Introduction to Wireless and Cellular Communication
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Lecture - 38
Fading Channels - Diversity and Capacity
Capacity of Fading Channels, Capacity with Outage

Good evening. This is lecture 37 and today we will be covering the concept of capacity. It is one of those fascinating topics from information theory, but actually it is used quite extensively in wireless channels. And how do we optimize and get the maximum out of a wireless channel. The goal would be to not only understand capacity, but also to understand; what are some of the challenges and how do we overcome that in the context of a wireless channel.

So, first let me quickly summarize what we one of the concepts that we have talked about. The last concept in diversity the key take away from the discussion on diversity is that we could think of the benefit of diversity in terms of 2 components one is in terms. Of an array gain the other one in terms of a diversity gain.

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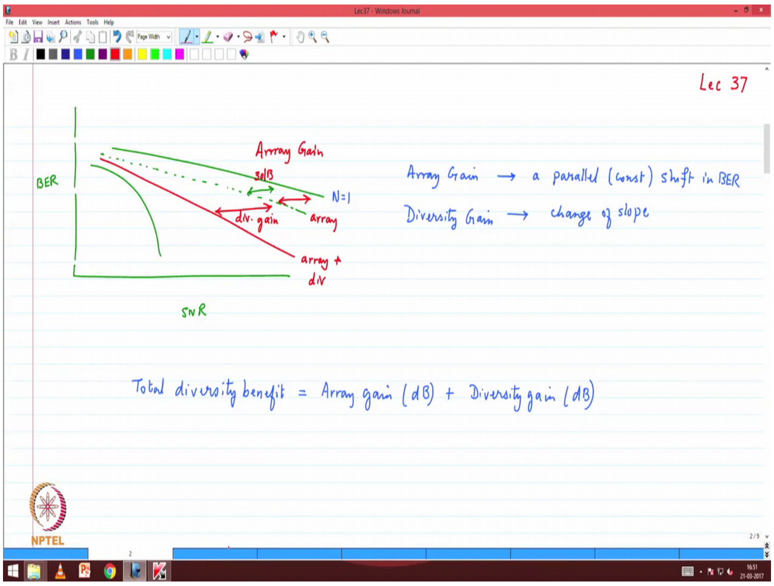

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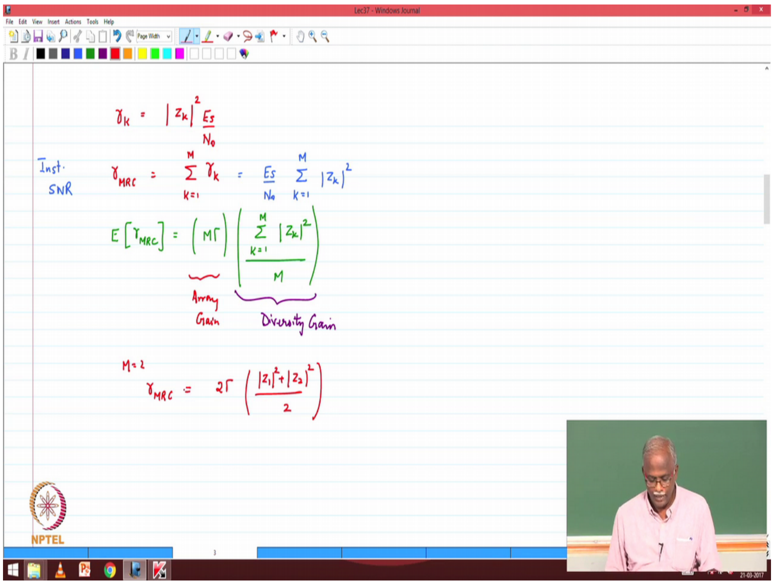

The instantaneous SNR gamma K can be written as mod Z K whole square that is the amplitude square of the channel gain, times E s by n naught what a nominal SNR multiplied by mod Z K square. We find that gamma MRC which is the optimal diversity combining gives us the sum of the instantaneous SNRs K equal to 1 through M gamma K, notice that this is the instantaneous SNR, because if I take average SNR I will get M times gamma.

So, this can be written as E s by n not the average SNR in the absence of fading summation K equal to 1 through M mod Z K square, again these are all just results from diversity combining. And the important result expected value of gamma MRC. This we write as M times gamma times summation K equal to 1 through M mod Z K square divided by M. And of course so, this is important form that we get, which says that I get a component which is array gain, array gain. And then the other one is the diversity gain that is an instantaneous benefit array gain is an average benefit. So, this is the diversity gain.

So, here is the way that we have visualized it. So, if I look at 2 branch diversity then the graph without diversity is the green line, if I have only the array gain which is the 3 dB advantage that will be a parallel shift of the BER graph, but then the MRC actually changes the slope as well and therefore, we get the additional benefit which we attribute to diversity gain. So, total diversity benefit is array gain plus diversity gain again keep that picture in mind.

So, for the special case of M equal to 2. So, for the special case of M equal to 2 the benefit due to gamma MRC would be 2 times gamma times mod Z 1 square plus mod Z 2 square divided by 2. So, this would be the SNR that we would get; so for gamma MRC ok.

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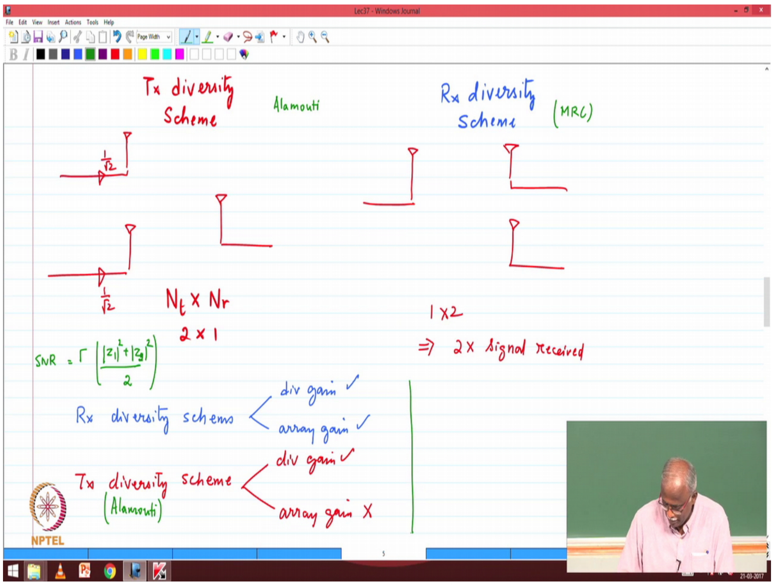

Now, if you go back and look at our Alamouti scheme, it was the Alamouti scheme was a scheme where we had 2 transmit antennas and one the receive antenna. Anti cross n r is 2 cross 1 as opposed to the MRC scheme this would be the MRC scheme, which will be 1 transmit 2 receive 2 received signals. So, this was the sixth scenario that we looked at. And we showed that the SNR for an Alamouti scheme the SNR, the SNR for the Alamouti scheme instantaneous SNR comes out to be gamma times mod Z 1 square plus mod Z 2 square divided by 2. Notice that 2 gamma is missing the 2 is there missing in the array gain.

So, that is why we said that the Alamouti scheme which is in general. So, we could write it here as Alamouti, Alamouti scheme it does not have array gain, but it has the diversity gain. This is mod Z 1 square plus mod Z, Z 2 square is the array gain. Z 2 square as opposed to the received diversity schemes like MRC which give you both diversity gain and the array gain. And may be the there is a simple way of visualization which I thought might be helpful for us to look at, let me just add a new page and ok.

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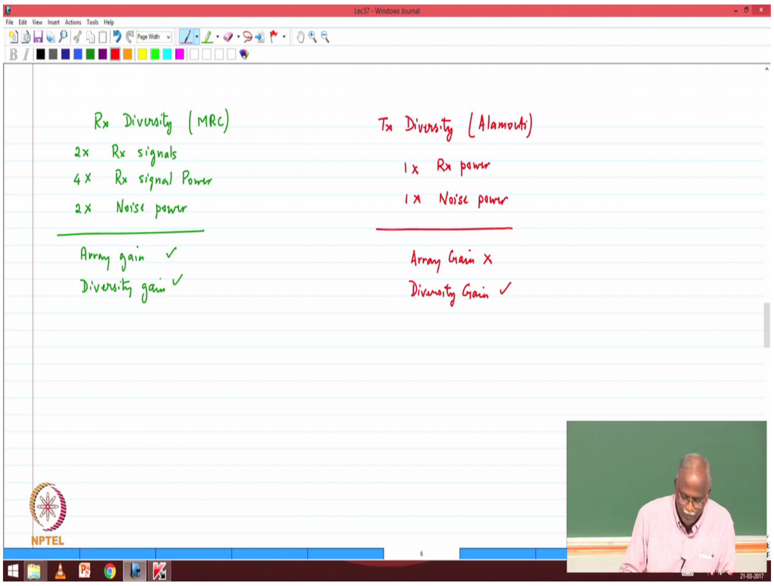

So, this is the traditional receive diversity scheme, receive diversity scheme let us assume you are doing maximal ratio combining.

So, we have 2 x R x signals there are 2 antennas. So, there are 2 x R x signals they have been combined in a co phased manner. So therefore, when it comes to computing the signal power I get a 4 x times R x signal power, is that correct? I have co pashed the 2 signals. So, effective I have got twice the signal level and when I when I take the signal power. How many noise terms? I have 2 x noise terms, noise power. So, the net benefit is 2 x 4 x and the signal component 2 x in the noise component. So therefore, this is a case where I get array gain array gain and diversity gain diversity gain. So, both are present.

Now, on the other hand if I had a transmit diversity scheme, transmit diversity scheme such as Alamouti, how many, what is the total receive power that I have? 1 because I have got 1 over root 2 which is a scale factor when I look at in terms of power is half on each antenna. So, I get 1 x times the R x power and of course, since there is only one antenna there is only 1 x noise source. So, noise power. So, the ratio is no array gain because it is a 1 x in the signal power 1 x in the noise power. So therefore, I do not have array gain array gain no, but diversity gain yes because I have actually combined mod Z 1 square plus mod Z 2 square.

So, again this is just repeating what we have said just now, but again to visualize saying where is the benefit coming from. So, in other words the advantage of the 2 antennas, the advantage of the 2 antennas is that is that I am actually intercepting twice the amount of received signal power, whereas if I have only one antenna then I get the the power gain benefit of one antenna. So, that is another way of visualizing it, but again I would like you to be very comfortable saying when do you get a diversity gain when do you get array gain and you know which contributions to which contributes to what and how do we exploit that in the system ok.

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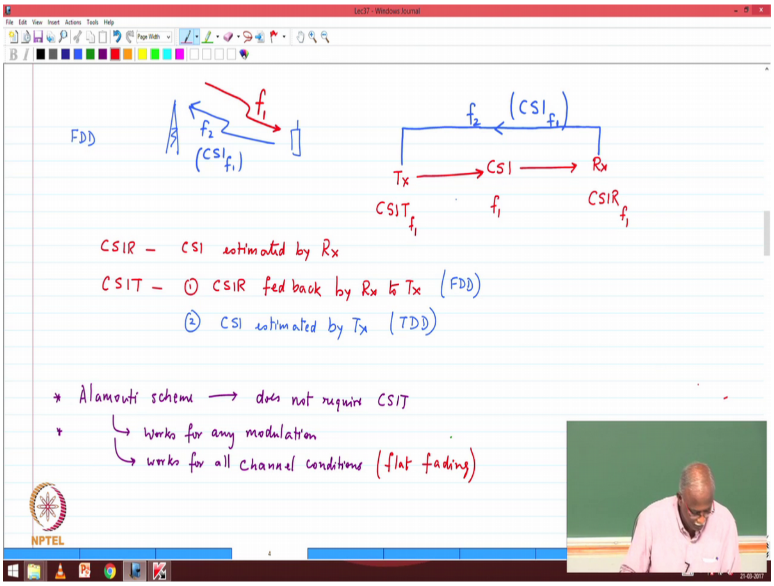

Another element that we had introduced in the last class was the notion of feedback of information about the channel, feedback to the transmitter. So, here is a TDD systems are very straight forward because of the channel reciprocity, we assume that when the receiver transmit is the sorry when the mobile transmit is to the base station base station figures out what the channel is. But FDD systems are a little bit more involved, FDD systems the downlink happens downlink happens on frequency f 1 up link happens on frequency f 2. So, transmitter is base station sends some information through a channel to the receiver, now the channel is the behavior of the channel at frequency f 1. So, this channel state information corresponds to f 1 frequency f 1.

So, the channel response is estimated at the receiver it is f 1 channel response corresponding to f 1, but this information has to be fed back to the transmitter. So, that the any communication between the mobile and the base station is happening on f 2. It may be sending other information, but it is also sending the channel state information for the downlink.

So, yes the channel state information pertains to f 1, but it is going to the base station on a different frequency it happens to be f 2. So, CSIT basically means that whatever has been the channel response that has been measured by the mobile has is been fed back to the transmitter and therefore, both transmitter and receiver have got knowledge of the channel. TDD systems are straight forward both uplink and downlinks are on the same frequency. The reason for reviewing this is to ask you the following question. Alamouti scheme does that require CSIT? Did it did it use information at any point? If you go back and look at the I have I do not have this slide from Alamouti scheme notice we it all it was doing was it was doing a time reversal and a conjugation of the symbols being transmitted, it did not require you to feedback Z 1 and Z 2.

So, may be a good point to note down is Alamouti scheme is does not require a CSIT. Alamouti scheme does not require CSIT. In fact, it is a it is so general that it works for any modulation it works under all channel conditions. You do not need to worry about it the only assumption that it is making is that it is flat fading because, it assumes that you the there is only one channel coefficient, but in the transmitter and receiver.

So, does not require CSIT that is one point. The other point is that it works the Alamouti scheme it works for any modulation scheme for any modulation we did not make any assumptions about the modulation scheme it works for any modulation method it can be QAM, it can be PSK and it works under all channel conditions, works for all channel conditions, all channel conditions. The only assumption being it is flat fading, channel conditions let me just put it with in bracket that in we have assumed that it is a flat fading channel. So, if you have flat fading channel then what you get is based on the transmission scheme that we have developed you get a channel matrix which happens to have the orthogonal property which you then can exploit to detect the signal did it require CSIR there is a channel scheme requires CSIR.

Student: Yes.

Yes, it has estimate the channel, only then it can it can do the conjugation and multiply it by the appropriate a things so, by the appropriate channel coefficient. So, yes it does require CSIR it is a coherent scheme in that sense so, but it does not require information at the transmitter ok.

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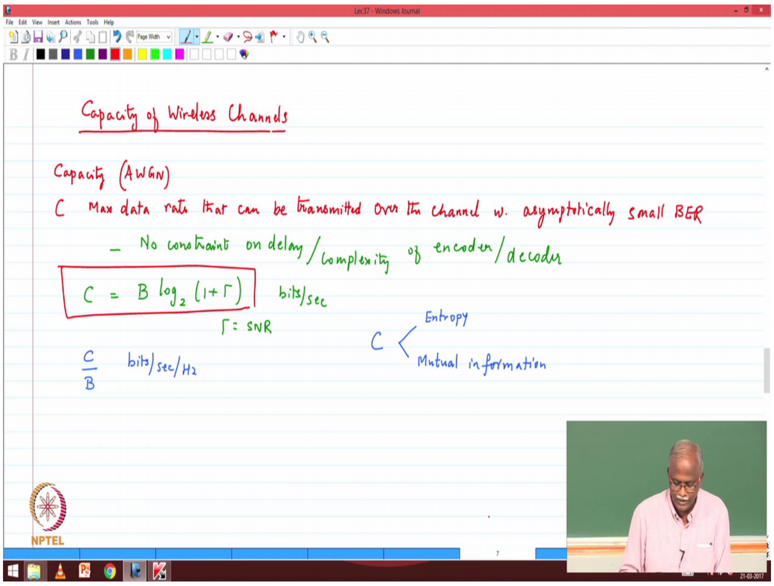

The last part that we had discussed in the in the previous lecture was our understanding of channel capacity. Again we just refer to the basic discussions on concepts of capacity that you would have studied in digital communications which is linked to the concepts of entropy and mutual information between transmitter and receiver. And based on this assumption without any constraint on the delay or the complexity of the encoder decoder the channel capacity according to Shannon is given by bandwidth times logarithm base 2 1 plus SNR.

So, again this is a very useful form if you take C by B which is a normalized capacity then it is the unit are bit is per second per hertz otherwise it would be in bit is per second gamma is in SNR a dimension less quantity. So, based on this we had a couple of examples may be just let me just mention,

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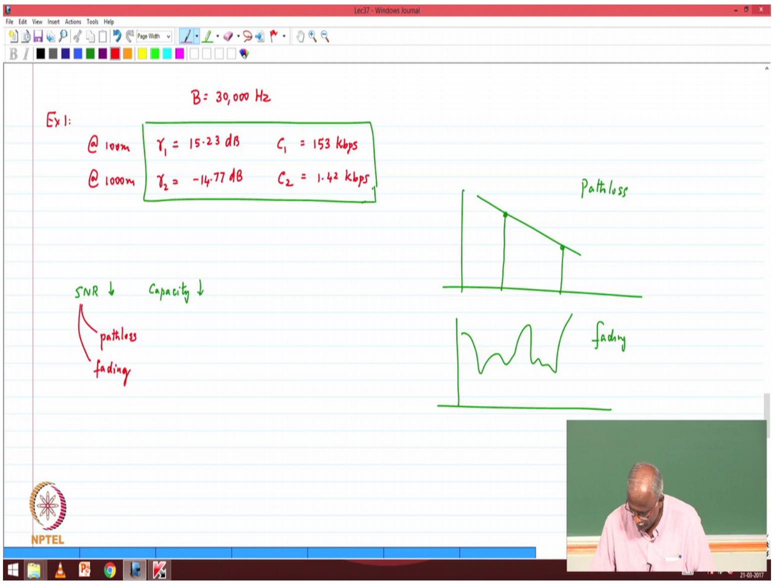

That the first example that we looked at example one, they were 2 cases at a distance of 100 meters. We found or we computed the SNR to be 15.23 dB and given that the bandwidth was 30 kilo hertz. We found that the capacity C 1 came out to be 153 kbps. 30 kilo hertz it is a according to Shannon's capacity you can actually pump a 153 kilobits per second through that, but at a distance of 100 meters for the same channel model the model itself is not that important.

The important one is the concept that we are taking away from there the SNR came out to be minus 14.77 dB and the capacity drastically drop to 1.42 kbps again the numbers are easy to plug in and substitute, but the most important thing is the observation, that SNR place a very, very important role in the capacity of the channel ok.

Now, this is a case where the SNR changed because of what? Path losses because, you moved away, but it could very well have changed because of fading. So now, the once you have understood this, now we then move the context saying do not worry about path loss it is not path loss anymore it is going to be in the context of fading. And what is going to happen the fading environment at least in the case of path loss I know it is it is going to I know I can predict, but in this is path loss path loss. So, the capacity at this point is very different from the capacity at this point because the path loss has changed. But in the context of fading I have a capacity that is fluctuating it is fluctuating. So, at each point based on the SNR then I get a certain capacity. So, it is no longer a fixed quantity it is something that is constantly changing, it is like the SNR in the in a fading channel.

So, that is where we are going to really engage with the concept of capacity. So, what we can summarize from this example if the SNR goes down then in this particular case it went because of capacity the capacity goes down. Now SNR can go down for 2 in 1 it can be because of path loss as in this particular example or it could be due to fading, really does not matter which is the what is the cause of it is going to affect eventually affect my capacity.

So, with that broad frame work in mind I would like for us to look at one more example. Again the purpose of this example will become clearer as we as we look at it and spend more time with it ok.

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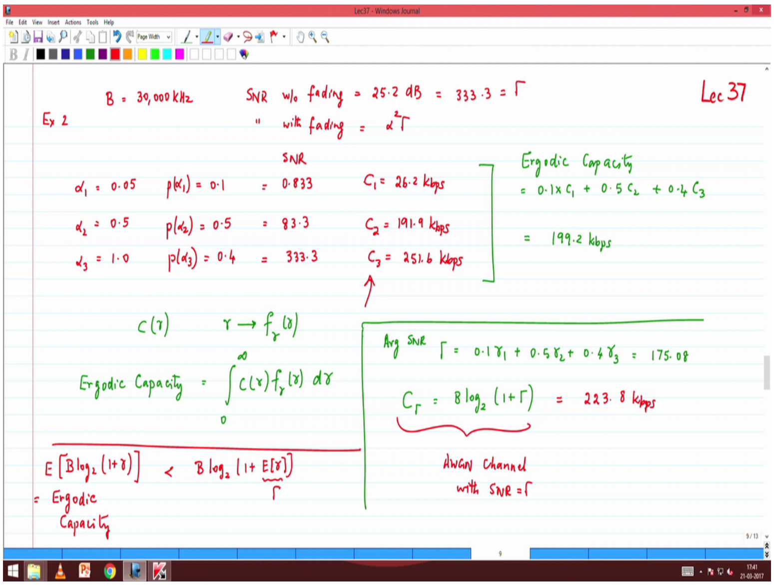

So, here is the example 2 the bandwidth is 30 kilohertz like the previous example. And you are going to look at 3 different SNRs you may wonder you know why there we already sort 2 SNRs what difference you gone make for 3, but the important thing is how we are going to interpret these results. So, if I will this is a channel where the SNR without fading; that means, in the absence of fading, is a high SNR system 25.2 dB. Now this on the linear scale corresponds to 333.3 and we can we will calculate.

Now, the instantaneous SNR, SNR with fading. So, we can call this as gamma this will be equal to alpha square gamma, that is the instantaneous SNR. So, this is an example where there are there SNR possible. Alpha 1 equal to 0.05, alpha 2 is equal to 0.5 and alpha 3 is equal to 1.3. So, it is either act the nominal SNR or worse the SNR again it is a constructed example it is more for illustrative purposes.

Now, what is going to be very important is the next column the probability of alpha 1 is 0.1 probability of alpha 2 is 0.5 probability of alpha 3 is 0.4 these are the 4 possibilities. Now just quickly compute the capacities for each of these. This corresponds to 0.833 and this corresponds to a channel capacity of 26.2 kilobits per second A because of the low SNR. This corresponds to 83.3. This and the capacity for this is 191.9 kbps and the alpha equal to 1 of course, we have already computed 30, 333.3. This is the SNR column; this corresponds to a capacity of 251.6 kbps ok.

So, the problems before us are the following. I have 3 states if you want to consider. It is not good state bad state there some intermediate state as well the capacities are 26 kilobits per second 192 kilobits per second and approximately 252 kilobits per second. Now the key concept is, how do how do we even deal with or come up with something which has a reasonable representation of the capacity of the system? So, this is where we go back and say that this column, this capacity column can actually be written as a capacity as a function of gamma which it is function of the instantaneous gamma.

Now, gamma has got a probability distribution f gamma of gamma. So, we can actually come up with a probabilistic capacity for this channel based on the channel states which is called as Ergodic capacity. Ergodic capacity it is a capacity it is a number that reflects what is the behavior of the channel. So, the Ergodic capacity is defined as over all possible SNRs. The capacity at that particular SNR, the probability of that SNR integrated over the range. So, basically what we are saying is if a particular SNR is very highly probable then the Ergodic capacity is going to be very close to that number, because that SNR is very likely. If a probability is very less than that is not going to that is not going to be as dominant factor.

So, this is for a continuous distribution of SNRs, but her this is an example where we have a discrete number. So therefore, I would like to calculate the Ergodic capacity. So, the Ergodic capacity for this channel will be 0.1 times C 1 plus 0.5 times C 2 this is

Ergodic capacity, plus point 4 times C 3. Again it is simple calculation that you can you can compute and verify and this comes to be 200 and sorry 175.0, 170 let me just, 19.2 kilobits per second, 19.2 kbps ok.

Now, the reason this the bad channel did not dominate or did not cause a significant degradation was because the probability is less it is only 10 percent of the time. So, it did actually pull down the overall capacity, but the better channels are sort of tended to dominate and therefore, we got a good number. Now it is a very interesting for us to do a very similar calculation. Now the question that we are going to ask is what is the average SNR? What is the average SNR of this particular channel? So, basically gamma will be 0.1 times gamma 1 plus 0.5 times gamma 2 plus 0.4 times gamma 3 right that is the average SNR in this particular in this in this channel.

So, if you were to look at this comes out to be 175.08. Very interesting question what would have been the capacity for this particular average SNR C subscript gamma. So, this would have this will be equal to B times logarithm base 2 of 1 plus gamma and so, what is the question that we are asking? We are saying that we are doing 2 types of calculation one is I have a channel that is varying I have 3 different SNRs. What I have done is computed the capacity for each of those SNRs and then done an average of it based on the probability. One is an averaging of the capacity, the other one is an averaging of the SNR and then computing the capacity.

So, this latter case which is in green is like an AWGN channel which is got average SNR equal to gamma. So, this is what would be the equation or a capacity equation for an AWGN channel with this with this gamma. So, this is this is like a this is like an AWGN channel there is only one channel number one capacity number channel with SNR equal to gamma. And it is very interesting for us to compute than this comes out to be 223.8 kbps. The observation and AWGN channel we know that in the context of fading even if it has average same average SNR AWGN channel is much better for us to deal to with, because the fading actually degrades a performance. It is seems to be doing something very similar for capacity as well. Because the capacity of the of a similar channel if you just take it on the basis of SNR is given you much higher capacity in the content of an AWGN channel as opposed to the Ergodic capacity ok.

So, the question that we would or the other way that we would make this statement would be as follows. At least in this particular example what we have found is that the expected value of the capacity is B logarithm base 2 1 plus gamma. That is what we that is Ergodic capacity. So, this is Ergodic capacity. It turned out that the Ergodic capacity in this particular example is less than B times logarithm base 2 of 1 plus expected value of gamma. And this is nothing, but gamma, one case it was 199 kilobits per second other one was 223 kilobits per second.

Now, is this always true and if so, what are the implications? Lot of very interesting questions that arise from our discussion, but it is very important that you know we sort of get a feel for what is the concepts that we are dealing with. So, here is a summary statement, we know that fading compared to AWGN in terms of SNR we have done fair amount of that we looked at the bit error rate. Now just like fading affected the bit error rate it seems to be affecting capacity as well. And the capacity impact also is along the similar lines as the impact that it had on BER that fading is actually makes the performance much worse than the corresponding AWGN average AWGN channel ok.

So, again how do we, how do we develop this? How do we build it up into a concept that we can work with? This is this is the crux of today's lecture. So, we know move into another important concept again which you would have seen in a information theory, but I would like for us to look at it again in the context of what we are studying.

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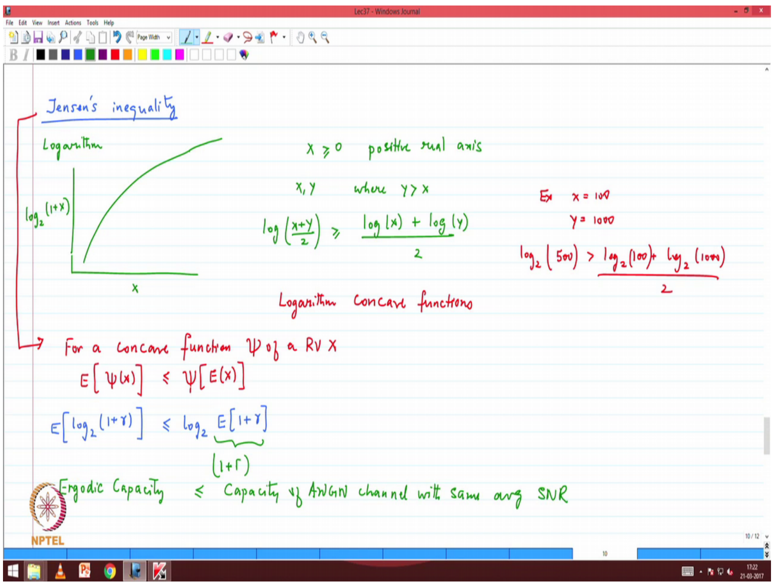

So, this is concept known as Jensen's inequality. Jensen's inequality will actually give us a definitive answer to the previous question that we said is it always true. That Ergodic capacity is worse than the capacity of an AWGN channel with the same average SNR. So, Jensen's in equality will give that, but before that let us first build up our intuition. Logarithm function, if I were to plot the following logarithm function x logarithm base 2 of 1 plus x, you will get a graph that looks like. This basically, you can you can us do a sketch and you will find if you and the following result is true. So, supposing you consider 2 points.

So, basically x is greater than 0 basically it is the positive real axis, because we are looking at can do greater than or equal to 0 positive real axis and if you look at 2 points x comma y, without loss of generality we assume y is greater than x. We can make the following statement about the logarithm function. Logarithm of x plus y divided by 2 is greater than or equal 2 log x plus log y divided by 2. Just to be sure that you are comfortable with the result, take x equal to 100 y equal to 1000 x plus y by 2 logarithm base 2 will be x plus y by 2 is log base 2 of 500 you can you can verify that this is greater than log base 2 of 100 plus log base 2 of 1000 divided by 2. Basically it log base 2 is really not immaterial you can look at it and in any base logarithm has this has this property.

Now, this property is called concavity property. Basically logarithm is a concave function; logarithm is a concave function and this property that what we have written down is the general property of concave functions. Here is the Jensen inequality. So, it is always good for us to have the context in which we are result is being stated, Jensen's inequality states that for a concave function and for us you are very specifically interested only in the logarithm because we are looking at capacity.

So, for any concave function psi of a random variable of a random variable of a random variable x the expected value of psi of x is less than or equal to the function of the expected value of the random variable. This is Jensen's inequality. Concave function expected value of the function is less than the function evaluated at expected value of the argument, expected value of x and this is the now the key things is what is it that we are trying to study or equate, I want to know what is the Ergodic capacity. Ergodic capacity is expected value of logarithm base 2 of 1 plus gamma logarithm is a concave function.

So therefore, it satisfies the Jensen in equality this is less than or equal to logarithm base 2 expected value of 1 plus gamma which will be the same as expected value or this will be the same as 1 plus gamma which is the capacity of a AWGN channel with the with the same average SNR.

So, Jensen's inequality more or less confirms that Ergodic capacity will always be less than the corresponding AWGN channel with the same average SNR. So, we can write down that statement. So, Ergodic capacity, Ergodic capacity is less than or equal to the capacity of AWGN channel, AWGN channel with the same average SNR, same average SNR. Now why is that very important for us? The capacity of AWGN channel very easy to compute you just calculate average SNR and you can compute that is a upper bound.

So, what you try to do is whatever system you design you want to see how close you can get to AWGN remember says, like BER graphs you want to know how close you can get to BER graph, because that sort of the performance bound if you will that you we that we have, but at the end of the day what is it that we can optimize? We can only optimize Ergodic capacity. So, that is where the challenge will lie that is where the interesting concepts we are going to learn.

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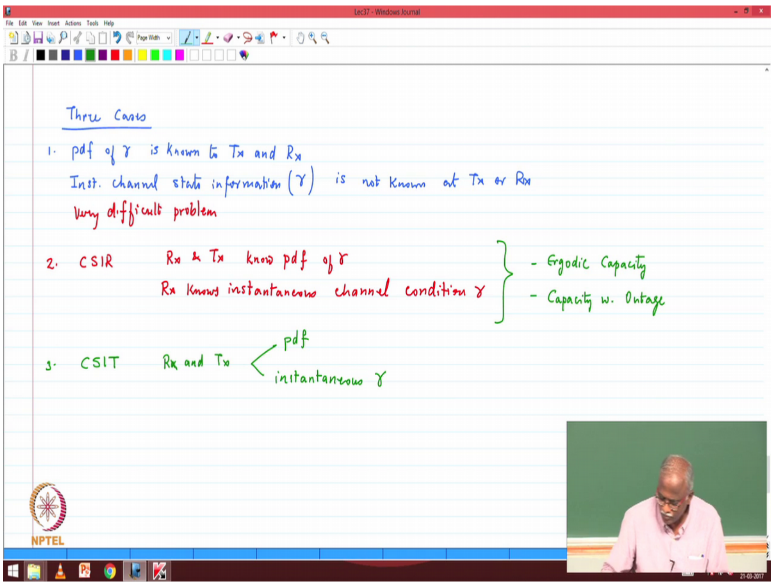

So, here are the 3 cases that we will that we 3 possibilities that we have for studying Ergodic capacity, 3 cases. And very specifically we will be focusing in one of them, but let us write down all the 3 cases. Case 1 the PDF of gamma, the PDF of the instantaneous SNR. For example, if it is our Rayleigh distributed Rayleigh channel than the SNR would be the chi square distribution. So, the PDF of gamma is known to the transmitter and receiver. Known to T x and R x, this is a case where the instantaneous channel, instantaneous channel state information, channel state information that is gamma information this is gamma is not known, is not known at T x or R x.

Now, if you do not know the SNR, what would you do in a practical scenario? You measure it, but in information theory it is perfectly fine to assume that you do not the SNR you do not want to measure it you just say if I have only PDF, what do I do how do I achieve capacity? And it turns out that according to the information theory experts this is a very, very difficult problem because you do not know what the instantaneous SNR is.

Therefore, you know it is one of those situations where you have to design such a complex system that it will it will you know it will survive even in all kinds of a SNR condition. So, number one is really not of much use to us. It is it may be we just make the statement that it is a very difficult problem. In fact, for most of the channels that we encounter it is still an open problem, very difficult problem.

So, maybe it is the question is you know what why do we assume you can always measure. So, the second channel option that we have is that I can measure the SNR at the receiver which means that this is CSIR situation. So, both R x and T x know the PDF like before they know that it is Rayleigh statistics or Rician whatever it is both R x and T x know the PDF of gamma. And R x knows the instantaneous value, R x knows the instantaneous value of the channel conditions or the gamma, instantaneous channel condition. So, it knows the PDF it also knows the condition for gamma ok

Now, this turns out to be a very, very interesting situation, because under these assumptions you can ask the following to be computed. One is Ergodic capacity you know the SNR you know; what is the distribution of the SNR. So therefore, you know what the Ergodic capacity is and therefore, you should be able to try to design systems that will try to achieve. So, Ergodic capacity becomes a interesting problem once you have a measurement of the instantaneous SNR. The second probably the more practical example or the result comes from what is called capacity with outage, capacity with outage.

And it is a very interesting concept. This is something that is at a heart of trying to design practical systems and it trying to get the best out of the wireless channel. Let us just finish the discussion the third case is probably the most interesting case where you have measured it at the receiver and you have fed the information back to the transmitter. So, the transmitter knows the instantaneous value. So, R x r x and T x they know the PDF they also know the instantaneous gamma, instantaneous channel state information gamma. Both of this information is known to the transmitter and receiver and under this conditions what is the best that that we can do.

So, again is the problem statement clear I have transmitter, I have receiver, I want to maximize the notion of capacity, now in the in the absence of knowledge of instantaneous SNR it is a very difficult problem not going to spend too much time on that. But CSIR is a very important case where we can compute Ergodic capacity, we can we can compute capacity with outage and of course, the best system is when you have knowledge at the transmitter and receiver. So, now, I would like us for us to look at a particular example again.

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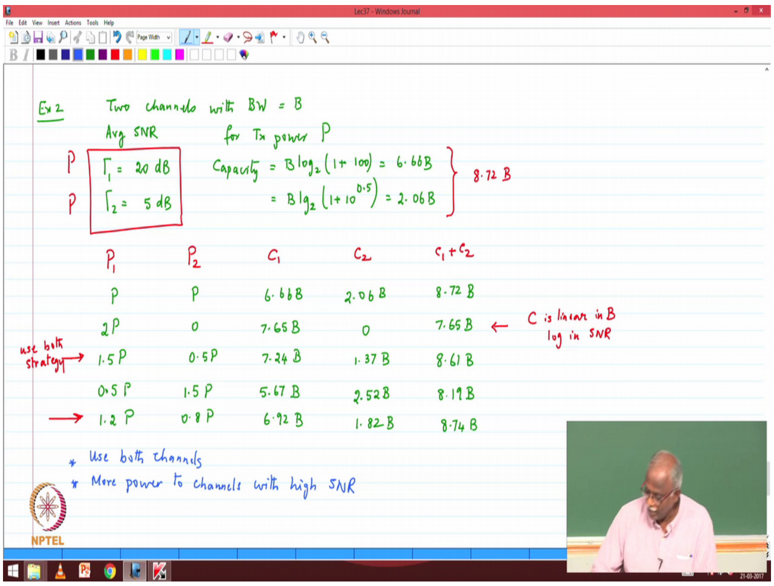

A simple example, but very illustrative in terms of the concepts that it communicates for us. There are 2 channels, 2 channels with bandwidth B width, bandwidth equal to B both channels are the same. The average SNR is the same for both, for T x power p. So that means, if I transmit with a power P on both of the channels the average SNR is going to be the same.

So, basically both are experiencing the same fading the statistics are the same. So, but here is the system, gamma 1 for SNR is a 20 dB the average SNR and gamma 2 is 5 dB the average SNR. So, the capacity for this particular channel capacity will be in terms of B, B times logarithm base 2 1 plus 100 this comes out to be 6.66 B capacity and this is going to be logarithm base 2 1 plus 10 raise to the power of 0.5 which is 2.06 B ok.

So, now if I were to feed P and P to both of these both of these, wait, wait, wait, wait I am I am sorry, sorry, sorry, sorry the average SNR for T x power P is given by this. It is it is not both have same average SNR. So, if I feed equal power P to both the channels. Channel 1 gives me 20 dB SNR channel 2 gives me 5 dB SNR and the corresponding. So, and I and I have I can give a P to each; so then the capacity that I will achieve as a combination of these 2 channels comes out to be the sum of these 2 capacities which is 8.72 B, 8.72 b. So, here is the interesting calculation. Instead of feeding P let us assume that I will feed P 1 to channel 1 P 2 to channel 2 with the assumption that P 1 plus P 2 must be equal to 2 P. I can distribute the power I can I can vary that based on that I will get C 1 and C 2, C 1 plus C 2 ok.

So, that is the assumption that that we are making, let us see how this how this place out in our calculations. So, the first line is the if I give P and P channel 1 gives me 6.66 B 2.06 B 8.72 B is the capacity that is the previous one now first of I am going to try an experiment I am going to double the power in one which means this is going to go to 0. So, what will happen to the SNR on channel 1 it will increase by a factor of 2 because I have would increase the power by factor of 2 and I now x want to see the what is impact in or capacity.

It comes out to be 7.65 B and for C 2 0, right? Because you took away all the power you did not give any power to this one. So, what is the net capacity that you achieved C 1 plus C 27.65 B? What is the logic here? I have a good channel let me give all the power to the good channel and therefore, you know hopefully I will get a better capacity. Well it turned out now that is not going to that is not the case you know giving equal power to both of them actually does much better.

Now, of course, you may want to say that well you know what take away power from the good channel give it to the bad channel the that you know you may want you may want to do a little bit of that. Let us if that that is going to help then it is instead of I am going to give 1.5 P and 0.5 p. So, basically we say the good channel give it a little more power, but do not take away all the power from the bad channel give it some of it and it turns out that this comes out to be 7.24 B 1.37 B again you will have to basically your plugging in different scale factors into the SNR and this comes out to be very close to optimal 8.61 B and never not optimal a very close to the case of equal distribution of power, ok

Now, let us reverse the logic saying well you know who needs more power the channel which is bad condition. So, why not take little bit power from here and give it to this one. So, 0.5 P and 1.5 P basically you are trying to equalize the powers is that the right strategy for us do. So, this comes out to be 5.67 B this one improves a little bit 2.52 B the net benefit or the net capacity is 8.19 B. So, well clearly taking power away from the good channel and giving to the bad channel is not strategy that you want to follow.

So, what have we learned So far? You must use both the channels worse case you give equal power to both of them may be if you play around with the power levels you may see some benefit and of course, if you try hard enough you will come to this particular case 1.2 P 0.8 P this comes out to be 6.92 B 1.82 B and this comes out to be 8.74 B, actually turns out it beats the equal power allocation.

So, may be here is the here is the points that we want to take away from this discussion. Why did not this case work? I gave 2 P to the good channel, right? Why did not it work? Why did not the capacity, why did not, I am surprised by that result I may not because I given all the power to the good channel. Capacity is a is logarithm it is a it is log of the power right. So, you know that band width term is sitting outside. So, if you have 2 times bandwidth that is going to play a bigger role because is power is embedded inside the logarithm function. So, here you can may be say C is linear in B linear in B it is logarithmic in SNR and therefore, it is not a good idea to take away a channel completely.

Now; obviously, the this one says that if you I you do not want to take it away from use both channels use both channels that is going to be the strategy, but do not try to equalize the power, because that is the that is again giving more power to the bad channel is not going to help us. The right strategy seems to be give more power to the good channel, but also use the bad channel. So, that you get the benefit of the linear term in band width.

So, observations use both channels, use both channels. Second give more power to the good channel may be that is a little counter intuitive, but that is a very important result that emerges from there now is this true in general it turns out that yes this is actually the insight that we need to work with the wireless channels. More power to channels with higher SNR, with high SNR that is a very, very power full result the other one also is a important result that we want to that we want to work with.

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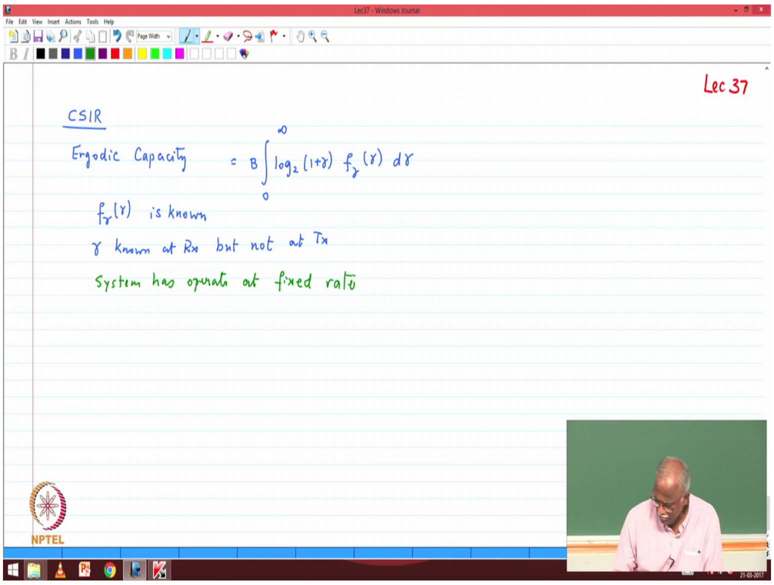

Now, let me just leave you with once you have this framework I want to leave you with the following summary. So, the case of CSIR, what can we, how can we summarize the CSIR? CSIR says that Ergodic capacity is what we are trying to achieve, Ergodic capacity is given by integral 0 to infinity B times logarithm base 2 1 plus gamma f gamma of gamma d gamma. That is our basic equation. F gamma of gamma is known to the transmitter and receiver. The instantaneous value of gamma is known at, known at the recover, but not at transmitter because you did not feedback this information, but not at the transmitter.

So, given this situation what can the transmitter do? Transmitter says I do not know what the channel is so what will it do? It says that you know sometimes it is good SNR sometimes bad SNR, but I must transmit error free right I must get the information across to the other side. So, what the transmitter has to do has to go for worst case design. Whatever is the worst possible SNR I have to transmit at that it because I do not know if it is better or worse than that?

So, the system has to because I do not know it at the. So, the system has to operate at a fixed rate, has to operate at the fixed rate. So, if that is the case, let me go back to the example that we calculated CSIR what rate will you achieve in this channel?

Student: (Refer Time: 48:56).

Because the worst case SNR right? You do not know what whether the SNR is alpha 1 alpha 2 or alpha 3 the channel condition and you have to transmit reliably under all the channel condition otherwise you will go into outage, you do not want outage. So, now, comes the important concept, if you want Ergodic capacity it will be based on worse case SNR because it transmitter does not know which is what instantaneous SNR is; however, if you now tell the transmitter by the way some outage is you transmit and what you think is a is a good scenario and if occasionally there the data gets lots it is what capacity can you achieve same example what is a capacity that I can achieve.

Student: 190.

I can do 199.9, 191.9 and what will be the outage probability? 0.1. 90 percent of the time the data will go through and it will go at 192 kilobits per second. So, we can see how if I allow a little bit of outage the channel is going to perform much better. What is the intuition? Do not try to communicate under all channel conditions particularly in the presence of fading. There may be some conditions under which the transmitter does not know it is such bad situation, but it is outage is fine, what will you do if outage happens? What will you do data is lost? What happens if you do data is lost?

Student: Retransmission.

Retransmission, no problem we have solution for that. So, you take outage, but the take it. So, that you get better capacity. So, that is the CSIR concept of thinking saying that designing for worst case the design has to be you know actually quite challenging because if you have a bad SNR like it 0.833 that is a less than 0 db. So, if you have such a poor SNR then you have to have a lot of coding. And basically you have you are code words must be very, very careful designed there will be very long code words the decoding is going to be very complex.

So therefore, this is a scenario where the design is going to be difficult and at the end of the day you are going to get poor capacity. So, a good engineering approximation says you know forget doing Ergodic capacity, but let us do capacity with outage and therefore, come up with some very practical examples. Now on the other hand if the transmitter knew that that the channel condition was bad, would you still have outage? Would you still have outage? No, what do the transmitter do? It will not transmit. It will not transmit.

So, that is the benefit of CSIT. So, basically whatever you can achieve with outage you can do better than that when you go into CSIT because the transmitter can just say oye this is really bad condition I am not going to transmit I am not gone waste my power. And the power that you saves who will you give it to you will give it to the good channel conditions you actually will get more capacity through more data through in to the into the system. So, that is the dimensions that we are going to study. And at the end of the day where did all of this come from? Let me just end with the little bit of history. So, that you can then think about and you really appreciate this is, ok.

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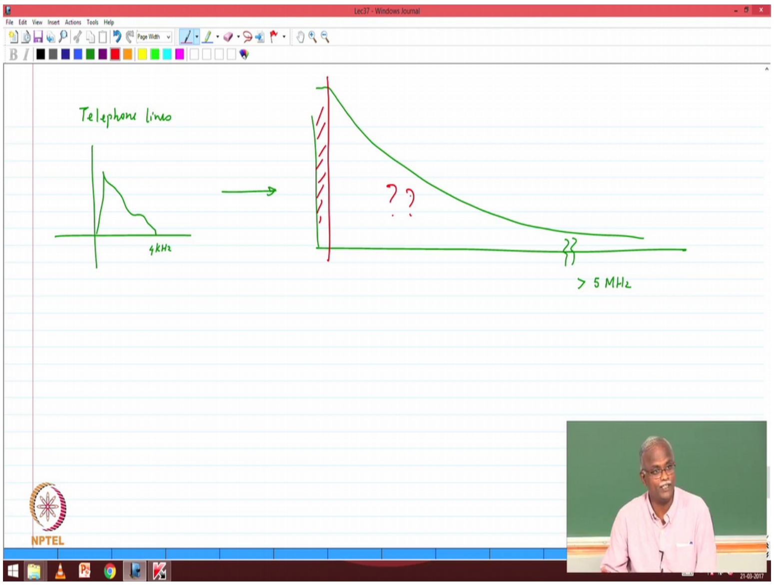

The most important system or things that we worked with in communications are the telephone lines. That is where most of the inventions telephone line analog telephone lines. Analog signal bandwidth voice 4 kilo hertz, 4 kilo hertz that is why we do 8 kilo hertz sampling and the total bandwidth that you are using is 4 kilo hertz ok.

Now, this is being going to be transmitted over a copper wire that is initially analog. The frequency response of an analog of a telephone line copper line is this axis is greater than 5 megahertz. It is got huge bandwidth, but the problem with this is this bandwidth has a response which is very bad the gain is very poor at high frequencies. So, what do we do we use only the useful portion and all of this is waste. You on a copper line we transmit only 4 kilo hertz when it has got So many megahertz of bandwidth ok.

So, the question that always was at the at the back of people's minds is, what is the capacity of this telephone line? What is the capacity of this telephone line? And how can we achieve or how can we improve, right? Now you are using it for 64 kilobits over second, what is the capacity of this one? It turns out that the capacity of a telephone line is in the tens of megabit is per second, but what you are doing is transmitting it you know at a very, very low part of it.

So, the question became, how do we achieve capacity for such a system? And that is how once you understand that there is a really problem that real life problem that needed to be solved. Then we know go back next class, when we come back we will talk about CSIT and all of that. So, at the back of you are mind you keep asking the question how do we achieve capacity of a telephone line channel. And the answer to this came from Shannon which says the way to do this is through his way of first of all computing capacity and then secondly, achieving the capacity for such a system.

So, again it is a beautiful concept which we I hope you will appreciate as we build upon it. So, the answer to this question comes in tomorrow's lecture will see you.

Thank you.