

Introduction to Wireless and Cellular Communication
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Lecture - 36
Fading Channels – Diversity and Capacity
Array Gain, Diversity Gain, Alamouti Scheme

Good morning. We begin with the review of lecture 34. But today will be studying some very interesting new topics. First of all I would like to introduce 2 terms or 2 concepts which are very important, very useful for us in a understanding of diversity. One of them is called array gain the other one is called diversity gain. It seems like we are using the same names over and over again, but there is some interesting and important differences that help us in our understanding of the whole concept of diversity.

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16/3/2017 EE5141 Lecture 35

- Recap L34
- Comparison - SC, EGC & MRC
- Array Gain & Diversity Gain
- Numerical \int for evaluation of BER
- Alamouti Code (1998)
- Channel State Information (CSI)

Wireless Channel Capacity (Goldsmith ch 4)

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Moving from receive diversity we will now talk about transmit diversity. Alamouti code is a very famous in discovery 1998 which showed us that you can achieve the benefit is of diversity without multiple antennas at the receiver and then we will move from that to the next unit of study. And the next unit of study is on understanding the capacity of a channel. This is very closely linked to Shannon's capacity of a AWGN channel.

However, we are now talking about a wireless channel and that is our goal is to study the capacity of a wireless channel Shannon capacity as applied to a wireless channel. So,

there are several interesting variations from what you would have already studied in a digital communication.

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Lecture 34 Recap

Diversity

Lec 35

$$Y_{MRC} = \sum_{k=1}^M Y_k \quad E[Y_{MRC}] = M \Gamma T_x$$

$$Y_{SC} = \max\{Y_1, \dots, Y_M\}$$

$$E[Y_{SC}] = \Gamma \left[1 + \frac{1}{2} + \dots + \frac{1}{M}\right]$$

E_{GC}

$$G_k = c \cdot \frac{2\epsilon}{|Z_k|}$$

$$Y_{EGC} = \frac{1}{M} \left[\sum_{k=1}^M \sqrt{Y_k} \right]^2$$

Diagram:

- Diversity
 - Tx
 - Rx
 - SC
 - pre
 - post
 - EGC (co-phasing)
 - MRC (optimal)

But first let us look at a quick summary of the points that we have covered so far. And in any concept it is always good for us to sort of have a big picture. So, the big picture of diversity looks like this.

So, the concept of diversity the benefits of diversity or the ways in which we will exploit diversity are 2 fold, one is transmit diversity the other one is receive diversity. So far we have been only talking about receive diversity, today's class we will talk about transmit diversity. But in receive diversity, we have talked about 3 different forms we have talked about the optimal form which is maximal ratio combining. The simplest form which is selection combining and then something in between which is equal gain combining. And in selection combining we said there were there was pre detection and then there was post detection ok.

So, several flavors that we have talked about; and we make the note that this is the optimal; this is very close to optimal. So, this is a something which is quite attractive we have also used the name co phasing for equal gain combining. So, keep the names and the relationships all together. This is the selection combining as you know is the simplest of the ways of we are doing it does gives us benefit, but not as much as the once that are close to optimal.

So, that is that is sort of the broad frame work the specific results from yesterdays lecture. The optimal combining method for diversity gives us the diversity SNR as the sum of the individual SNRs K is equal to 1 through M γ_K . Expected value of γ_{MRC} is equal to M times γ , that is a very important result. Then we saw that selection combining we choose the maximum of the antennas that are SNRs that we have available to us γ_1 through γ_M and we have shown that expected value of $\gamma_{selection}$ is equal to average SNR of one antenna plus 1 by 2, all the way to 1 by M since that are M antennas.

And in the last class we also said that we would like to look at a sub optimal method which did not involve the estimation of the noise variances because MRC requires us to have the optimal combining coefficient equal to $Z K^*$ divided by σ_n^2 whole square. It requires us to have that quantity and therefore, the estimation of the noise was the challenge

So, equal gain combining did avoided that it said that we will use gain terms which did not involve the noise variance. So, some constants scale factor does that does not affect us, $Z K^*$ divided by σ_n^2 . So, basically that is only the phase and because of the conjugations it is a negative of the phase of the gain term. This does the co phasing for us, we showed that the SNR of EGC through the steps that we showed in the last class this comes out to be an interesting number 1 over M summation K equal to 1 through M square root of γ_K whole square.

And specific examples we can show what this value is and so the, concluding statement about received diversity. We can say as follows, in general γ_{MRC} is the best.

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Lec 35

$$\gamma_{MRC} \geq \gamma_{EGC} \geq \gamma_{SC}$$

$$\sum_{k=1}^M \gamma_k \quad \frac{1}{M} \left[\sum_{k=1}^M \sqrt{\gamma_k} \right]^2 \quad \max \{ \gamma_1, \dots, \gamma_M \}$$

$$M \Gamma \quad \Gamma \left(1 + (M-1) \frac{\gamma}{\gamma} \right) \quad \Gamma \left(1 + \frac{1}{2} + \dots + \frac{1}{M} \right)$$

$$\int_0^\infty p_E(\gamma) p_Y(\gamma) d\gamma$$

depends on type of diversity

So, we will say that it is greater than or equal to gamma EGC, in general is strictly greater than no harm in saying that it is or always as good or better than selection combining. This obtains the sum of SNRs K equal to 1 through M gamma K this one achieves 1 over M summation K equal to 1 through M square root of gamma K raise whole square. And the last one is maximum of gamma 1 through gamma M.

So, M antennas gamma 1 through gamma M instantaneous values, some of them may be at a good value some of them may be not. So, good MRC combines all of them to get the maximum benefit. EGC does something very similar except it makes the assumption that noise variances are the same. It is not a correct assumption in all scenarios, but it is a good enough for it is an approximation and of course, selection diversity says I will only pick the best. I do not worry about the ones that are not so good.

So, again each of them have got certain advantages disadvantages, but at the end of the day, what is the net benefit or average benefit? I will see an average SNR of M gamma here, in this case I will see something which is close to M gamma 1 plus M minus 1 into pi by 4 and this one would gamma into 1 plus 1 by 2 plus 1 over M that is a summary statement.

So, hope your now confident that once we have the antennas then we know of the benefit is are going to come about, we know how to get the probability of bit errors. So far we have looked only at the maximal ratio combining, but it does not matter which ever is the

method of diversity combining that is given to us. The key thing that we will need is the probability of error, probability of error as a function of gamma. Whichever is the combination scheme that you are using f gamma of gamma integral 0 to infinity d, d gamma this will be the probability of error in fading.

So, this depends on the type of diversity that we are going to be using depends on the type of diversity. So for selection diversity the PDFs can be obtained for the equal gain combining we have not derived it in the class, but the corresponding PDFs are available likewise for MRC as well depending upon the type of diversity.

So, once you are given that you can then perform the integral or you can use the moment generating function and all of these methods are now tools that are available to us. There may be times when you want to do numerical integration because the integral itself is very difficult to solve and you may say I need to get a quick feel for the numbers, so therefore, you may do numerical integration. So, here is a useful tool for doing numerical integration.

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The image shows a handwritten derivation on a digital notepad titled "Numerical Integration". The derivation starts with the BPSK error probability $P_{e,BPSK} = Q(\sqrt{2\gamma})$. It then expresses the Q-function as an integral: $Q(a) = P(X > z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{x^2}{2}} dx$. An alternative expression for the Q-function is given as $Q(a) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{a^2 \sin^2 \phi}{2}} d\phi$, with notes indicating a finite range $[0, \frac{\pi}{2}]$ and that the range of integration does not depend on z . The derivation then substitutes this into the error probability formula, leading to $P_{e,BPSK, fading} = \int_0^{\frac{\pi}{2}} Q(\sqrt{2\gamma}) f_\gamma(\gamma) d\gamma$. This is further manipulated to $P_{e,BPSK, fading} = \int_0^{\frac{\pi}{2}} \left[\int_0^\infty \frac{1}{\pi} e^{-\frac{\gamma \sin^2 \phi}{2}} d\gamma \right] f_\gamma(\gamma) d\gamma$. The inner integral is identified as $\Psi_\gamma(s)$ with $s = \frac{1}{2} \sin^2 \phi$. The final expression is $P_{e,BPSK, fading} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Psi_\gamma\left(\frac{1}{2} \sin^2 \phi\right) d\phi$, with a note stating " $\Psi_\gamma(\gamma)$ is known". The slide is labeled "Lec 35" and "4" in the top right corner.

So, again this is only meant as a to complete the picture and our understanding that, yes integration is good integration is gives us close form, but you do not necessarily have to do it you can do numerical integration. So, here is a method that is quite useful for us. Now supposing you were asked to find out the probability of error of BPSK. So, probability of error of BPSK is Q function, Q of root 2 gamma. And Q itself is an

integral. So, Q of Z is a probability that a normalized random variable is greater than Z that is given by $\frac{1}{\sqrt{2\pi}} \int_Z^\infty e^{-x^2/2} dx$.

So, basically Q itself is an integral and if I want the performance in fading it is going to be probability of error of BPSK in fading. I will now have to include the PDF of the fading, fading SNR and compute another integral. So, this will be equal to 0 through infinity Q of $\sqrt{\gamma}$, again this is all familiar stuff just want to refresh, $\int_0^\infty f(\gamma) d\gamma$, ok.

Now, actually these are a little bit tricky to interpret in the context of (Refer Time: 11:06) numerical integration, because these are limits that are going towards infinity. So, it is always good for us to have finite bounds when we want to do an integration. So, here is a way by which we can achieve the finite integration. So, here is the first step. Step one says that there is an alternate expression for Q of Z , So alternate expression for Q of Z .

The alternate expression says Q of Z can also be written in the following fashion. It is an exact expression $\frac{1}{\pi} \int_0^{\pi/2} e^{-Z^2 \sin^2 \phi} d\phi$. Now we will take this expression without proof, because again the intent is to show that there is a difference the first form is an infinite integral, the other one is this is a finite integral and these are exact expressions.

So, the advantage is that this is a finite range, finite range 0 to $\pi/2$, 0 comma $\pi/2$. So, again if you want to implement it in some form of numerical integration like the trapezoidal rule, this would be that and the range does not depend on Z . So, that is another important so, the range of integration the previous one the range of integration depends on Z . Range of integration does not depend on Z . That is also another observation does not depend on Z , Z is your argument of the Q function ok.

Therefore, this is a form that is useful for numerical integration. So, here is what we would do, we would now see if we can substitute this. So, integral 0 through infinity that is the outer integral the substitution for Q of Z $\frac{1}{\pi} \int_0^{\pi/2} e^{-Z^2 \sin^2 \phi} d\phi$. Just needed to make sure because the $\sqrt{\gamma} \sin^2 \phi$. That is the expression for Q of Z and then the other terms, let me write that in blue $\int_0^\infty f(\gamma) d\gamma$, ok.

So, I just substituted the expression for the new second alternate expression for Q of Z. Interchange the limits what you will find is $1 - \sin^2 \phi$ comes out, it is a constant. Integral 0 to $\pi/2$, the inner integral is 0 through infinity, $\int_0^\infty \Gamma^s E^{-\Gamma} \sin^2 \phi d\Gamma$ and then $d\phi$ outside. So, take a closer look at the integral, what is this? Looks like the moment generating function, right? Except that you have chosen a specific value of s . So, this is nothing but the moment generating function of Γ , $\int_0^\infty \Gamma^s E^{-\Gamma} d\Gamma$. Evaluated at s is equal to $1 - \sin^2 \phi$. So, the integral now becomes, this becomes equal to the integral $1 - \sin^2 \phi$ integral 0 to $\pi/2$, the moment generating function Γ gamma of $1 - \sin^2 \phi$ and it is $1 - \sin^2 \phi d\phi$.

So, if you know the moment generating function of the SNR then choose values of ϕ in the range 0 to $\pi/2$, whatever's integrals that you want a spacing that you want and evaluate the moment generating function at these points, at $1 - \sin^2 \phi$. And basically that becomes your numerical integration step and it is very straight forward because once Γ of Γ is known. So, we assume Γ of Γ is known, for us it is known for most of the common PDFs that we encounter, the types of fading that we encounter and also known for all the different forms of a diversity combining. So, once this is known ϕ is fixed range from 0 to $\pi/2$, you can quantize it in whichever number of steps you want and then implement the numerical integrations ok.

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The slide displays the following mathematical expressions:

$$\Psi_{\Gamma_{MRC}}(s) = \left(\frac{1}{1 - s\Gamma} \right)^M$$

$$P_{e, BPSK, MRC} = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1 + \frac{\Gamma}{\sin^2 \phi}} \right)^M d\phi$$

Goldsmith 6.2 Q - alternative exp

The slide also features the NPTEL logo in the bottom left corner and a small video inset in the bottom right corner showing a lecturer.

So, this is a good way for us to, for us to implement, so just as one last observation- ϕ gamma MRC of s , what is that? What is the moment generating function of maximal ratio combining? We derived it in the last class. It is $1 / (1 - s \gamma)^M$. So, if somebody were to ask you probability of error of BPSK in the presence of maximal ratio combining of course, you can do some very elegant integration we will show, you show how that can be done. But you can also do simple numerical integration of the form $\frac{1}{\pi} \int_0^{2\pi} \frac{1}{1 + \gamma \sin^2 \phi} d\phi$, right? Few lines MATLAB program and you will get the performance of the bit error rate in maximal ratio combine ok.

So, again it is for you to have all the tools available. So, that you can quickly estimate what the bit error rate is, what the performance is and for those of you who would like to study this in more detail please look at Goldsmith chapter 6.2 and also Goldsmith chapter 7, these are some very useful the Q function approximation is in 6.2 this is where the Q alternative expression is and then 7 talks about the diversity methods.

So, hopefully this is this sort of completes the picture, we can do integration, we can do simulation, we can do numerical estimation all of the ways in which and all of them should give us the same result and all of us all of them gives us a very good insights into the topic that we are trying to study. If not we move into another very interesting concept the concept of today lecture. So, the first question that I want you to think about diversity just following statement is beneficial in fading true.

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Diversity is beneficial in fading ✓ T

AWGN $r_1(t) = s(t) + \eta_1(t)$
 $r_2(t) = s(t) + \eta_2(t)$
 $r(t) = r_1(t) + r_2(t) = 2s(t) + \eta_1(t) + \eta_2(t)$

Avg SNR $\frac{4E_s}{2\sigma_n^2} = 2 \left(\frac{E_s}{\sigma_n^2} \right)$ 3 dB benefit! Array Gain

Single antenna $\frac{E_s}{\sigma_n^2}$

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You would agree always yes under any condition is it not useful, always true, diversity in AWGN is there a benefit.

Student: Cannot get better than AWGN.

Cannot get better than AWGN; so why do you want to have put? So, let us see if that is statement is correct I agree with you AWGN is a mildest of the forms of channel. So, here is a AWGN scenario, $r_1(t)$ is $s(t)$ plus $\eta_1(t)$, that is my first antenna. Second antenna also AWGN system so, is equal to $s(t)$ plus $\eta_2(t)$.

Supposing, I were to add $r_1(t)$ plus $r_2(t)$, what will I get? I will get 2 times $s(t)$ plus $\eta_1(t)$ plus $\eta_2(t)$, may be our mind is already thinking well you gained a factor of 2 for the signal, but you already added 2 noise terms may be the net result is useless let us see what is the average SNR. Average SNR I must square the coefficient of the signal. So, therefore, that gives me 4 times the whatever is the energy of the signal E_s total noise power is 2 times σ_n^2 2 noise terms that is equal to 2 times E_s by σ_n^2 , this is the SNR of a single antenna of a single antenna. So, what did you get you got a 3 dB benefit.

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The slide contains handwritten notes on a lined background. At the top, it says 'Multiple Antennas'. Below that, it lists '1. Array Gain (Beam forming Gain)'. A green arrow points from this section to a diagram of an antenna array with a main lobe. The notes include:

- * Inc avg SNR (even w/o fading)
- * Occurs for all diversity techniques (SC, MRC, EGC)
- Max for MRC

 A formula is written: $G_{array} = \frac{\Gamma_{div}}{\Gamma} = \frac{\text{Avg Combined (div) SNR}}{\text{Avg Single Branch SNR}} = \frac{M\Gamma}{\Gamma} = M$.
 Below this, it says '2. Diversity gain' with notes:

- * Over and above array gain
- * fading environments
- * Combining the antenna signals \rightarrow more favorable pdf for Y

 At the bottom, it says 'pdf "improves" $P_e \downarrow$ Outage \downarrow '. There is an NPTEL logo in the bottom left corner.

So, why not take 3 dB? 3 dB is quite valuable in any channel. So, there is a 3 dB benefit. So, even in AWGN channels there is some benefit for the diversity. Now these benefit which comes whether you have fading or not is what we refer to as array gain. Why because I have an array of detectors, they add they all of them are picking up the signal I will explain in a minute so let me just write it down, this is what is called array gain. Now when we do averaging, supposing you have a signal that is very noisy, how do you remove the noise? You average it, right? What do you do you are adding several versions and then dividing when the scale factor does not matter.

Now, what happens, what is a, what is the process that is happening? The signal is adding coherently the noise is non-coherent. Therefore, the net benefit is that the noise variance goes down by a factor of M. So, instead of in so, you can think of this you can interpret. Let me just give you an interesting way to look at this, this expression I can write it as E_s divided by σ_n^2 by 2, right? How will you interpret it? You will say same signal was transmitted, but it looks like the noise variance went down by a factor of 2. So, if I have M antennas what will be the expressing it will be E_s divided by σ_n^2 by M, as if your noise were this is exactly what averaging is, that is why averaging works.

So, the array gain is this is another way of saying averaging; the benefit of averaging is what is what you need. In diversity if I just add the coefficient yes I do not get that

benefit because the fading statistics did not change, but there is still an understanding of array gain and that is very important for us. So, the 2 types of gain that I want us to talk about when we have multiple antennas multiple antennas are the following. The first one is I want to talk about array gain. In classical adaptive signal processing literature this is also called beam forming gain. And the historical where does this come from beam forming gain.

So, if I have an array of detectors and if I want to focus on an object that is coming in a particular direction, I must co phase all of my received signals. So, that I get a beam in the direction of my desired signal. So, my antennas are going to give me gain in a particular direction that is achieved through appropriate phasing not, not co phasing appropriate a phasing. So, that is why is called a beam forming gain. Now for us we refer to it as array gain. And we will see in a minute how these 2 are related.

So, array gain basically tells us that there will be an increase in average SNR, there will be an increase in average SNR even if there is no fading. Increase in average SNR because of the array benefit even when there is no fading, even without fading and that is what we saw in the case of the AWGN channel. And it occurs for all diversity techniques, occurs for all diversity techniques, diversity techniques we are talking primarily about receiver diversity techniques here.

So, it can be selection combining or maximal ratio combining or equal gain combining all of them will achieve it; however, the array gain is maximum for MRC, maximum for MRC. The definition of array gain, array gain is g_{array} this is equal to average SNR under diversity conditions divided by SNR of average SNR of a single antenna. So, this is average combined that is combined under the diversity method.

So, let me just combined under which ever diversity method you are using average combined or average diversity SNR divided by average single branch SNR, single branch SNR. We already know what the average SNR is for the different diversity schemes we already made the statement that average SNR of MRC is greater than or equal to equal gain combining greater than or equal to average SNR selection combining. So, therefore, this statement where that the maximum occurs for MRC is sort of more or less something that is obvious to us, because for MRC this will be M times γ divided by γ that is equal to M .

So, in other words the maximum array gain possible is a factor of M and that is achieved by MRC. Now comes the second one, than what is diversity gain. You told me now we just now made statement saying that there is gain even in the absence of fading. So, what happens in the presence of fading? In the presence of fading, that is when we get the benefit of diversity gain. Now diversity gain is over and above array gain, over and above array gain and this will manifest itself only when there is fading.

Because is no fading there is no need for diversity, basically all you can get is the array gain and this is primarily in fading environments. And what this is referring to is not the 3 dB or you know the gain that you get in the improvement in SNR, this gain is talking about changing the statistics of the gamma. There is a minimum benefit that you will get through the array gain, but what more can you get by changing the statistics.

So, basically this is focusing on a slightly different aspect it is says I am combining the antenna signals, combining the antenna signals with a specific intent to manipulate or improve or change the statistics of the distribution of the SNR. Antenna signals to get a more favorable PDF, it is maybe it is you understand exactly what we are referring to more favorable PDF.

Favorable PDF means I want to get a better BER for gamma. So, in some sense the more your PDF improves mathematician will be very angry when we write statements like this, but engineers are if the PDF improves the probability of error comes down, that is what we mean by our in our case this also means that the probability of outage will also come down if your PDF improves, that is what we mean.

So, this is where, I believe once more the presentation of a Tse and Viswanath comes in really handy in the way they have explained it.

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Handwritten derivations on a whiteboard:

$$\gamma_k = \frac{|z_k|^2 E_s}{N_0}$$

Instantaneous SNR: $\gamma_{MRC} = \sum_{k=1}^M \gamma_k = \frac{E_s}{N_0} \sum_{k=1}^M |z_k|^2$ (1)

Avg SNR: $E[\gamma_{MRC}] = \Gamma E\left[\sum_{k=1}^M |z_k|^2\right]$ $E[|z_k|^2] = 1$

Instantaneous SNR from (1): $\gamma_{MRC} = \underbrace{\left(\frac{M E_s}{N_0}\right)}_{M\Gamma} \underbrace{\left(\frac{\sum_{k=1}^M |z_k|^2}{M}\right)}_{\text{diversity gain}}$

Comparison: $\frac{1}{M} \sum_{k=1}^M |z_k|^2$ vs $|z_1|^2$ as $M \uparrow$

So, you can look up the corresponding one in Tse and Viswanath the basic definitions same as what we have been using. So, we will derive or show the result that they have given.

So, the instantaneous SNR of the K-th antenna is given by $\gamma_k = |z_k|^2 E_s / N_0$. That is the average SNR multiplied by the magnitude square of the channel gain. γ_{MRC} is equal to summation γ_k from $k=1$ through M and this can be written as E_s / N_0 times summation $|z_k|^2$ from $k=1$ through M . This is instantaneous SNR, this is instantaneous SNR, instantaneous SNR. Average SNR can also be obtained for completeness, let us write that down also average SNR is expected value of γ_{MRC} , this is nothing but γ times E_s / N_0 I have written it as γ , expected value of summation $|z_k|^2$ from $k=1$ through M take the expectation inside. We know that we have if we have normalized our fading coefficient then expected value of $|z_k|^2$ is equal to 1. So, this will actually be equal to M times γ .

So, now there is nothing new that we have written down. So, what exactly did Tse and Viswanath, what is the insight that they give? So, they said let us examine the instantaneous SNR, instantaneous SNR. We said already we already looked at it what is there to look again let us look. At it one more time instantaneous SNR, instantaneous SNR for is given by γ_{MRC} , going to write it in a particular form that is very beneficial for us, M times E_s / N_0 I am going to multiply and divide by M the

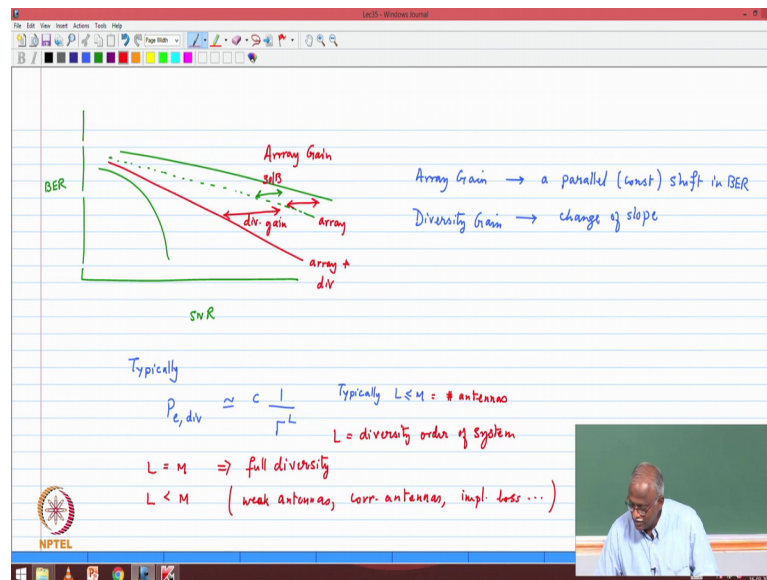
expression for instantaneous SNR. And going to the remaining term is K equal to 1 through $M \bmod Z$ K^2 divided by M is a second term. So, basically this is I am rewriting equation 1, instantaneous equation from 1. What we have done is just multiplied and divided by M , but notice that this is pointing us in the direction of M times E_s by N not is the same as M times γ , M times γ So this is array gain.

So, the net gain that we get from maximal ratio combining, they have shown it is we can split it as 2 parts: one that comes from array gain and the other one that will come from diversity, gain diversity gain. Now if I average the diversity gain, basically if I take the average, what will happen? 1, because average of the numerator is 1, when you add M of those you will get M divided by M it will become equal to 1, but where did you get the benefit because instantaneously one of some of the Z K s may be bad some of them may be better. Because you are able to combine them you would have gotten a better statistics in the fading coefficient that is where the benefit is coming from.

So, basically what we want to now compare is, how does 1 over M summation K equal to 1 through $M \bmod Z$ K^2 how does that compare with $\bmod Z$ 1^2 ? $\bmod Z$ 1^2 square here only one term if it goes in to a fade your stuck, but when I have this type of an expression, then I am much more robust against fading. This is what is changing the statistics in terms of the statistics of the SNR. So, as M increases, as M increases then what happens? We find that there is the probability of this being better than the diversity being better than the one without the case without diversity or becomes more and more probable, because this is the one that is going to give us the advantage.

Now very interesting for us to now is this part clear. So, there is the array gain which is which will there even in the even when there is no fading. There is a component that seems to come in only when fading is present and that to instantaneously it changes something instead of working with one antenna it is trying to do a combination of antennas. And the argument is this combining is much more robust because some Z K s may be down some Z K s may be up on average, I am up that is the argument that is being made, ok.

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So, but how do you visualize this; back to our BER graph. The BER graph basically says this is BER versus SNR; AWGN graph is like this fading graph is this. If I had only array gain, what would be my graph look like? 2 antennas only array gain it will be shifted by 3 dB, but it will follow the same graph, right? That is 3 dB benefits this is array gain, 3 dB that is 3 dB and that is array gain 2 antennas, but no benefit in terms of fading I have not, but what is the graph that you get for 2 antennas? A maximal ratio combining, you find that the graph actually is this.

So, what is happening there is of course, array gain 3 dB and there is diversity gain this is diversity gain. And the combination of these 2 is the net benefit that I receive and this is why it is so important. So, what this graph is showing is array plus diversity. This graph will be only array gain, array gain will just be the same line shifted because it is only you know 3 dB or 6 dB whatever is the number of antennas that you have and this is the benefit that we achieve, ok.

So, in Layman's terms array gain is a shift a parallel shift of the of the BER graph. Array gain will contribute a parallel shift well will take the parallel shift if that is all you get that is, but that, but very important thing is to recognize that diversity gives you more than that. It is a parallel or constant shift in the BER, the diversity gain on the other hand changes the slope diversity gain is a change of slope. And what we are seeing when we do things like MRC is a combination of array plus diversity and the reason we want to

separate it and not just call both of it diversity the just as one diversity gain term is because there is a notion of how when we combine multiples sensors and you do being forming you get array gain, that is an SNR gain.

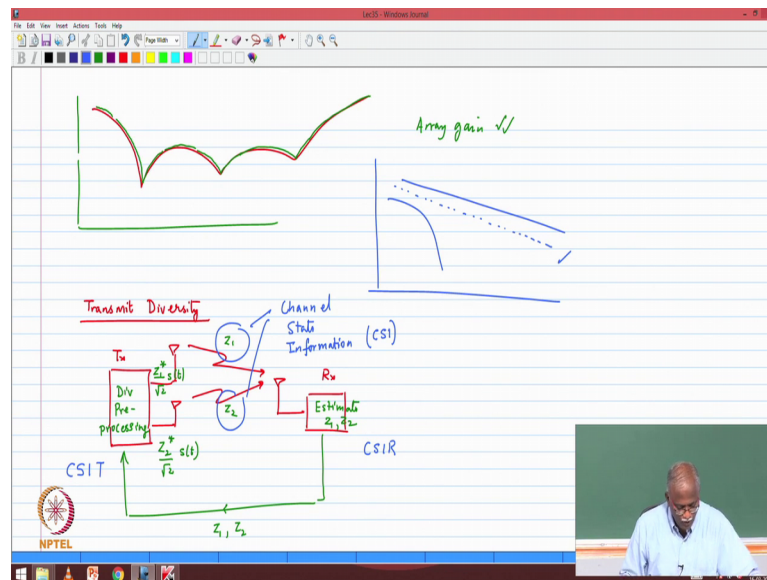
So, there is the notion that the what comes from adaptive signal processing it says, hey there is a underlying array benefit, but over and above that you have managed to change the statistics of the of the SNR and therefore, you get that. So, the change of slope is the important one which of them is more important, I would not argue you know take both, both of them together gives us the benefit.

So, typically what you will see when you are looking at the BER performance you will find that the probability of error with respect to diversity, typically will be of the form some constant 1 over average SNR raise to the power l . l may be not be an integer, but the slope will be some number. And typically l will be less than or equal to M , less than or equal to M . Where M is the number of antennas, M is equal to number of antennas. If you managed to show that l is equal to M you have achieved it full diversity. That is the best that you can achieve, but so, l is called the diversity order of the system. l is the diversity order of the system that is the slope of the graph. Diversity order of the system and if you have a weak antenna or if you have a correlated antenna then you will not be able to achieve full diversity order, but it will be less.

So, typically l equal to M implies you have achieved full diversity for your full diversity benefit and full diversity performance gain for your system. Typically, if l is less than M that is probably due to either you have some weak antennas or you have some correlated antennas or some whatever reason you have not be or you did not implement your little lazy and you did not want to do MRC you did EGC you did not get the full benefit that is because of the choice that you have made but at the end of the day.

The slope is what matters you have either correlated antennas or due to implementation losses, whatever may be cause for this implementation loss your channel estimation was not good whatever it is you achieved something that is less than M . But that is your diversity order of the system. That gives us a complete picture of what we need in terms of the understanding of diversity. Any questions?

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Now, comes the acid test now this is crucial question, let say that this was going to determine the grade for the course. The question is I have the following situation, fading environment antenna 1 antenna 2. 2 students, student a says it is a waste throw away one of the antennas no benefit it is fading environment correlated antennas I know no use, right? Throw it away. Even under this environment array gain will still be there. So, do not throw away the second antenna you will still get 3 dB antenna gain array gain. Whenever the original signal faded this will still fade, but you would have boosted your average SNR by 3 db.

So, that is very important so, but. So, in this case diversity gain nil you did not change the slope of the diversity, but you still got array gain. Now if it so happened that the 2 antennas where not so identically correlated then you would start to get some diversity order and then if they are totally uncorrelated you will get full diversity order, good very happy with that. So, this will be a case where if you if you actually implemented it you will get BER slope equal to 1 you will get if you if you took the array gain you will get the benefit you will see a graph which has got the same slope, but 3 dB better. So, this is why and have you pass with full marks ok.

Now, one last piece of information or probably some notation that is very important for us. So, transmit diversity we already gave an example, but let me just introduce some notation for us before we got in to one of the most interesting case studies in transmit

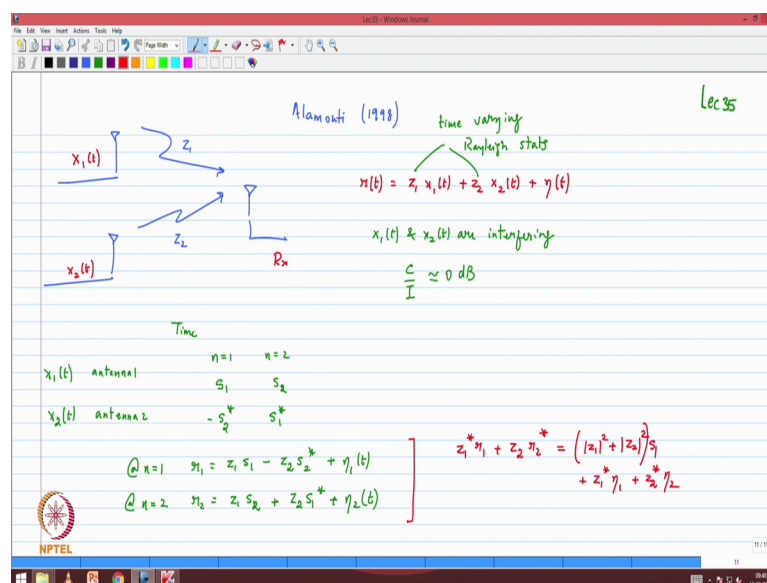
diversity. Transmit diversity basically says that I have a transmitter which has got multiple antennas let me just show the case of 2 antennas and I have one antenna at the receiver. So, this is my transmitter this is my receiver of course, this need not be limited to 2 I can have anything more than 2 as well. The channels are uncorrelated Z_1, Z_2 we know that we have to split the power. So, $1/\sqrt{2}$ has to be applied to each of these terms.

Now, we will do this block we will call it as diversity preprocessing, which means that you have to do something to the diversity signals before transmitting. So, that you get the benefit. So, what exactly what was the thing that we said that we will not transmit just $s(t)$ from each of these branches we will transmit $Z_1^* \cdot s(t)/\sqrt{2}$ from this branch we will transmit $Z_2^* \cdot s(t)/\sqrt{2}$ from the second branch and that is when we will see co phasing of the signals at their receiver. Now for this we needed to this block on the right hand side to do the following. It needed to estimate Z_1 and Z_2 and it also had to feedback this information, and that was the feedback channel.

So, this was it is feeding back Z_1 comma Z_2 . Now this frame work has got some notation that is very, very useful for us to remember. Now Z_1 and Z_2 are referred to as channel state information, channel state information. The channel conditions are described by Z_1 and Z_2 , channel state information which is popularly denoted with the acronym CSI channel state information. Now the minute the receiver is estimating CSI. We referred to the information here as CSIR, because what is the true channel is CSI. What I estimate at the receiver is CSIR and so far the transmitter does not know what is happening. But if I do feed this information to the transmitter then I say that channel state information is also available at the transmitter.

So, CSI in the channel CSIR in the receiver if I feed it back it is this information that is sitting at the transmitter is now referred to as channel state information in the transmitter. That is notation, but it is useful for us to keep that picture in mind.

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Now, I want you to think about a transmit diversity scheme which was which a very powerful scheme. So, transmitter 1 transmitter 2 this is receiver channel $Z_1 Z_2$.

Now, Alamouti is name of a person 1998, he proposed the following. He said I am going to transmit 2 different signals X_1 of t from antenna 1 x_2 of t from antenna 2 and I am going to receive them at the receiver. If you had put X_1 of t and X_1 of t what would you, have told him do not do that it is a waste of time because there is no benefit from doing that because we already know that.

You might you would if you are smart you would do co phasing by pre rotation he says no. I am not interested to know the channel they will say wait a minute; you do not want information at the transmitter than I think you are in trouble. Because I do not think you are gone get any benefit out of it because unless Z_1 and Z_2 this really not we are not sure or you can get any benefit. So, this is what Alamouti said, he said r of t is going to be Z_1 times X_1 of t plus Z_2 times x_2 of t plus eta of t ok.

Now, Z_1 and Z_2 I have written them as if they are constants, but these are actually time varying Rayleigh coefficient. One particular snapshot is what we have shown. So, therefore, these are Rayleigh time varying random variables with a Rayleigh statistics. So, that is something that you keep in mind, ok.

Now, X_1 and x_2 are 2 different signals. So, if I transmit both of them at the same time they will interfere with each other. So, X_1 of t and x_2 of t are now interfering with each other. And x_2 of t are interfering now, what is the c over i ? C over i carrier to interference ratio forget the noise. This you ask the same level 0 t . So, c over i is approximately 0 db. So, which mean that no detection is going to be possible at ask Alamouti are you sure you know what you are doing.

So, this is what he says he is wants us to do. So, this is time this is antenna 1 this is what he is going to define as X_1 of t this is X_1 of t x_2 of t which is antenna 2. And he said this is time n equal to 1 n equal to 2. And he says antenna 1 you transmit s_1 and s_2 whatever you are going to transmit normally symbol number one symbol number two. But antenna 2 he says you transmit minus s_2 conjugate and s_1 conjugate. And then we say well I do not mind you have ask me to do this let me. So, he says write down the equations; so at n equal to 1 r_1 equal to $Z_1 s_1$ minus $Z_2 s_2$ conjugate plus η_1 of t . Correct? What was transmitted by antenna 1 at time 1, what was transmitted by antenna 2 at time 1, multiplied by the corresponding coeeficnce. And then added together then at time n equal to 2 r_2 equal to $Z_1 s_1$ $Z_1 s_2$ $Z_1 s_2$ plus $Z_2 s_1$ conjugate plus η_2 of t .

So, now the statement that was being made is; obviously, these are now I have 2 equations and 2 unknowns. So, you know I may be there is some hope for us what we are doing. So, he said do Z_1 star times r_1 plus Z_2 times r_2 conjugate. You can easily verify and do that and you will find that what you get is $\text{mod } Z_1^2 \text{ plus } \text{mod } Z_2^2 \text{ square times } s_1$. What is that? That is like maximal ratio combining, times s_1 plus noise terms $Z_1 \text{ star } \eta_1 \text{ plus } Z_2 \text{ star } \eta_2$. So, basically there are some couple of noise terms that are present and likewise you can write a combination that gives you s_2 also with the diversity expression in front.

So, what is Alamouti actually doing? This is the first transmit diversity or the space time code where you transmitted different signals, but managed to get the benefit of diversity and interference cancellation because only if you cancel the interference you can detect the signal. You cancel the interference and you got the benefit of diversity that is space time coding.

And this was the first illustration of space time coding that was demonstrated which then of course, opened up the entire field. But how did Alamouti work on it? What is the array

gain? That he achieves is there are diversity gain. Let us answer those questions in tomorrow's class.

Thank you.