Introduction to Wireless and Cellular Communication
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Lecture - 37
Fading Channels - Diversity and Capacity
Alamouti Scheme - Part II, Channel Capacity

Good morning. We begin lecture 36. In todays lecture we will be covering the first space time code that has been proposed in 1988 that is the referred to as the Alamouti code. We will also talk about the channel state information, the different forms and how it plays a role. And this will build up our understanding to talk start beginning our discussion on Channel Capacity.

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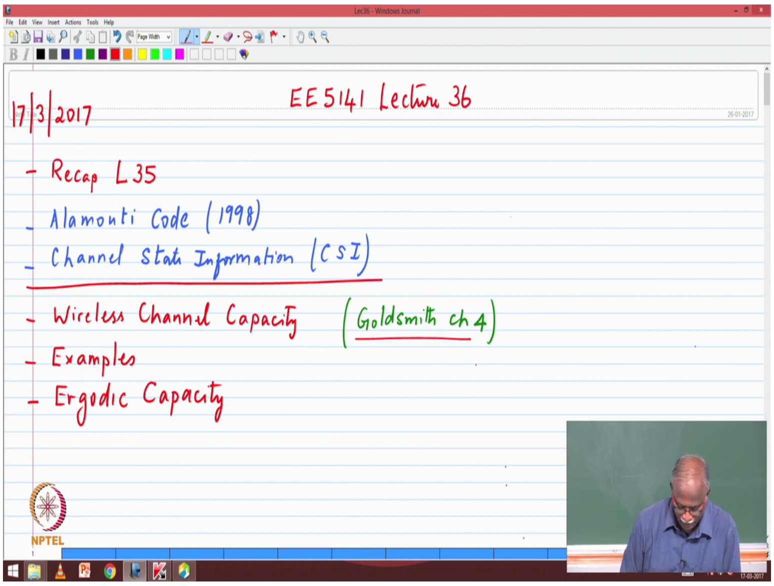

So, the channel capacity is the next unit that we are addressing a very good reference for that will be goldsmith chapter 4, I would very much encourage everyone to read this. Because this material is not there in some of the other references. So therefore this a good place to begin.

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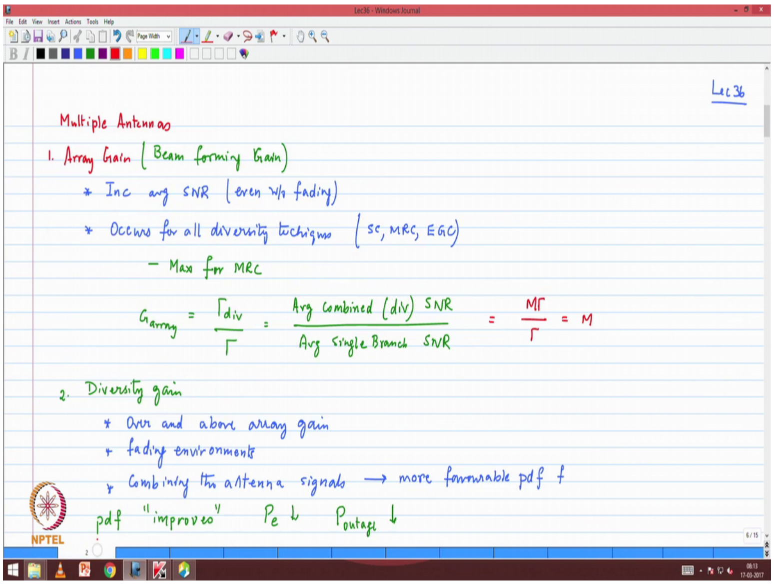

But to summarize the yesterday's discussion they were 2 key concepts that we introduce that, one was the understanding of the benefit of diversity being split into 2 components. One is the array gain the other one is a diversity gain. The array gain defined as M the average SNR with diversity divided by the average SNR of a single antenna.

So, in the case of the MRC we got a value of M it turns out that is the maximum that you can get in terms of array gain if you have M elements you can improve the get an array factor of M. And that is the for example, the benefit that you would get in terms of the improvement in SNR. The diversity gain by the way array gain comes whether there is fading or not. The diversity gain comes in primarily when there is fading, because you are trying to improve or change the statistics of the fading distribution.

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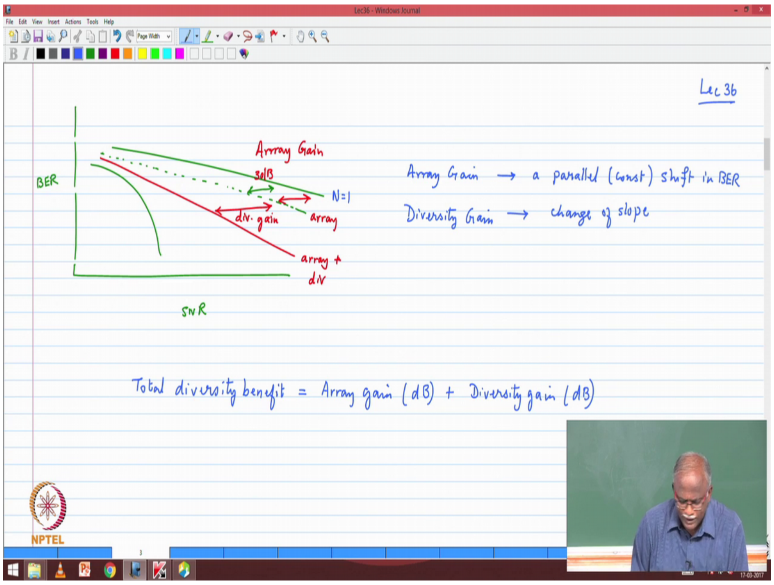

So, array gain and diversity gain probably, the best way to remember it is the graph that we have drawn yesterday. So, I would like you to review that once more. This is the bit error rate of the of a any modulation scheme in a AWGN. The performance in a single antenna channel, that we can even mark it down, mark it down as a n equal to 1 single antenna. Now if I move to n equal to 2 I saw different slope. That difference between the 2 performances, you have split up into the array gain which will be a factor of 3 dB because of the second antenna being optimally combined. Again it may be less than 3 dB if you have done not maximal ratio combining, but you have done equal gain combining for example.

So, this shift you should, you should not always mark it as 3 dB. It is the shift that you get is because the some array gain that you have obtained; it if you have gotten 3 dB that is the best that you can achieve. So therefore you are doing something in terms of the optimal performance now what is the balance is the over and above the array gain is the benefit called as we refer to as the diversity gain. The combination of the 2 is the net benefit. Therefore, you know when you think about it in terms of the net improvement in performance we do not say I got. So, much BER improvement due to array gain. So, much no it is more of intuitive understanding saying maximize your array gain. Then you will see that the maxima over all benefit you have maximized as well. And the diversity gain is something that will show up when the fading is severe and therefore, you will get a significant improvement in terms of the performance.

So, the way to summarize would be is the is the total benefit, the total diversity benefit, diversity benefit is a benefit will be a combination of the array gain, array gain in dB. Because if it is in linear scale it will be a product of the 2 gains because it is in dB it is an addition of the 2 gains plus what we get as the diversity gain. And we mentioned in the last class that things that can reduce the diversity gain are weak antennas, correlated antennas you did not implement maximal ratio combining. So, several things that could reduce the benefit of diversity gain. Those some of those could even reduce the benefit you know on the other side as well, but important thing is to know that it is a combination of these 2 systems that we are working with and this is what we would like to maximize.

The other important result that we discussed in the last class was a numerical integration again.

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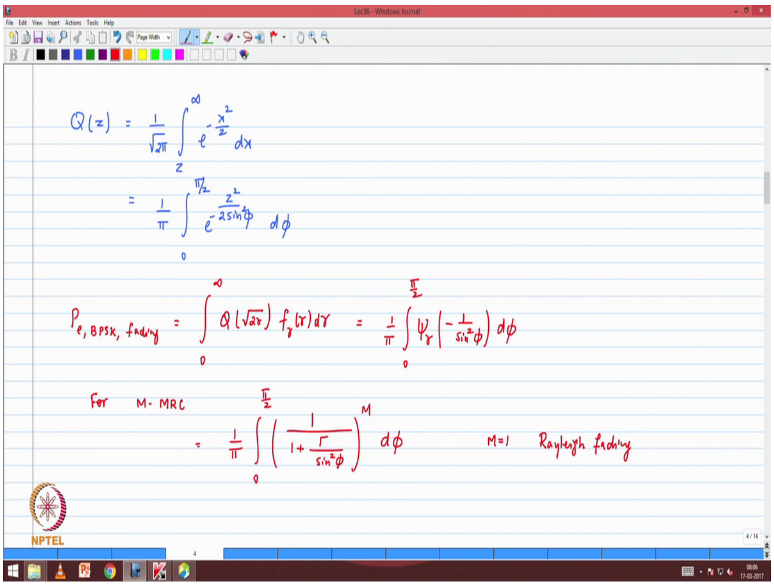

Just as a quick refreshing, refresher of what we have talked about in the last lecture Q of z normally defined as the complementary error function which is given by 1 over root 2 pi integral z 2 infinity e power minus x square by 2 dx. Now this form is very convenient for several of the integration problems that we encounter, but if we want to do numerical integration 2 things one is the limit is are infinite the other one is it is it is dependent on the integrand itself.

So, in alternate expression for Q of z which we said would be more useful from a numerical integration point of view was 1 over pi integral 0 to pi over 2 e power minus z square by 2 sin square phi d phi. So, that would be our equivalent expression that we want to want to leverage. Now we said that the probability of BPSK is one where you would have the Q function. And likewise there are several modulation schemes. In fact, most of the Q a M modulation a 16 QAM 32 QAM all of those will have a Q function as part of the expression for error.

So, any of the general family can be handled in the following fashion. As an illustrative example we took BPSK, probability of error of BPSK in a fading. This would come out would the basic expression would be integral 0 to infinity Q of root 2 gamma. You can take any Q any function any a function of the BER which involves Q you can do the same method. F gamma of gamma d gamma and we showed that once you substitute the new form for the for the Q function this can be written as 1 over pi integral 0 to pi by 2 psi of gamma, that is your moment generating function evaluated at minus 1 by sin square phi d phi. Moment generating function we assume is a is known that can easily be programmed. So, you basically feed in different values of phi the compute the argument minus 1 by sin square phi compute the moment generating function and then perform the numerical integration.

So, then integrand is well defined the integral is well defined. So therefore it is easy for us to work with. And as a special case we said for M branch MRC combining the this particular expression would be 1 by pi special case for would be 1 by pi integral 0 through pi by 2 1 divided by 1 plus gamma by sin square phi raise to the power M d phi, where gamma is your average SNR that is provided for us. Again this is in terms of the numerical integration, once you have given gamma once your given M this should be something that we can readily program. And special case of the special case if you take M equal to 1 you basically get single antenna Rayleigh fading, single antenna Rayleigh fading and you can verify that your getting the forms that we are interested in ok.

So, that was primarily the key concepts that we developed in the last lecture. Now we also talked about channel statement information. I want you to start thinking about channel state information in the following manner. So, if I have a transmitter and my transmitter has many antennas.

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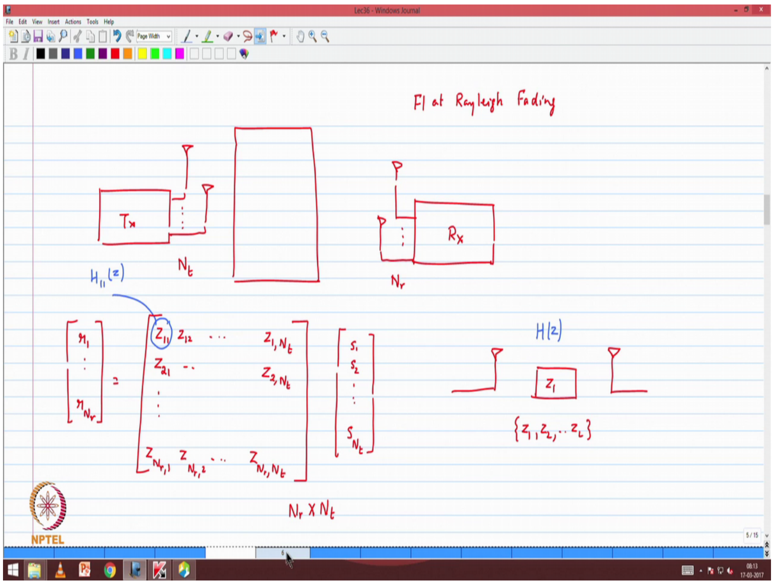

So, let me say that it has got N t antennas. So, this is my transmitter N t antennas and I have a receiver which has N r antennas. This is the most general type of situation that we will encounter N r antennas. This is N t transit antennas N r receive antennas at the receiver. Let us take the simple case of flat fading, flat Rayleigh fading channels, Rayleigh fading ok.

Now, describe for me what is the channel between the transmitter and receiver? What, how would you characterize the flat fading? Means it is between every transmit antenna every receive antenna it is a single Rayleigh fading coefficient. Now there are N t signals that are getting transmitted each of those N t signals will be picked by each of the N r antennas. So, this channel that is in between here is a matrix channel. Because it connects t anti transmit antennas to N r receive antennas. So, if I were to write it in matrix form the transmitted signal from the different anti antennas would be s 1 s 2 dot, dot ,dot s of N t and I would pick on N r received signals. R 1 to r n r So, N t inputs N r outputs.

So, my matrix is a N r cross N t matrix wh.ere each of these coefficients let me call this as z 1 1 z 1 2 z 1 comma N t. What is the definition of that? The, that is the channel between the z I/ j is from the jth transmit antenna to the ith receive antenna. So, basically all of these coefficients are the ones that will contribute to r 1. Similarly there will be z 2 1 to z 2 comma N t. Again this is notation that will be very helpful for us to visualize.

And the last of these will be z N r comma 1 z N r comma 2 dot, dot, dot z N r comma N t ok.

So, keep in mind that this is the channel that if it is a flat fading channel. Now if I were to ask you single transmitter single receiver SISO channel, what is your matrix look like? 1 cross 1. So, basically it collapsed in both dimensions to come become a 1 cross 1 system. So, if I were to tell you I have one transmit antenna 2 receive antennas this becomes a 2 cross 1 matrix if I tell that is receive diversity. If I tell you it is transmit diversity system 2 transmit one receive it will be 1 cross 2 matrix is what represents the channel. So, all special cases all combinations SISO, MISO, SIMO, and MIMO, all of them can be covered by once you understand this basic picture, good.

Now, just to make it a little more interesting and challenging, what would happen if this were a time dispersive channel? What would happen if it is a time dispersive channel? Let us say that it is a discrete channel that is between the transmitter and receiver. So, in a SISO case if it was just a flat fading channel you would represent it by some coefficient z 1. Now if it was a time it is a basically it is a time dispersive, then it becomes z 1 and then there is a z 2 with the different delay there is a maybe there are some number of taps that are present in the system.

So, if we were to represent this as a filter H of z, H over z is not basically if you were to represent this as a filter. Let me just denote it in DSP terminology if I were to denote it as a filter I will I will call this as some H of z, it is got it got some impulse response and it is got errors. So, what would happen in this if this became a MIMO system with time dispersion then each of these from being a single coefficient will become a polynomial. Become H 1 1 of z, each of them becomes a polynomial says becomes a polynomial matrix. But we would not complicate things today we will just treat it as a flat fading channel which means it is at any given time if you take a snapshot it will be an N r cross N t matrix with N r times anti Rayleigh random variables where each of them are independent of each other that is present in the system. Now given this frame work I would like you to follow along on in this fashion; the whole notion of CSI. CSI channel state information means you know this is z matrix, that is all it; that means, that part of it is no.

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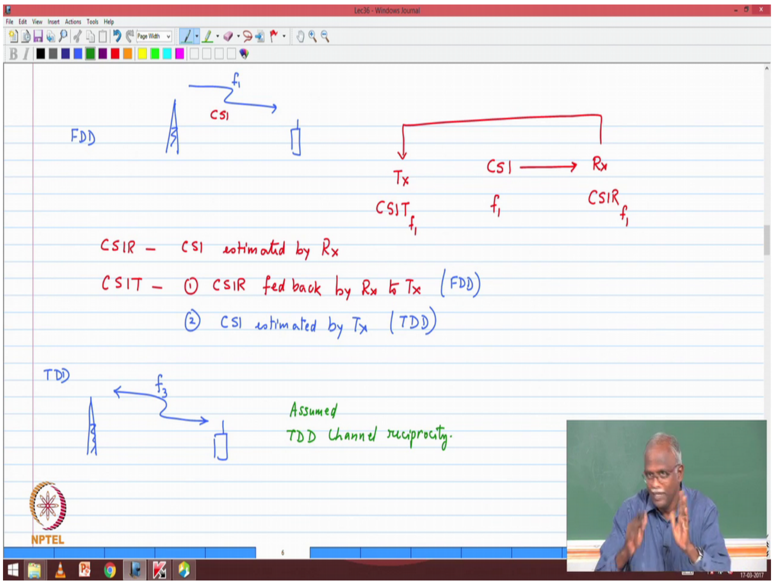

So, now the following are important. Let us take the case where you have an FDD system. Always to visualize think of it as a base station and a mobile the downlink and the uplink are or on different frequencies f 1 and f 2 because they are on different frequencies the channel seen in the uplink and down link are different. So, when I look at the information, I am going to worry about f 2 for the moment. I am going to remove f 2 from the picture keep in mind that the uplink is a different frequency. Now channel state information is an understanding of what is happening in the channel at this frequency f 1 ok.

So, CSI at this frequency f 1 is happening in the channel. Now what happens when my receiver receives the signal? So, at the receiver I will try to estimate the channel. So, then what I get is CSIR our estimated the channel state information at the receiver. Now does transmit know what the channel state is absolutely no idea it has no clue because f 1 is it only sees f 2. So, it knows channel state information at f 2, but it does not know channel state information at f 1. So, the only way that the transmitter is going to know; what the channel state information is if the receiver passes that information. So, this then becomes may be to clarify this is CSIR at f 1 the downlink frequency and one what was fed to the transmitter was the channel state information also for frequency f 1, but it was now the information is available at the transmitter.

So, this is the setup that we have for feeding back the information. So, what information does the transmitter need to know? It needs to know what was the channel experienced by the signal when it transmitted. So, that information can only be given by the receiver on the feedback path. So, we can write down the following CSIR is channel state information estimated by the receiver, estimated by the receiver. CSIT the there are 2 cases one when CSIR is fed back by R x to T x and this is true for FDD systems.

What is the difference? Now comes the TDDs case, TDD case I have the transmitter I have the receiver, TDD system there is only one frequency that is going between transmitter and receiver both are transmitting. So, when the mobile transmit is the base station already knows; what is the channel state information of f 3. So, in the case of TDD that is a second scenario CSIT is actually estimated by the transmitter itself.

So, CSI estimated by the transmitter itself. So, this is for the case of TDD is it clear why TDD is different from FDD, why a CSI has to be fed back in one case and in the other case it does not need to be fed back, is that clear? What is the assumption that you have made? That TDD transmission is reciprocal, what the base station sees channel is the same as what the mobile sees and to a very large extent that is a. So, we can also may be make a comment that you have assumed TDD channel reciprocity, which is a fairly good assumption there is nothing wrong in that channel reciprocity ok.

So, in the nut shell, what are we trying to do? We want the transmitter to know; what is the channel to which it is transmitting, at the end of the day that is the simple thing. While the reason for that is if the transmitter knows it is a good channel maybe it can send more information. If it knows it is a bad channel maybe it sends less information or sends less important information or introduces more array correction code. See that knowledge of how to transmit the information is very critical and that is only possible if the transmitter knows what the channel conditions are. So, when you start now saying I want to understand the capacity of the channel, capacity says- what is the maximum information I can pass through the channel.

So, then you start to think if my transmitter has no information what the channel is it, could be very good channel or very bad channel because of fading it has no idea. How will it design the system? It will design it for worse case it will say I do not know whether is good or bad. So, I will design it. So, that it will it will survive even the bad channel, but that means it is going to send only a limited amount of information, but if it knew in a dynamic fashion what is the current channel conditions then it can optimize the rate it will not transmit at a fixed rate it will be adjusting the rate and because it is able to adjust the rate it will achieve better capacity.

So, CSIT is very, very important because that is what will help us achieve capacity but that is the topic for our discussion today. But before that let us first look at one of the space time codes this is a case where I have 2 transmit one receive now I hope you now you will know what the dimensions of the channel matrix are.

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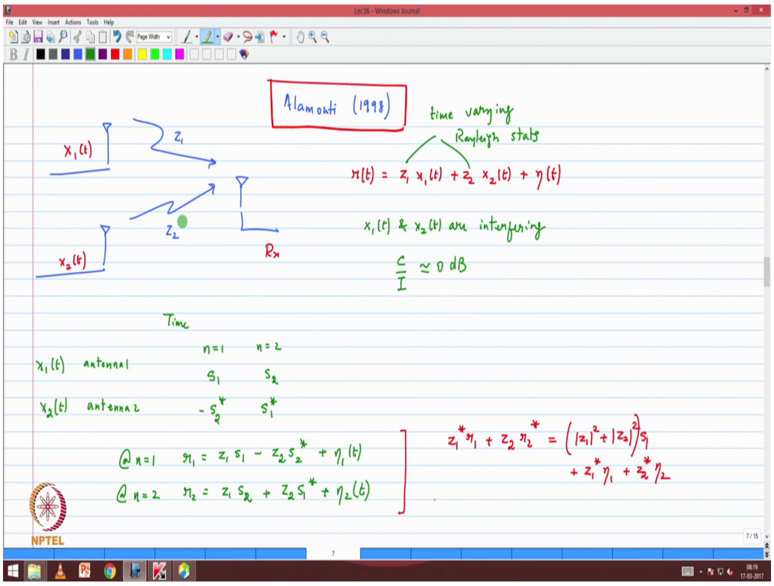

So, that is the Alamouti code, this is where we left off in the last lecture. So, let me pick it up from here 2 transmit antennas each of which are transmitting different signals they are experiencing different independent Rayleigh fading coefficient z 1 and z 2. It is looks like a problem situation because these 2 signals are going to interfere with each other; however, the algorithm that was proposed saying take z 1 star times r 1 and z to times r 2 star seems to have done something, we may be you already have gone through it and figure out what exactly is happening, but basically this is what we got as the output.

So, this is where we pick up our discussion today and we write down the second of those 2 equations; so in the same manner as the previous one.

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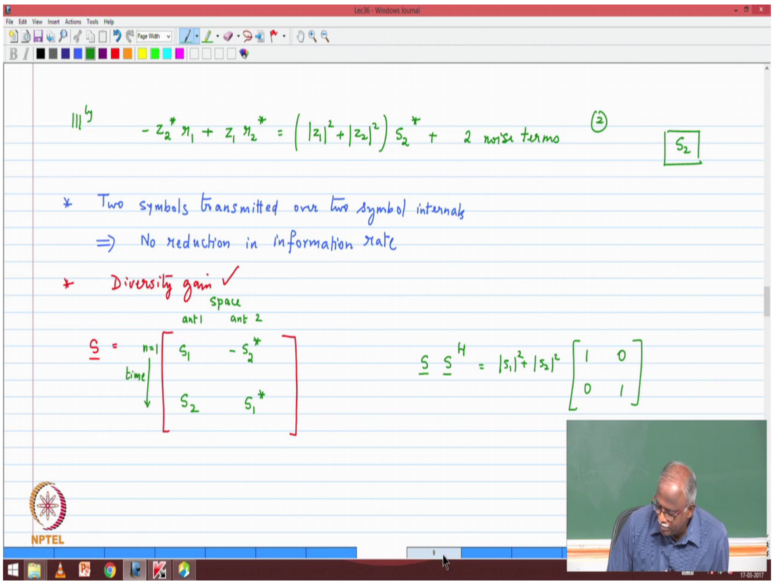

So, there we managed to get s 1 out of the 2 equations. So, if you do minus z 2 star times r 1 plus z 1 times r 2 conjugate, what you will find is that you will get mod z 1 square plus mod z 2 square times s 2 conjugate plus 2 noise terms. Again the noise terms are not important for us all we will make a note is that the noise variance I going to increase by a factor of 2.

So, equation 1 gave us s 1; so this gave us s 1. So, this helps us to detect s 1 and equation 2 helps us detect s 2 helps you to detect s 2 conjugate it is it is trivial basically you can get; that means, you can get s s 2 as well, but notice what is your channel coefficient. The channel coefficient is mod z 1 square plus mod z 2 square even if one of them is in a fade you will still be able to get robust detection.

So, you have gotten diversity gain diversity gain is what helps you when one of the channels is under fading conditions. So, this is a very important result because, what is says it typically we think of diversity as something you can get when you have multiple antennas at the receiver. What this technique said was no that is not necessary you can actually transmit 2 signals from transmit on the transmit side. Traditionally what you would think if I transmit 2 different signals on the same frequency they will interfere each other I will have a big mess at the receiver.

But what this one says is no if you design your transmission carefully then you actually get 2 branch diversity in this system. Everyone is very happy with this result first question they will ask is can I extend it to more if I if I use 3 antennas can I have 3, 3 branch diversity 4 branch diversity. So, it turned out that this was not easy to generalize.

So, again, but first let us first understand what are some of the key benefits, why was this such a important contribution to the field of communications? So, first thing we always ask is when you get something you say- what is the price that I paid. Is because there is a basic tenet which says you do not get anything for free somewhere the there is a price you have to so, what is the price that we pay. So, here we said how many symbols did you actually transmit 2 symbols in 2 instances 2 symbols transmitted over 2 symbol periods. So, I did not compromise on the symbol rate right that is very, very important transmitted over 2 symbol periods, that is very important because if I transmitted 2 symbols and I took 4 symbol periods then that is I wasted bandwidths.

So, this is something which is an important. So, over 2 symbol intervals. So, I have not increased by bandwidth I have not lost rate. So, that is an important observation, there is no reduction in the information rate, in information rate that is an important one. But I obtained diversity gain. So, where did it come from? Where did it come from? But diversity gain yes diversity gain is very much present ok.

So, we then go back and ask the question, now where is the secret what is what is happening? This turns out to be the insight and the power of space 10 codes. So, let us take a look at so, if you want to visualize it, this is what is happening. So, the signal that was transmitted is the following. So, let us draw it in an expanded form. So, that it will easy for you to make the additional notations. So, I am going mark time on this axis time and it is increasing from top to bottom. This is space so, whatever I am writing on the first column is the symbol transmitted by antenna 1 transmitted by transmitted symbol by antenna 2.

So, please fill in this matrix. So, antenna 1 at time instant n equal to 1 transmitted s 1 at time instant n equal to 2 transmitted s two. But on the other hand antenna 2 transmitted minus s 2 conjugate and transmitted s 1 conjugate. So, instead of just transmitting s 1 and s 2 you over a 2 symbol period you seem to have transmitted a 2 cross 2 matrix. But it is very important to note that this matrix is not just any matrix it is got a very important structure which is the orthogonality structure. S times s hermitian or if you want to take s hermitian times s you will find is equal to mod s 1 square plus mod s 2 square times the diagonal matrix it is got a very, very important structure.

So, it is a non-trivial structure it is may not seem very obvious, but this is a very important form that we know in signal processing as a orthogonal matrix. So, basically if you take one as so, cosine one as sin and the other as cosine you will find that it satisfies this type of a structure ok.

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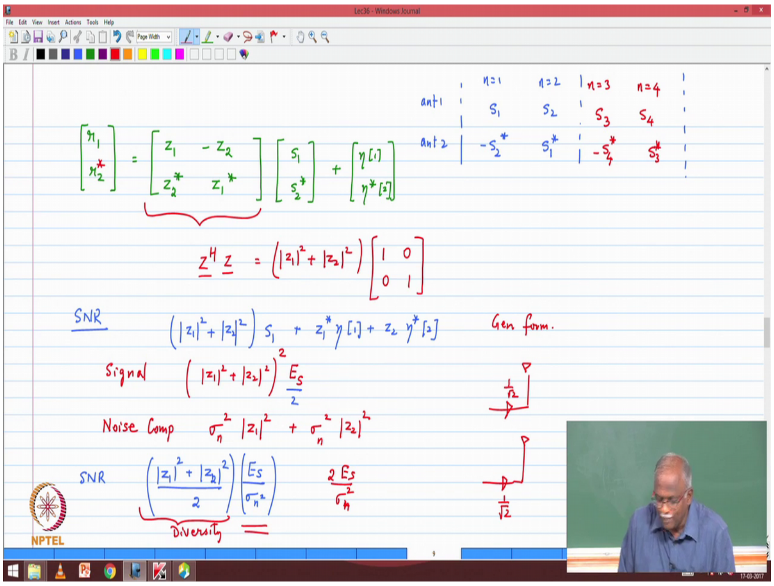

Now to do some little bit more linear algebra because lot of this insights come in become very handy. I want you to write down r 1 and r 2 the 2 signals that are received by the antenna, receive antenna there is only receive antenna at 2 instances of time I want you to write it down in matrix form. And the matrix form representation is very useful and important for us.

So, r 1 if you go back and look at it I want you to write it down in terms of s 1 and s 2 conjugate. This turns out to be z 1 minus z 2, r 1 is z 1 s 1 minus z 2 s 2 conjugate which is what you would if you go back and look at it plus eta 1. That is the first equation. The second equation r 2 basically we saw r 2 conjugate, but you can write down r 2 becomes equal to z 2 conjugate z 1 conjugate and s 2 s 1 s 2 conjugate and this one becomes n 2 conjugate n, let us put with in brackets n 2 conjugate. So, please verify that the equations that we got are of this form. Notice that what we have constructed also is a matrix that has that orthogonality property built in to it because if you do z hermitian times z what you will get is mod z 1 square plus mod z 2 square into the identity matrix 1 0 0 1.

So, what did we actually do in the process of re detection process? We took z hermitian and then multiplied it with the r matrix and we found that the equations that that aware of interest are you know basically at one time instant you get s 1 the next time instant you get s 2 and you get the a desired detection mechanism. So, this is this is a very simple 2 2 cross 2 system where orthogonality was built in 2 the transmission process. So, that you could then detect the signal without any complicated signal processing without having to do any interference cancellation basically it a the orthogonality structure gave you the interference cancellation as part of the computation.

Student: Sir.

Yeah.

Student: First equation which you teach over yesterday, sir.

Is it not? Let me just take the look at it.

Student: Yesterday (Refer time: 30:35).

Z 1 s 1 minus z 2 s 2 conjugate.

Student: Sir, second equation sir r 2 conjugate.

R 2 is z 1 star z 1, this should be a conjugate. Thank you yes. Is that correct everything? Yes, it should be r 2 conjugate you are right. Because only then I will get z 1 star s 2 star z 2 conjugate s 1 yes correct it is a r 2 r 2 star that is correct, but the z matrix is correct and so I probably you will have to work with I will have to I will have to check this equation there is this receiver equation I would not write this down, but basically the the this that the received signal is correct if you put the conjugate sign I am thank you for pointing that out.

Now, I want us to analyze the system that we have designed. First of all I want to understand what is the I have basically I am trying to get the signal to noise ratio. So, I want to compute the SNR, compute the SNR. So, if this is the final form that we are getting the form that we are getting is of the following structure where let me just take one of the 2 equations what we have is mod z 1 square plus mod z 2 mod z 2 square times s 1 plus z 1 star eta of 1 plus z 2 eta conjugate of 2. Now I want to basically it you get several equations. Now what happens in time instances 3 and 4? In case we so, time instant one and 2 this is what we transmitted.

So, antenna 1 antenna 2 time instant n equal to 1 n equal to 2 that is all we have specified so far it was s 1 s 2 minus s 2 conjugate s 1 conjugate. Repeat the same structure going in to time n equal to let me use a different color n equal to 3 n equal to 4, this would be s 3 s 4 minus s 4 conjugate s 3 conjugate.

So, you would detect 2 symbols at a time and basically you get the benefit of diversity. It is not the symbol by symbol detection, but it is 2 symbols at a time, but basically every time you want to detect the symbol your equation is of this form. This is the general form of the equation. So, there is a signal term which is mod z 1 square plus mod z 2 square. Then there is z 1 conjugate times a noise term z to conjugate times a noise term and I want to now understand what is the signal component. Signal component will be mod z 1 squared plus mod z 2 squared if I take it as a scaling of the amplitude of the signal when it comes to energy or power it will become square of that times whatever was the energy of the signal s 1.

So, whatever was the signal energy scale factor times the square of that. What is the noise component? Noise component it is sigma n square that is the noise term, but because you multiplied it by mod z 1 I have to scale it b mod z 1 square the variance will go by the by the magnitude square. And similarly there is the second noise term sigma n square mod z 2 square that would be the effective noise variance.

Now, one thinks that we should not forget. What did we do I transmitted from 2 antenna. So, I must not forget this 1 by square root of 2 because that is what makes that I am transmitting with the same total transmitted power. That do not just keep in mind that factor of 2 should not be should not be omitted. So now, comes the competition of the effective SNR that we will. So, by the So, this should be E s by 2 E s is when you transmitted all the power through one antenna this one would have an s 1 by root 2 sitting there, because you did the scale factor of root 2 and then when you actually compute it will be E s by 2. So, SNR will be the ratio of the signal component to the noise component which comes out to be mod z 1 square plus mod z 2 square divided by 2 that is that by 2 factor E s divided by sigma n square.

So, we have shown the following result that it has diversity gain, Alamouti scheme is the transmit diversity scheme. It enables you to get the diversity gain without any complexity at all. Basically you have to detect 2 symbols at a time and you are able to get it with simple matrix manipulation. However, did you get array gain? So, this is the diversity gain, right? This is diversity gain, diversity gain. If you had to if you had to obtained 2 times E s by sigma n square the sigma n square then that would have been array gain as well, but definitely you did not get there is no there is no 2 in the numerator is basically this looks like the SNR of a single antenna.

So, what is happening to array gain? So, this is where I want you to think about array gain in a sort of an intuitive manner.

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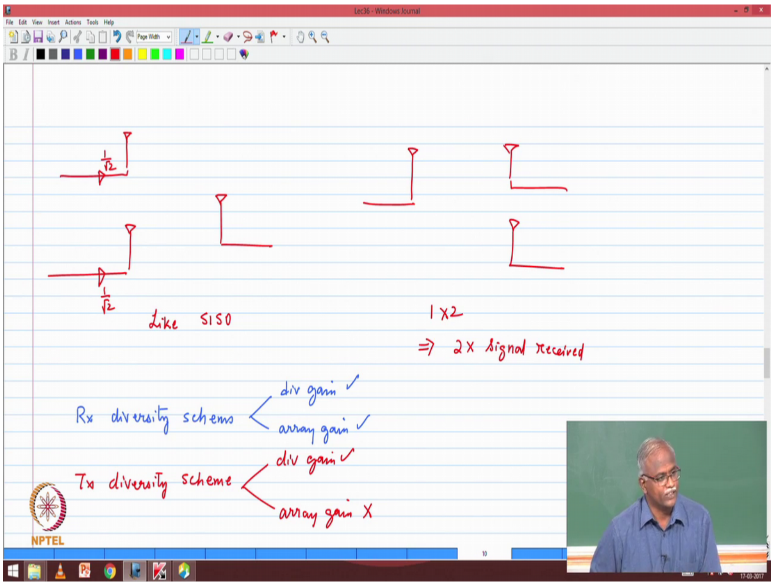

So, what did what did Alamouti scheme do like most transmit diversity schemes? It had to preserve the total transmit power. So, that is why this 1 over root 2 is there. So, the total transmit power that is into entering channel is constant. And how many antennas do we have? Only one antenna whatever I can intercept with one antenna is what I can benefit from the receiver. The total transmit power has not increased.

So, effectively this looks like a SISO channel single input single output system from a power point of view, right? Because the total power coming in is the effective power of one antenna what is a getting used at the receiver is that that of a single antenna. On the other hand if I had received diversity I have one transmitter I have 2 receivers is this any different. From a diversity point of view no I still will only get 2 diversity channels and if I can get mod z 1 square plus mod z 2 square diversity benefit is obtained, but notice I transmitted with a certain amount of power. But I have antennas that are listening to the signal so; that means I can pick up twice the amount of energy that I will pick up with the single antenna.

So, this is a 1 cross 2 systems which basically implies 2 times the signal that I can pick up. Signal that is intercepted or that is received, intercepted is because you are antenna is intercepting or may be just write signal received. 2 times by signal received by single antenna and that is why received diversity gives you array gain as well. Because there are more antennas picking up energy it is picking up energy. So, it gives you diversed array gain because there are uncorrelated antennas it is also giving you diversity gain, but if you cannot afford or complexity wise you cannot put the multiple antennas at the receiver Alamouti scheme says no problem I can do it for you at the transmitter I can get you the diversity gain, but I cannot give you array gain that is that fair enough, because you know I could not put in the multiple antennas.

Therefore, a good statement to summarize would be R x diversity schemes. R x diversity schemes will have diversity gain yes, they will have array gain yes. Now how much array gain? How much diversity gain depends on which diversity scheme you have implemented? On the other hand T x diversity schemes because I have to conserve power I have to do the scaling of the signals. This diversity gain yes, array gain no. Only diversity gain no problem it is it is a, but still never in a fading channel diversity gain is a huge advantage any way.

So therefore we will take that as you know bonus. Any questions? Topic that kind of emerges more or less in a very natural fashion from what we have been discussing so far.

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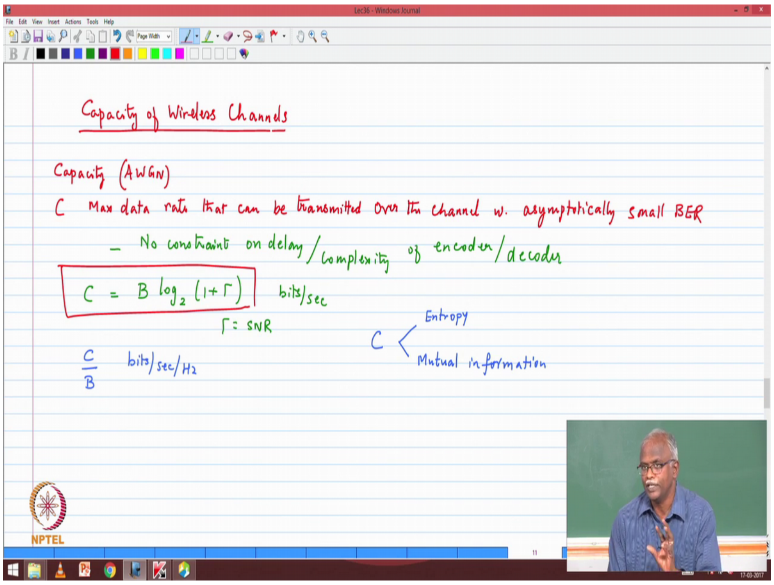

It is a topic of capacity of wireless channels. A quick quiz on capacity that you have would studied in digital communications, contributed by Claude Shannon in the year 1948, 49 what is the 2 lab papers mathematical theory of communication and communications in the presence of noise, those where the 2 papers. What is a if you were to give me the channel capacity theorem what is that?

Student: (Refer Time: 41:05).

There is a number c below which you can transmit with almost 0 probability of error, but complexity is not specified it can be very, very complex system, but it if you cross that capacity c there is no way you can get low probability of bit error basically you will get very high bit error rate ok.

So, very good; so the capacity our notion of capacity is primarily from the context of AWGN channels. The understanding is that there is a capacity number c which represents the maximum data rate at which you can transmit you can communicate. Reliably maximum rate that can be transmitted over a channel over that particular channel AWGN channel transmitted over the channel, over the channel with asymptotically small BER, with asymptotical small BER, ok so almost 0 BER.

Now given that and of course, there is no guarantee no nothing has been mentioned. So, there is no constraint on how much processing you will have to do and what delay it would cause no constraint on delay or complexity. And this delay and complexity can be at both ends it can be at encoder or it can be at decoder. These are you know it is a very, very open ended statement all it says is you can communicate reliably.

So, always we are interested to know what is the capacity of a given of a given channel. So, we know from that Shannon's channel theorem, c is given by it is proportional to bandwidth, b is the bandwidth of the channel logarithm base 2 1 plus SNR. Gamma is your SNR and the unit is are bit is per second that is the capacity theorem. And very often we also look at c divided by b capacity per unit bandwidth and in that case, it will become bit is per second per hertz and that is a good way to compare different systems because of them, may be 30 kilo hertz the other one may be 2 100 kilo hertz. How do you compare the 2? Look at the normalized capacity per unit bandwidth ok.

So, again the derivation of the capacity involves 2, 2 quantities. One of them is the entropy. So, the derivation of capacity depends on the definition and understanding of entropy and also how entropy is related to mutual information. I will assume that this part of it is a familiar that in of course, for us the key equation that we are interested in is what is the capacity number. So, first and foremost what is the impact of this capacity I want to get want to want you to have a intuitive feel for that.

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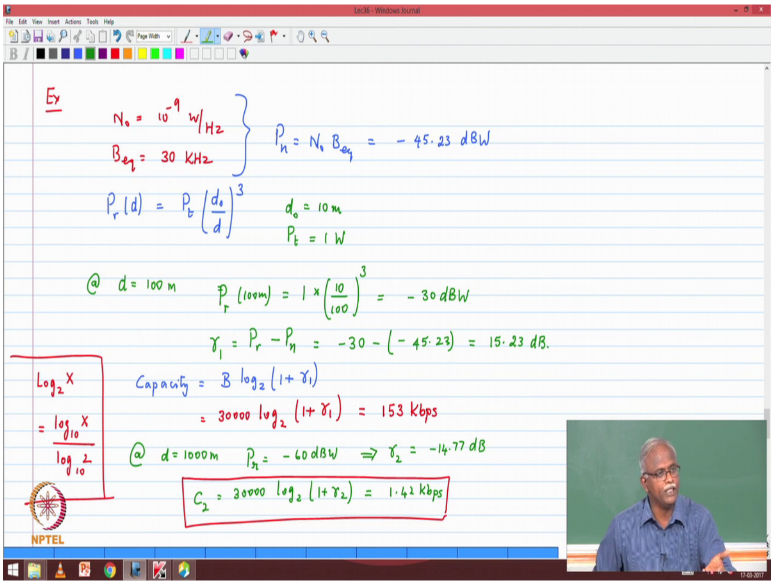

So, let us take an example, examples always help us capture the importance of a particular concept. So, the first thing is supposing I were to give you I want to translate

Shannon's capacity in from a theoretical concept into very practical concept. I set up a transmitter I have a receiver there is a certain distance between them propagation loss some SNR is received at the receiver that is the scenario.

So, here it is a very simplified model the noise spectral density is 10 power minus 9 watts per hertz. The equivalent bandwidth of my system is 30 kilo hertz. The combination of these 2 tells me that I know what my noise power is, because noise power is n not into b equivalent, into b equivalent that is given I have multiply the 2 minus comes out to be minus 45.23 dBW that is my noise floor that is my noise floor. Now we are told that the propagation equation that received signal at a distance d is given by basically the break point model P t times d not by d the whole cubed path loss exponent is 3. And we are given that the break point is 10 meters and the transmit power is one watt ok.

So, now simple calculation I want to know what is my SNR at d equal to 100 meters. I would expect that this would be something that you will be able to do very easily. So, this would be equal to I am first of all I need to compute the received signal power p r at 100 meters that would be 1 watt in to 10 divided by 100 meters raise to the power 3, that if I convert into dBW ill be minus 30 dBW . So, my SNR gamma 1 will be signal power minus the noise power which will be minus 30 minus of minus 45.23 which is 15.23 dB. Up to this nothing new. Basically go back to the propagation model plugged in the values obtain the SNR of the channel.

Now, comes the important one. Now if this is the channel that you have what is the capacity of the channel. So, than we say I can tell you that not a problem because capacity is equal to the bandwidth times logarithm base 2 1 plus SNR gamma 1 log 2 we have to calculate everywhere log 2. So, log 2 log x base 2 is the same as log x base 10 divided by.

Student: Log 2.

Log 10 log 2 base 10 please that is easier way make sure that. So, you have to calculate log base 2. So, it will be 30 kilohertz logarithm of 2 15.23. I have to convert it into linear scale and then basically compute the logarithm base 2 of 1 plus gamma 1 and please verifies that this comes out to be approximately 153 kilobit is per second ok

Now, why are we spending So much time on this? For the following simple reason. Supposing I moved my receiver to a distance of 1 kilometer one 1000 meters basically, from 100 meters I moved it to 1 I am transmitting with on watt right this I am I am not transmitting with low power. So, now, calculate the capacity. So, what you will find is that the received signal power is now less it turns out to be minus 60 dBW. So, which means my new SNR let me call it as gamma 2 comes out to be minus 14.77 dB it is actually negative now.

So, one this new position is not a good one at all for me from a transmission point of view, but did the channel Shannon’s capacity also indicate that yes, if you if you calculate c 2 as 30000 logarithm base 2 1 plus gamma 2, it comes out to be 30 can we use 30 kilohertz what would you expect as the nominal rate of the symbol at least 30000 kilo bit is per kilo bar right. And if you use one bit per second 30, 30 kilobit is per second please note it is 1.42 kbps as predicted by Shannon’s capacity and this 2 with infinite complexity

So, why is this discussion very important, because if somebody gave me this problem statement and said- hey I know large scale propagation. I know propagation I know channel models I know how to calculate break point model no problem, I would have said hey no big deal I can predict for you what is it you want to transmit 20 kilobit is per second no problem 1 kilometer 1 kilometer. What is the capacity it is 1.42 kilobit is per second even if you do infinite complexity you will not get any transmission useful transmission.

So, this is why capacity is very important we have been able to predict SNR, but we now need to know how much information can I transmit. And how do I maximize that information that goes through, why because this is only the average SNR, remember?

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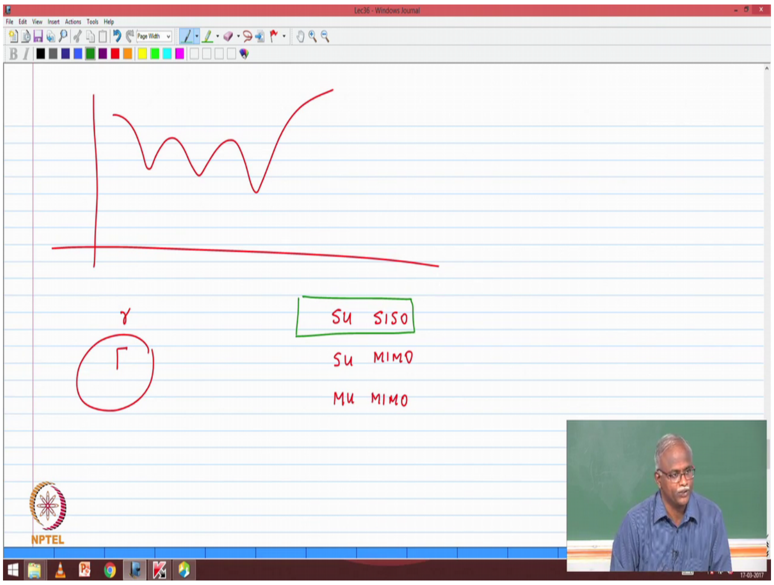

What is the scenario in Rayleigh fading? Does not a constant SNR; so if I were to ask you what is the capacity of this fading channel, what will you say? Tell me what is a instantaneous SNR, instantaneous SNR is gamma average SNR is upper case gamma. This is of no use at all for me in calculating capacity because SNR is fluctuates.

So. In fact, a Rayleigh fading channel capacity, what is the capacity? High now, low now, high now, low now, that is that is a channel that you have. Now the problem statement is get me the maximum capacity out of this channel. That is the, now we start to say I know how Rayleigh fading fluctuates, I know I know Shannon's capacity theorem and then now I say put this 2 together now maximizing capacity in a wireless channel is not at all trivial. It is going to be something which is very dynamic, I must know how to work with this channel.

So, first of all if it is a SISO channel; it is a tough problem, if I now say it is a SISO channel but with multiple antennas. Basically it is a sorry, it is a single user single users SISO channel. That is that is the simplest of the cases that we can deal with that is what we will deal with first. Second complex the difficult situation will be single user MIMO, if I allow the user to have multiple transmit multiple receive, because you know already Alamouti showed us that you can you get a certain benefit from doing the MIMO system.

And then of course, the most complex which will be in an advanced course not in this one will be multi user MIMO. If you know have multiple users who are accessing the channel and then you get the benefit. So, what we will be focusing on is the single user SISO channel and single user MIMO channel, but that itself is a very interesting problem by itself.

So, that is what we will address in the course of next week, and in the lectures to come.

Thank you.