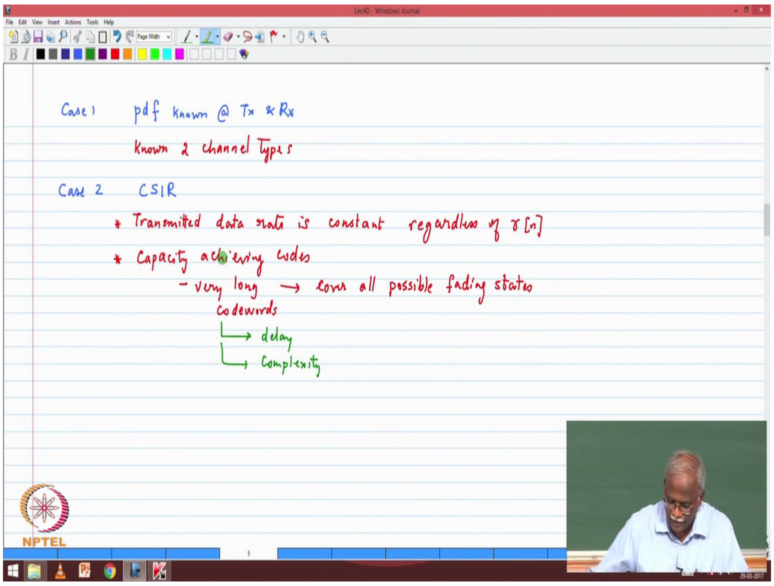

So, now, we let us go little bit deeper just to sort of quickly run through, the 3 cases are when I know only the PDF then the when the channel receiver knows the channel state information, and the way we would achieve a capacity in this case is to have very long code words. Again we talked about the delay and the complexity of such systems.

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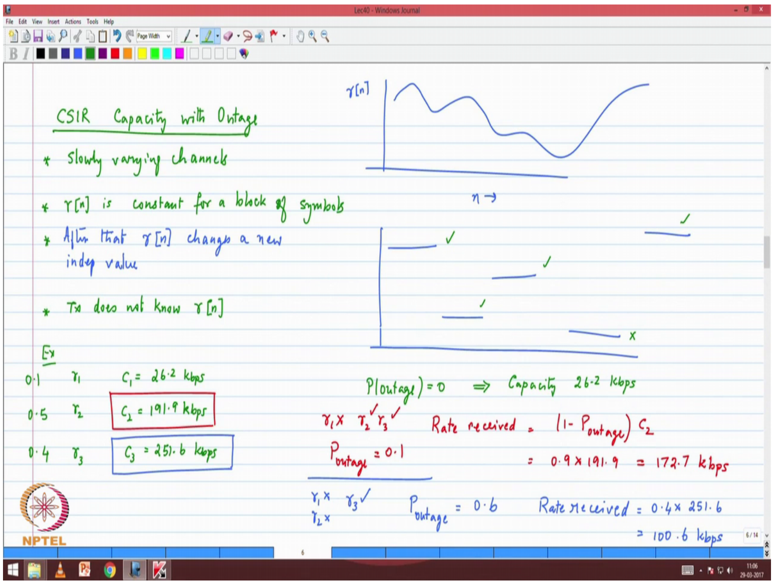

Now, capacity with outage starts to build bring in to the realm of practical implementation. So, if I have a system where the channel SNR is changing, I am going to modulate as an equivalent channel where the SNR remains constant for a block of data, and then between blocks of data the SNR can have an arbitrary change. So, under this set of conditions I know have to pick which SNR will be my threshold, above that SNR I must guarantee that the information is communicated reliably. Below that I will consider it outage.

So, if I consider gamma 1 as my threshold, anything below gamma 1 at gamma 1 or below is going to be an outage then we showed that we could actually achieve a fairly high data rate without the transmitter knowing the information. All we are saying is and that bad channel conditions I am going to consider it as an outage. So, that is a good data point for us to keep in mind ok.

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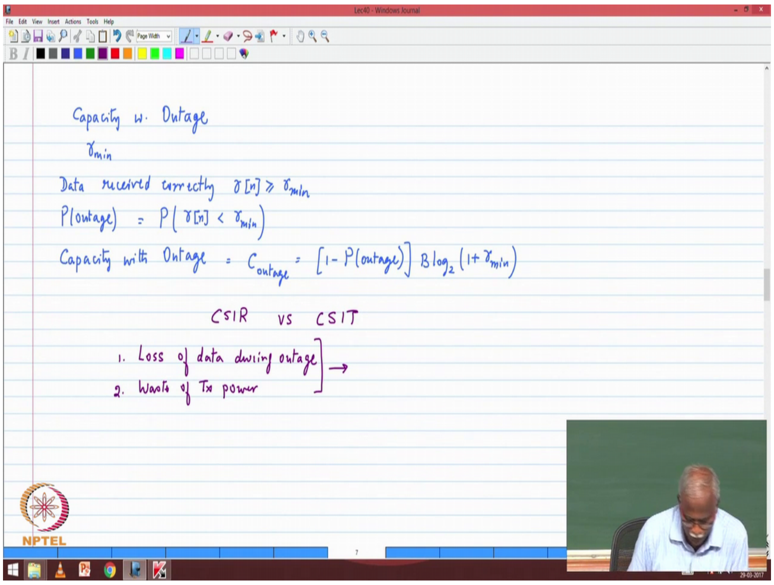

So, to summarize outage will be if the SNR is below the threshold that we have set. And the capacity with outage will be the capacity at the SNR for which I have designed into 1 minus probability of outage. Because there are certain percentages of the time my SNR will be below gamma min and that that is an outage scenario. And please not forget this because if you do not if you forget it you will get a more optimistic result. Because since the transmitter does not know it will continue transmitting even when the channel is below gamma min SNR is below gamma min and that data will be lost. So, the may be a point that we can we can make about So, CSIT versus CSIR, CSIR versus CSIT. What are the penalties if I do not have CSIT? What if I basically in this scenario where I am transmitting a designed for some gamma min and I am transmitting.

So, I believe the points that we can make in terms of the disadvantages, are the first of foremost there is loss of data, right? There is loss of data, loss of data during outage. And we said I do not care about I do not I cannot do anything about it. So, I am helpless. So, there is a loss of data during the periods of outage. Probably another related element which we may sort of omit, but it is going to be important in the context of today's lecture is that we have also wasted power, is that a correct statement; because this data did not go through.

So, there is a waste of power, T x power. Now if I had had CSIT, then I could have avoided both of these things. One of one is I would not have transmitted. So, there would not be any you know there is no notion of loss of data second is I would have used this transmit power for better scenarios to transmit more data. So, you know CSIT sort of avoids both these things and actually takes benefit is from both of those ok.

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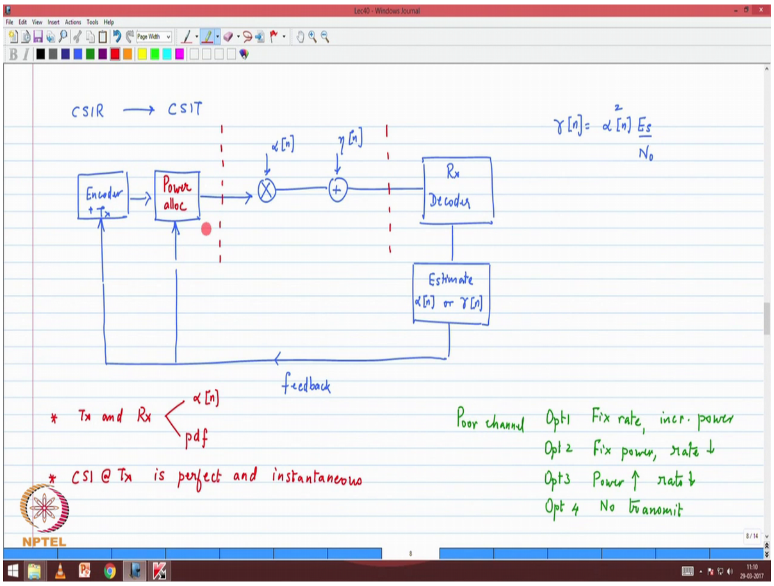

So, the context of CSIT is as follows, we have the channel as before a fading channel. Fading coefficient plus AWGN have to design the encoder and decoder the only difference is now is that I can estimate SNR and I can feed it back. I can feed it back for 2 purposes one is one is to choose encoder decoder, but more than that to also do power allocation because now I can do differential allocation of power for the different channel realizations.

So, the this power allocation block is what we are going to focus on that is where we had stopped at the end of last lecture saying, yes in principle if you give me an SNR I know how to design the encoder and decoder. But how do I allocate power? What is the optimum power allocation? What is the intuition behind the optimum power allocation? And then how does it have ramifications in terms of our understanding of wireless capacity as a whole ok.

So, we have several options for when the channel is a is poor, our assumption is going to be that under certain conditions you will not transmit power you will not transmit at all. And when the channel conditions are according to channel condition if it is above the threshold then you will adapt both the power and the transmission rate. So, basically the coding decoding as well as the power allocation will be will be adapted. The assumption is that the channel information at the transmitter is perfect.

That means no errors in estimation and is instantaneous, see because these 2 things are not are ideal assumptions because typically you are estimating it. So, estimations can have estimation errors associated with it and then there is a finite delay in feeding back. So, in that if it is a very fast changing channel then by the time the feedback reaches the transmitter something could have changed right so that we have assuming that under ideal situation that the knowledge at the transmitter is perfect and is instantaneous. So, that is another important element.

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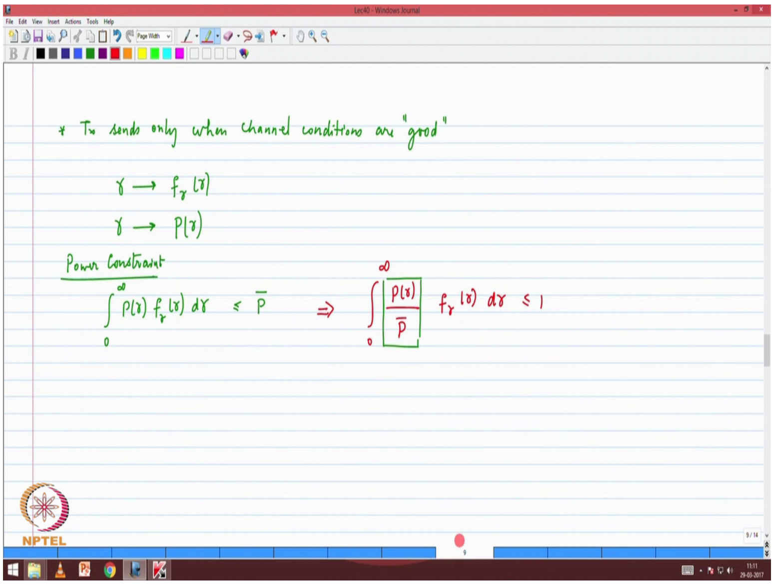

So, now comes the power optimization part. And the power constraint is at I have to allocate power for each of the SNR scenarios. So, basically that is P of gamma is the power that I am going to allocate for a particular SNR. And I have to ensure that I do not do arbitrary allocations. So, there is a constraint. So, the average power must be maintained across all SNRs. So, the power allocated for different SNRs times f gamma of gamma d gamma is must be bounded by P bar or if I divided by P bar I get a normalized constraint.

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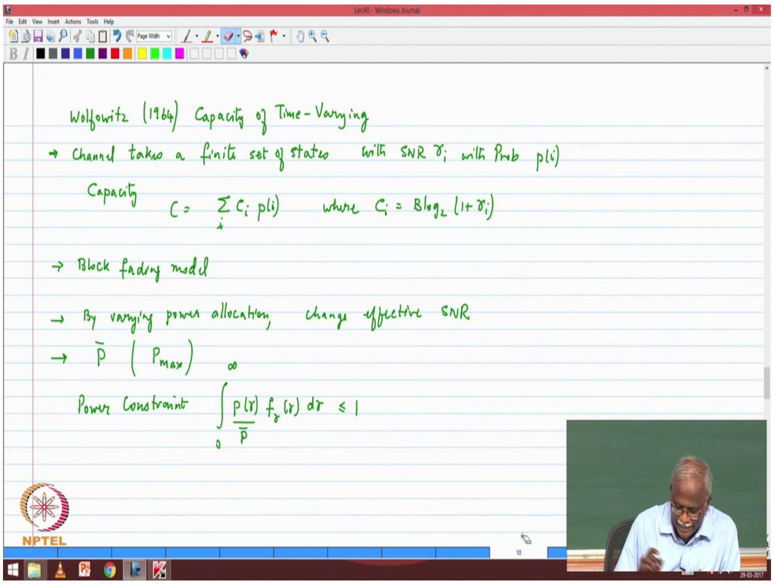

So, with this we rode down the capacity requirement. So, we are looking at the capacity or to compute the capacity of a time varying channel. We want the information is available at the transmitter, and I want to do optimum power allocation subject to average power constraint that is given by this expression. So, the statement of power of the problem that we now have is I want to maximize capacity; I want to maximize it under optimum power allocation.

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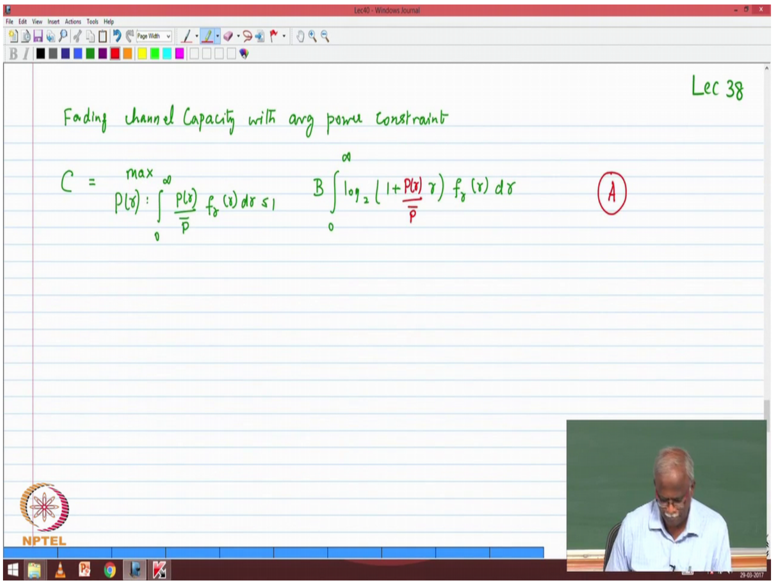

So, the power allocation if you denote it as what we have to indicate by red, there is a normalized coefficient; it either in boost gamma or decreases gamma that is the decision that you have to make.

Now, you are going to maximum this capacity over all possible power allocation schemes. So, it is maximization of capacity no I keep hope fully there is no confusion about what you are maximizing. So, you are maximizing capacity, under all possible options for P gamma. And P gamma must satisfy the average power constraint which is given by this expression ok.

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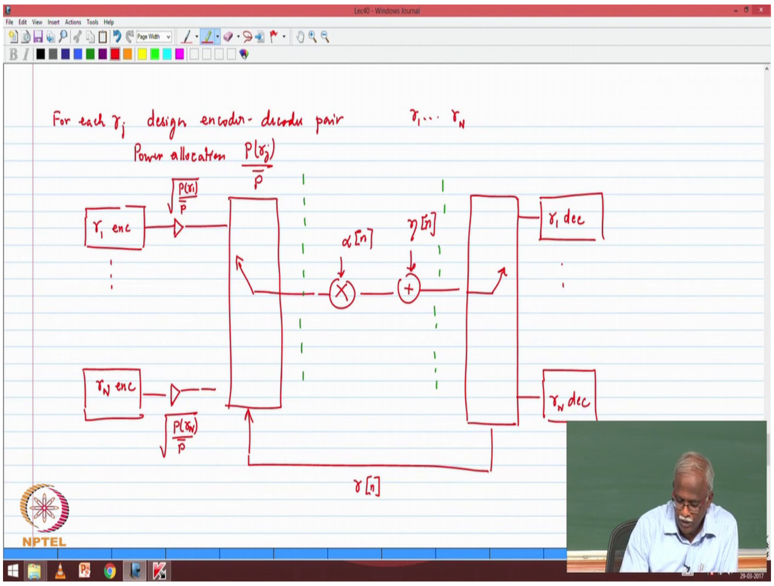

So, the way to visualize once you have obtained the optimum power allocation, what will I do? What will we do? We will design the different encoders we will take the channel conditions we will quantize them in to different SNRs. For each of the encoder decoder pair the minute I know that the current channel conditions has got gamma j then I pick the corresponding branch, put the apply the a power location and then feed the information into the channel.

And assuming that the channel is constant for a period of time this packet of data will go through and I will achieve the best possible performance for all SNR conditions. So, it is a combination of the concept plus a practical realization, but the assumption is that somebody has told me; what is the optimum power allocation.

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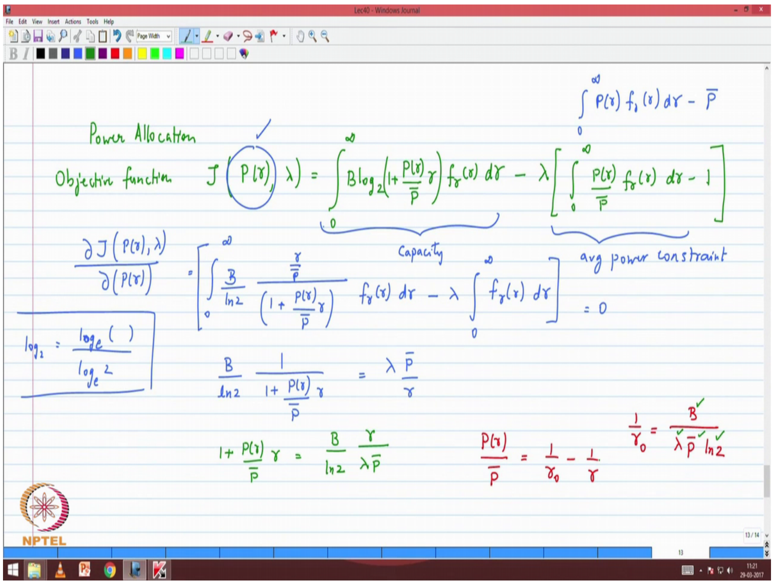

So, today’s class is all about how do we find out the optimum power allocation and how to leverage that. So, let us focus on that, typically our optimization problems you have an objective function and the Lagrangian technique also incorporates in to the objective function, the constraints on the on the variables of the optimization. So, one thing that we are going to do which you may not be familiar with is to treat see basically P of gamma is a function, right? For each gamma you have to specify p. So, I am going to treat P of gamma as if it were a single variable. So, again it is a permissible method and it actually gives us a very good insight into the method into the technique that we are trying to.

So, the objective function is for us to maximize capacity. So, this is the capacity part. I want to maximize the capacity this is the power constraint, average power constraint. It is a constraint optimization. So, basically I will differentiate with respect to the variable P of gamma and also with respect to gamma, and also with respect to lambda if I when I differentiate with respect to lambda I will get the average power constraint. So, that is already one equation that is available to me. Now the key things I want to differentiate with respect to P gamma ok.

So, the partial derivative of the objective function, objective function is a function of P of gamma and lambda I would like to differentiate this with respect to P gamma. As I said I am treating that as a variable. So, we talked about the differentiation of an integral, fortunately neither the upper limit nor the lower limit is has the variable or the off with which your differentiating. So, only one term which is the derivative of the integrand comes into play. So, in it is easy for us to write down this differentiation. So, basically it will be integral 0 to infinity, B times I cannot logarithm base 2 I want to write it as as we indicated before logarithm base 2 we write it as logarithm base 10, log base e of the quantity divided by log base e of 2.

So, rewriting this becomes B by lawn 2, I write the numerator as in terms of the of as the natural logarithm, which allows me to differentiate when I differentiate I get the argument in the denominator 1 plus P of gamma divided by P bar gamma. Then I have to differentiate with respect to P of gamma. So, that gives me gamma by P bar in the numerator, f gamma of gamma that is that is not does not affect the and then we have minus lambda times, may be just to avoid confusion, let me just write it as 2 separate quantities. This as d gamma again lambda times integral 0 2 infinity f gamma, I am going to rewrite this as a does that make a difference it should not, but I am just going to rewrite this as 0 to infinity P of gamma f gamma of gamma d gamma minus P bar ok.

So, basically when I differentiate with respect to P gamma i get f gamma of d gamma. So, this is the value that I get after differentiation, please make sure that and I would set this equal to 0. Notice that we are dealing with the probabilities and we are dealing with power allocation. So, all the quantities inside are positive. So, if the integral has to go to 0 the integrand must be 0.

So, basically we will set the integrand to be equal to 0 under that condition what we will get is B divided by lawn 2, one divided by 1 plus P of gamma by P bar gamma. This is equal to lambda times P bar divided by a gamma. Please check that I have not made a mistake, rewriting this what we will find is that one I will write it in a different color, 1 plus P of gamma by P bar of gamma i have also done the inversion, this comes out to be B by lawn 2 gamma divided by lambda times P bar ok.

One step away from the answer: P of gamma divided by P bar which should give me the optimum solution can be written as 1 by gamma naught by 1 minus gamma where 1 by gamma naught is a constant, which is given by B divided by lambda P bar lawn 2, 1 by P bar 1 by B by lambda P bar lawn 2. Why do I say that this is a constant? What are all the things that are constants? Let me tick the band width is a constant, lawn 2 is a constant, average power is a constant, the Lagrangian multiplier is a constant. So, all of these are constant. So, I treat 1 by gamma naught as a constant itself. And you may wonder why did you call it 1 by gamma naught and not as gamma naught, the answer will come in a moment with the.

So, I just want to make sure that we are comfortable with this expression. So, what have we done? We wrote down the capacity with the a constraint differentiated with respect to the function treating that as of single variable, and showed that the optimum solution should satisfy this condition and the condition is that P by gamma by P of gamma by P bar is 1 by gamma naught minus 1 by gamma. Now very important the interpretations: so, first of first and foremost this has a notation that I would like to introduce now.

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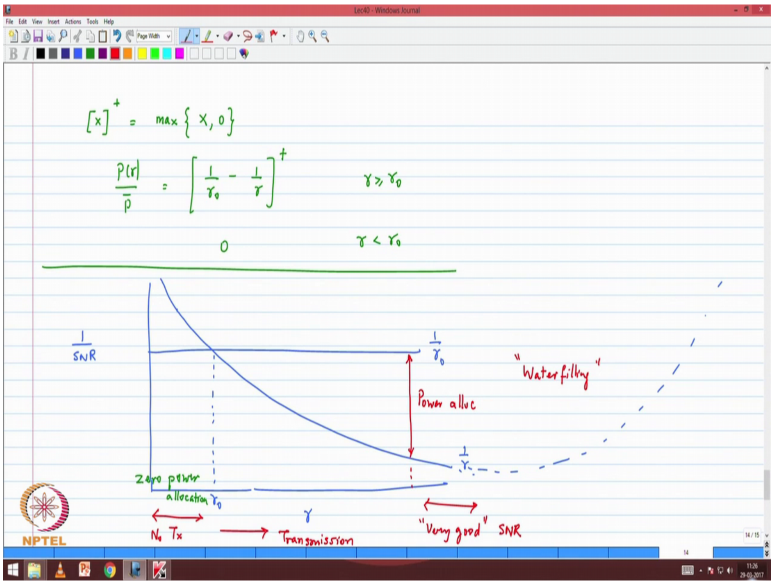

When we say x of plus, this notation basically says you take the maximum of x or 0. If x is the positive quantity you take it is value if it becomes negative basically it is it is it is bounded by 0 on the lower limit ok.

So, P of gamma by P bar can be written as 1 by gamma naught by minus 1 by gamma with a plus. Because power allocation cannot be negative you cannot say allocate negative power. So, I can either or allocate 0 or some positive value of power. So, this is the condition and this is true if gamma is greater than or equals to gamma naught, because under only under that condition the argument will be. Anything other than that it will be 0, for gamma less than gamma naught ok.

So, what I want you to do is just take a moment to just sort of look at the expression that we have obtained. Is it is it intuitive? Is it is it does it agree with what we at least have understood about capacity So far? It is says that the power allocation that I am going to do, is going to be 0 for certain channels below some gamma naught which is very intuitive because if the channel gets very bad I do not want to allocate any power. And as I get to better and better channels, I am going to allocate more and more power. Again very, very intuitive so, but the most interesting thing is the interpretation when we actually draw this graph. And many times this is the graph that a sort of enables us to sort of say oh now the whole things sort of makes sense, makes sense.

So, if you draw SNR on the x axis and very interestingly 1 by SNR on the on the y axis it is there is no information in this graph, but actually it does have, basically this it is it is 1 over gamma. So, it is a graph of 1 over gamma. That is we are what we are plotting. Now I want to interpret this result it is says that there is some SNR gamma naught, gamma naught which corresponds to 1 by gamma naught as a from if once you know gamma naught 1 by gamma naught is a value that is present it is a constant. The interpretation of the optimum power allocation says, that in this region below gamma naught there is no transmission, no transmission and above this portion there is transmission ok.

This is where transmission occurs. Where are the very good channels they are extreme on to the right here. These are the very good channels have using qualitative statement, but basically and how much power do they get? 1 over gamma naught minus 1 over gamma notice they get the maximum power allocation. So, this is some sense 1 over gamma naught by minus 1 over gamma this is the power allocation for this particular value. So, you can take any SNR as you go to higher and higher SNRs you are getting better and increasing levels of power allocation. No transmission basically means 0 power allocation in this region 0 power allocation. So, when the channel is very bad I do not allocate any power I do not transmit at all I take the power that is available and then do a basically, it is it is a progressive power allocation ok.

Now, very often if we want to visualize this looks like a bowl or a basan, where you are filling water; and this power allocation is a interpreted in terms of water filling because inside the bowl you when you fill water, the depth of water tells you how much power you are going to allocate for that SNR. So, this is where the name water filling comes from. Basically think of the blue line you have 2 visualize it as you know complete the bowl and then water filling, but. So, this is where the famous term water filling comes from. You are looking at an inverted bowl and or a bowl and then the SNR is being filled ok.

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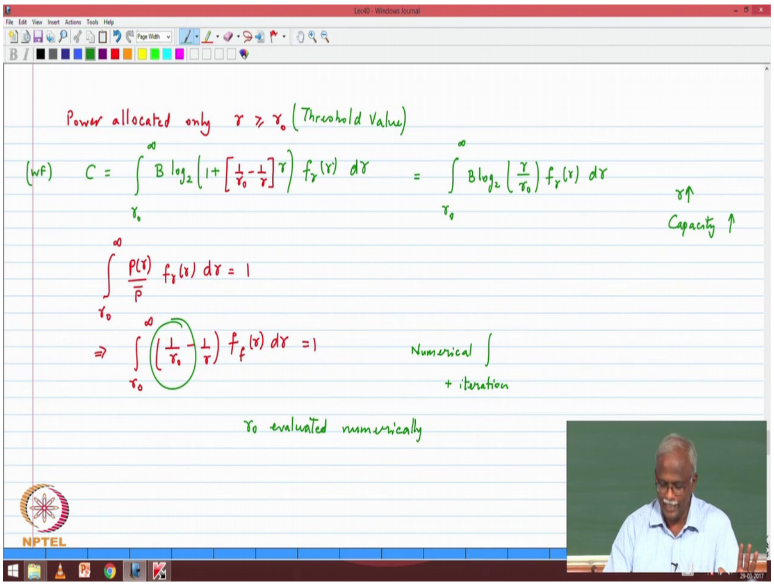

So, let us quickly consolidate whatever we have understood so far in this in this discussion. So, the summary is that power allocated, only power is allocated only if the SNR is greater than or equal to gamma naught, SNR is greater the threshold. Below the threshold we do not allocate power. So, this is like a threshold gamma naught is like a threshold value. Now the key thing is yes it is very nice the interpretation is good intuition is fine, what is gamma naught we need to calculate gamma naught. So, that is what we are going to be doing.

So, under this assumption what is your capacity; under optimum power allocation. So, basically under water filling as the assumption it is not 0 to infinity it is now gamma naught to infinity, B times log base 2 of 1 plus the power allocation times gamma. So, the power allocation is d1 by the following algorithm 1 minus gamma naught by 1 minus gamma that is the power allocation component. This multiplied by gamma that would be my SNR, multiplied by f gamma of gamma d gamma. That is the equation for the capacity of course, you can easily simplify just take the expression you can write this as gamma naught to infinity B times logarithm base 2 gamma by gamma naught f gamma of gamma d gamma ok.

So, it is a difference of powers when I do logarithm it become basically it becomes the ratio of the 2 values one. And that is what I have got here as the term and that will directly impact my capacity. So, as gamma increases the capacity will increase. So, my basically I would come up with the optimum power allocation, which introduces a variable gamma naught. And now how will I estimate gamma naught? That is the last step. Gamma naught comes from the other constraint on the average power constraint. So, integral gamma naught to infinity, P gamma by P bar f gamma of gamma d gamma equal to 1. So, this can be rewritten as gamma naught to infinity 1 by gamma naught minus 1 by gamma f gamma of gamma d gamma equal to 1 ok.

So, it is an integral it should satisfy this condition, and usually this is the quantity that we are interested in we obtain it through numerical integration, numerical integration. And it is an iterative process you assume some gamma naught you calculate it comes out now greater than 1 then you know I have to adjust it becomes very small then you kind of do iterative and then this.

So, numerical integration plus iteration and again if this is something that is not difficult at all to do you can easily find out. So, gamma naught evaluated numerically, evaluated numerically. So, the final statement is that the optimum power allocation goes via water filling. It has to have a reference point, what is the water level? And that water level has to be obtained through a numerical calculation which takes into account the average power constraint that you have, but once you have that it is something that is easily achievable ok.

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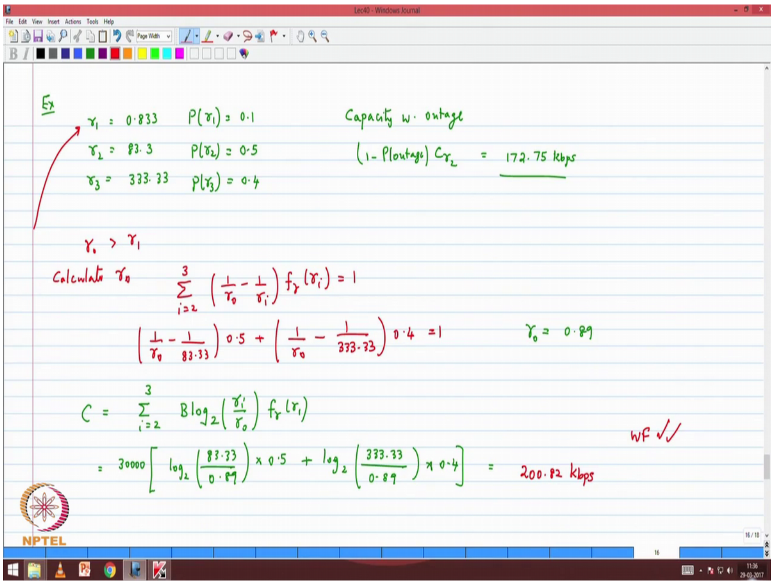

Let us relook at the example that we have been studying, it is the case of the not so bad channel, the over this channel with a 3 SNR states. So, gamma 1 equal to 0.8333 P of gamma 1 equal to 0.1 gamma 2 is 83.3 probability of gamma 2 is 0.5 and gamma 3 is 333.33, probability of gamma 3 is 0.4.

Now, this is the channel condition that we have done. what we have already computed is what is the capacity with outage, capacity with outage. What is the best that we could in terms of capacity with outage? Assume 10 percent outage and therefore, you will get the capacity of whatever is capacity corresponding to 83.3 multiplied by 1 minus probability of outage. So, 1 minus probability of outage into capacity of gamma 2 and this gave us a number which was 172.75 kbps. Now the question is will water filling beat it the question is and by how much, and as I mentioned this is not a very bad channel. So, getting capacity out of this is with outage was somewhat straight forward, but even under this average channel condition, we will we will let us see what sort of benefit we get from.

So obviously, what we would like to say is, I would like to exclude gamma 1. I want I do not want to transmit at gamma 1. So, my gamma naught the thresholds I am going to say is greater than gamma 1, I am right that is the threshold. I do not want to do any transmission at gamma 1. So, I am going to set my threshold greater than. So, under this condition I have to calculate gamma naught, calculate gamma naught. This is a discrete state it is of integral I will get a summation. So, i equal to 2 to 3 that is gamma 2 gamma 3 are expectable states under that the power allocation condition is 1 minus gamma naught minus 1 minus gamma i in to f gamma of gamma i should be equal to 1. So, basically I have written the average power constraint for these 2 SNRs.

So, this becomes 1 by gamma naught minus 1 by 83.33 into 0.5 plus 1 by gamma naught minus 1 by 333.33 into 0.4 equal to 1. That is the average power constraint the basically this is how much power allocation we are doing and based on the probability. So, based on this please verify that gamma naught comes out to be 0.89, 0.89 ok.

So, now the capacity with outage comes out to be again only the SNR states that we are considering gamma 2 and gamma 3, it will be B times logarithm base 2 gamma i by gamma naught. Remember, that is the capacity with water filling, f gamma of gamma I. So, this will be 30,000, that is common will take it outside. First one is logarithm, base 2 of 83.33 divided by 0.89 times 0.5, that is the probability. Second one is logarithm base 2 of 333.33 divided by 0.89 times multiplied by 0.4 please do complete this calculation, verify that this answer comes out to be 200.82 kbps

So, even under a nominal channel conditions water filling extracted almost thirty kilobit is extra kbps extra that is a significant percentage. So, actually water filling can in many condition it can do much better so, but we still agree that water filling has a benefit therefore, I know this is something that we would definitely like to keep and exploit to the maximum. Any questions?

So, water filling is all about finding the water level, allocating power such that the better SNR channels get more power under the constraint that there is a satisfying over all power constraint, and this enables us to get the maximum out of in terms of the capacity from the channel; always good to look at a few exceptional situations, first exception situation.

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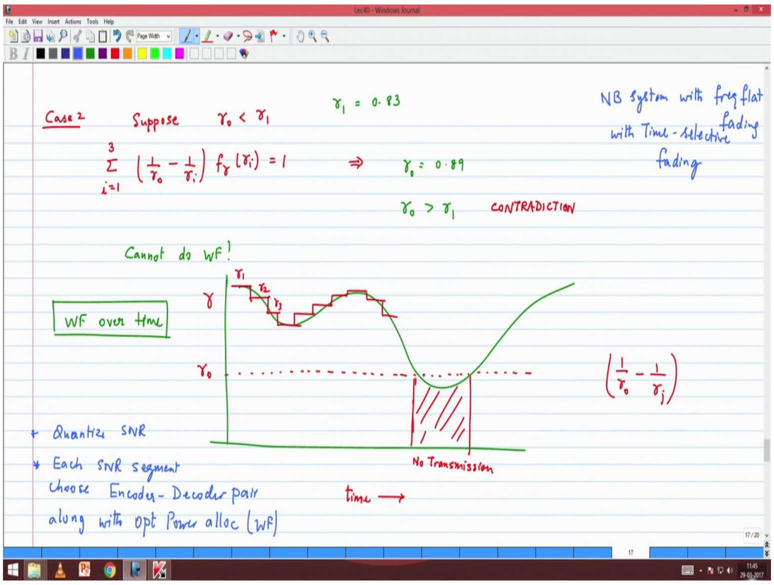

So, this is like case 2 of the same problem. What would have happened just for, just for this thing if you had gamma naught less than gamma 1, what would have happened? You are telling that you are going to use gamma 1 also into your equation and you are going to do water filling. Nothing says that you cannot do that. So, basically you are it is a perfectly legitimate because you are saying fine gamma 1 is not. So, good channel, but I, but I still want to do power allocation for that.

So, now this condition will be summation i equal to 1 to 3, 1 by gamma naught by 1 minus gamma i into f gamma of gamma i this equal to 1. You solve this of course, you will get an answer, gamma naught is equal to 0.89 comes out to be the answer. What is gamma 1 0.83. So, actually gamma naught is greater than gamma 1, and you say there is a problem there is a contradiction. So, under this and condition of trying to include gamma 1 also you find that actually there is no water level that will satisfy the condition. So, which means that I cannot in principle do water filling for this, because I cannot find a gamma naught.

So, it is very important that we choose gamma naught such that I get a valid gamma naught and then, because only getting a valid gamma naught is a is a prerequisite for me to do water filling. Because if I do not get gamma naught then basically I do not get a water level then I cannot do water filling. So, in this case I cannot do water filling, because I cannot estimate gamma naught which satisfies the condition that I am trying to achieve because if I try to do this under this I will get negative power allocation, which is not permitted. So, cannot do water filling in the conventional way.

So, keep in mind that it is all about finding the optimum gamma naught and finding the best way to do it. So, we have now come to the last stage of saying, what is this thing about doing water filling over time? What is thing about water filling over time, that is one of the key concepts and the concept can be interpreted as follows. I have a channel a time varying channel like a Rayleigh fading channel. And the SNR is going to fluctuate right, something it keeps going up and down. And the way to visualize this channel is that there is probably a threshold below which I would not want to transmit.

So, when the signal is in the deep fade I will not transmit. So, there are there are zones of no transmission. So, in this time period; so on this axis is time this is gamma. And so, time on this axis. So, during this period there is no transmission because it has gone below and for my from our discussion on water filling, there is some threshold gamma naught below which we will not do any.

Now, about the possibility above the above this point; so, what we do is we can treat this as basically you do a stair wise approximation of the, of the SNR. And treat each of these as the corresponding let us call this gamma 1 gamma 2 gamma 3 those are the SNRs. Map these gammas 2 the corresponding encoder decoder pair where is that figure, corresponding encoder decoder pair. So, you know gamma 1 means this encoder this decoder, what is the power allocation the power allocation is d1 by a water filling ok.

So, the power allocation is d1 by a water filling. So, we know that the minute, I have an SNR I have a encoder decoder which I can work with all I needed to know was the power allocation. And the power allocation will basically come as 1 by gamma naught minus 1 by gamma j will become my power allocation for that, and how did I get gamma naught? You by computing the average power constraint ok.

So, how can I get the maximum out of a fading channel, is by discretizing my SNRs and for each of these discretized values of SNR I have an optimum encoder decoder pair. Water filling tells me based on my average power allocation I can tell me how much power should allocate for the different SNRs. I use that information also and get the maximum in terms of the capacity. So, the solution that what we are talking about is for you quantize the SNR, quantize the SNR values and for each SNR segment each SNR segment choose the appropriate encoder choose the encoder decoder pair, encoder decoder pair along with power allocation, along with power allocation, along with the optimum power allocation via water filling, allocation via water filling. That is what will give me the capacity of a wireless channel. Well can it beat the Gaussian channel with average SNR no, but this gets me pretty close to the, but this does require me to have knowledge of the channel at the transmitter CSIT is absolutely essentially for me to get this one. Any questions? Last and the most important concept; water filling over frequency.

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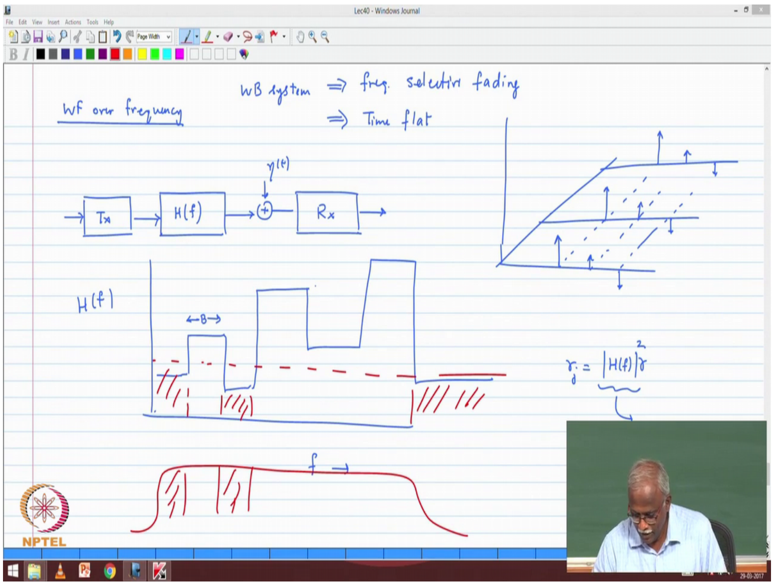

Now, what is this? I am going to pose you, a particular a particular framework, but for that the frame work that we have considered here, if I were to described this. This is a narrow band system with frequency flat fading. Because what did I do? I said I have an SNR and I said my entire channel is going to experience the same fading conditions. That is possible only if it is a narrow band which is experiencing flat fading, but it is a system with time selective fading, time selective fading. Because the channel changes with time, the SNR changes with time and I have achieved capacity for a narrow band system under time selective fading.

Student: Is this n continuously over time or between chunks of time?

Well you would have to design your system looking at all time all of time as a constraint, because you know the power average power allocation has to be done according over the entire duration. So, you must have an idea of how the SNR is going to vary over the period that you are going look at only then you can satisfy the average power constraint. But once you have done that you in when you actually are implementing it will be more on currently it is gamma 4, I take the gamma 4 then it is gamma 5 or gamma 10. So, basically once I have design the system it is based on each segment you can make a decision, you do not have to you do not have to design it based on the data, based on the statistics that are available to you design the system. And then you implemented it in practice in this fashion ok.

So, it this is a narrow band system with frequency flat fading; that means, the whole band is experiencing the same fading, but it is time selective. Now I want to change the assumptions. So, it is a wide band system, wide band system; that means, immediately it is frequency selective fading, correct? Frequency selective fading to make matters simple we are going to say that it is time flat; that means, it is not varying in time, now how does such a channel look? I have the 3 dimensional axis. Frequency selective fading means there is multipath components, correct? And if I look at it across time there is no change. The multi path components are remaining the same. So, the first tap remains the same value, second tap remains the same value, third tap remains here I look at it at beyond some point again same frame work I obtain ok.

So, this is a frequency selective fading channel, but it is time flat because nothing is changing as far as time is concerned. Now how would you model such a channel? It is like saying I am taking my data T x it is passing through a wide band channel with a frequency response h of f note keep in mind the h of f is no longer flat, because this is because there is a frequency selective fading.

Now, to this I add noise eta and then I have to design my receiver. Now the question is, how do I achieve capacity for such a channel? We then say let us look at what h of f looks like h of f is a continuous way form, but I can discretize the frequency response of h of f into some chunks. I am going to use uniform each of them has got a band width B. So, this is h of f this is as a function of frequency are basically have quantize the response ok.

Now, I want to ask you what is the SNR of this of this particular bandwidth B? Now this band width B is a flat fading channel. So, if the SNR of this particular channel, let us say gamma j is actually the SNR that I would have had in the absence of fading, multiplied by mod h of f whole square where f corresponds to basically which the this gain, corresponds to the particular segment with the particular gain ok.

So, the gamma j is not a constant cross all the channels. It is something that depends on the channel gain and depending upon the frequency selectivity I will get a channel gain simple statement. So, what does this look like? This looks like large number of narrowband channels with bandwidth B each of them with different SNRs. How do I get capacity out of it water filling? Water filling over time over frequency; so basically again decide that there may be some threshold below which you do not want to use any transmission, you do not want to use any transmission.

So, I do not wanted to transmit in this region, I do not want to transit here, but I want to transmit in the other portion. I do not want to transmit here also, but I want to transmit in the in the rest of the spectrum. Now you go back to provoke us and say I want to transmit over this band, but I do not want into I do not know transmit here and here. So, sorry I do not have a modulation scheme that does that, because once you choose the bandwidth the what is the, what is your spectrum look like? Your bandwidth spectrum looks like this write basically this the signal spectrum that you have we cannot say I do not want any spectrum I do not want any information content here and here that is not that is not possible.

So, this is what drove or lead to OFDM. Where we said do not transmit one wide band carrier instead, transmit narrow band carriers. Once you have narrow band carriers I can achieve the capacity of the system. I can achieve by doing 2 things, one is I can do power allocation I can do no transmission on some frequency bands, which I cannot do in a single carrier system. So, it only possible in a multi carrier system; and this is what will achieve capacity ok.

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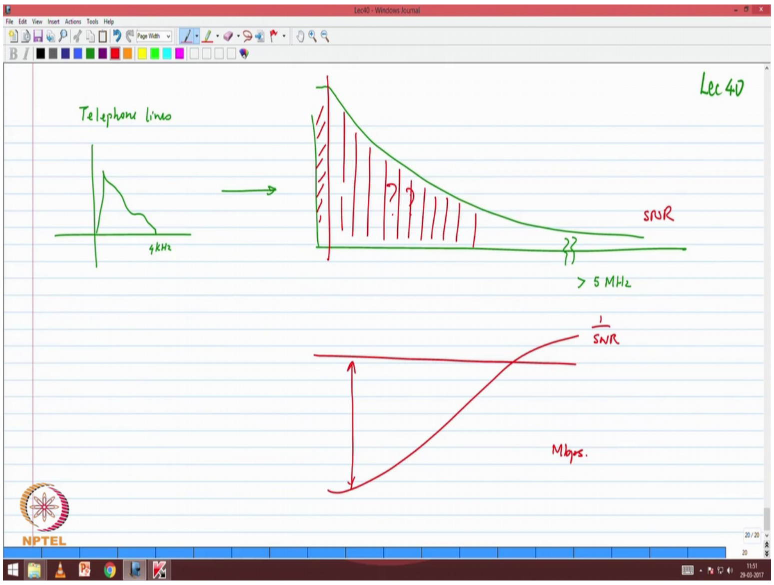

So, final thing, how did Shannon want to achieve capacity of the telephone line channel? What is this? A telephone line channel is nothing, but let us assumes it is not time varying it is a wide band channel with frequency selective. So, what should you do divide it up into narrow bands and do what; water filling water filling. So, what should you do for water filling? If this is the SNR plot, 1 by SNR will be this. 1 by SNR, this is the threshold and this is 1 by SNR. This is SNR and water filling is done like this. So, you can do 64 QAM on the first carrier. And then BPSK on the last carrier and with appropriate power allocation. And this is what gets you many megabits per second on a telephone line, when all you are using it for was 64 kilobit is per second ok.

So, again water filling one of the most powerful concepts in capacity we have water filing over time is what we were trying to solve, but it helps us address the broader issue of what do you do with wide band signals, which have frequency selective fading. No problem I do water filling across the different frequencies, choosing to do no transmission on some carriers. Now the most important thing, that I want you to think about I will just mention it in passing in tomorrows, what happens if you have a wide band time varying w selective channel what do you do you will do water filling over?

Student: Frequency.

Frequency and time: that is it that is how you get capacity of a fading. Fading channel a wireless channel for a broad band system ok.

Thank you will see you tomorrow.