Introduction to Wireless and Cellular Communication
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Lecture - 34
Computer simulation of Rayleigh fading, Antenna Diversity
Statistical Characterization of Antenna Diversity, Optimal Diversity Combining

Good evening. We begin lecture 33. The flow of today's lecture will be completion of our discussion on selection diversity. We concluded it by doing the mathematical characterization. Antenna selection is one way of benefiting from multiple antennas. The better or more enhanced ways of exploiting diversity would be to combine the signals. In fact, we have made a statement without validation that the optimal combining should give us the sum of the SNRS of each of the different antenna signals.

So, today's lecture will validate or will prove that result. So, signal combining is the way of exploiting diversity in a better manner, better than selection diversity and we will ask the question what is the optimal combining that we can do.

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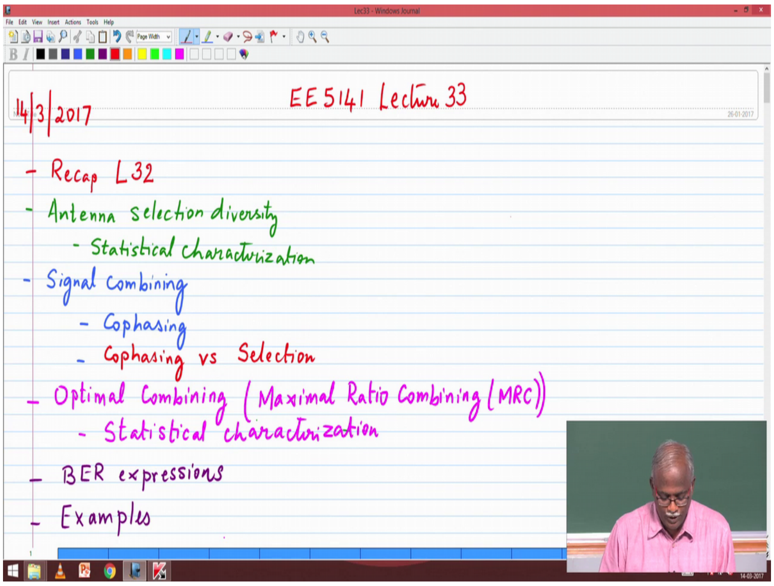

In the diversity literature, the optimal combining is also known as maximal ratio combining and then, again we will justify why it is optimal, why is it called maximal ratio and then, at the end of it, we will look at some BER expressions which will be using the mathematical characterizations, but first we begin with the quick review of lecture number 32.

Again if there are any questions, we will please do raise them. We will assume that 32 materials is comfortable to everyone.

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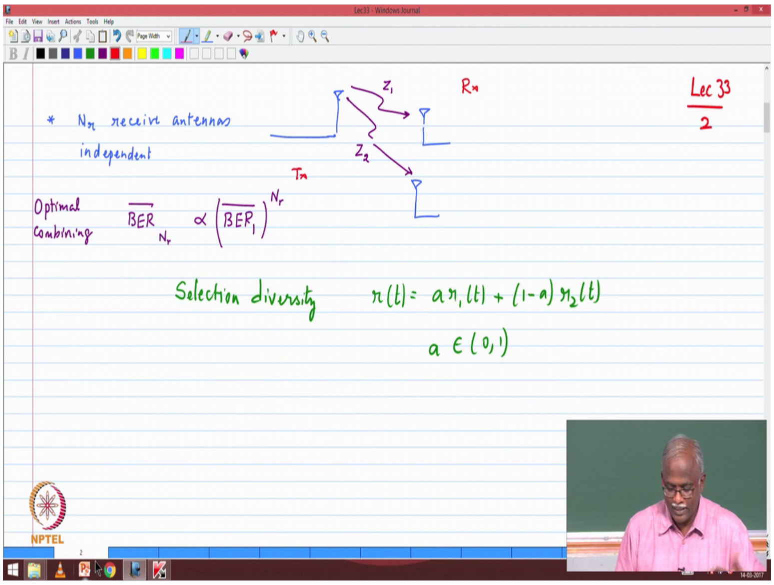

So, the basic frame work is that we have one transmit antenna, multiple receive antennas and the signal that is transmitted by the transmit antenna is picked up by the different received antennas. So, we have shown a case of two examples i.e. the general case would be N r antennas and selection diversity basically says you just give weightage of one to one of the antennas and then, zero to the others and that is what this signal represents.

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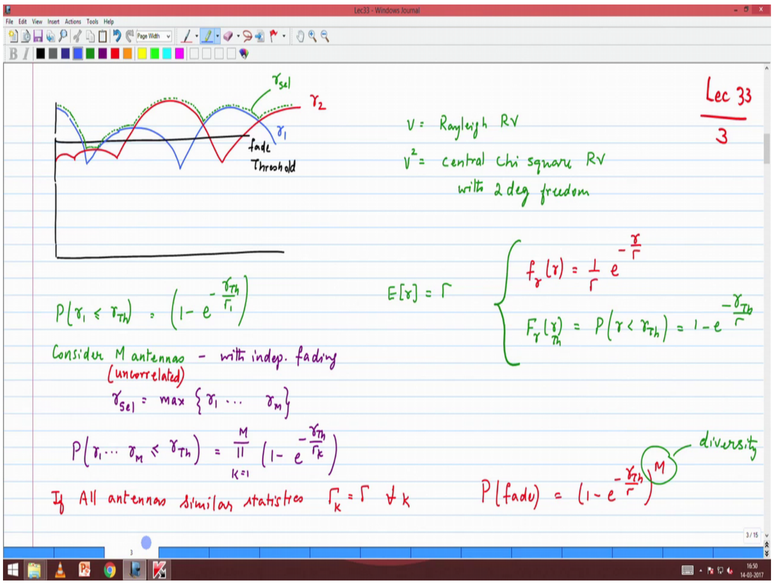

So, selection diversity is really not combining. It is actually selecting, but in the different antennas. The benefits of selection diversity we saw through two independently fading antennas and giving the benefit of choosing the better of the two in every situation and we can see that definitely we will do better than either of the two antennas individually, and we showed that the cumulative distribution function can be expressed as a product of the individual CDFs and in the case where all of them have identical statistics, it would be raise to the power m is sort of start to see the benefit of the diversity factor.

Then, we went on to look at the effect of a weak antenna and then, showed that if you have M antennas M minus one strong and one of them weak. It is effectively like having M minus 1 antennas because the selection process will not pick up the weak antenna. It will always favor the strong antenna, ok.

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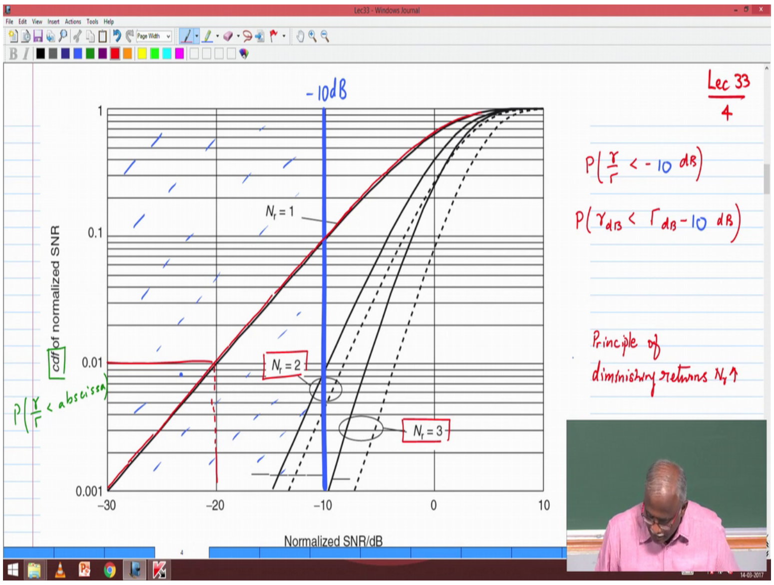

Now, just one very quick input regarding the benefit of selection diversity. The solid lines that we showed here are the cumulative distribution functions which is what we have shown in the last graph raise to the different powers of m and m being 1 2 and 3, 1 is no diversity, 2 and 3 are selection diversity and the solid lines are CDF functions. I have drawn a dark blue line to indicate the minus 10dB. So, if your fade margin was 10 dB, last time we discussed it as 20 dB just as an illustration.

So, basically this normalized parameter gamma by upper case gamma less than minus 10dB says this is what it means. It is same as saying what is the likelihood of CDF lying being less than or being in this range below 10dB or less and that is denoted by the intercept on the y axis. So, for the case of one antenna, it is around 0.1 dB, sorry 0.1 is the probability for two antennas. It is about 8 into 10 power minus 3 for 4 antennas. It is not even there. Basically it is less than 10 to the power of minus 3. It is something 10 to the power minus 4.

So, you can see that the benefits of the selection diversity already are quite substantial. When you go from 1 to 2, you can see there is a huge gain that you get and then, you start to see diminishing returns as you go to more number of antennas and that is what we may we can just mention that the principle of diminishing returns as you increase principle of diminishing returns as M increases or as N r increases. Yes, you do get benefit, but increase in the performance becomes less.

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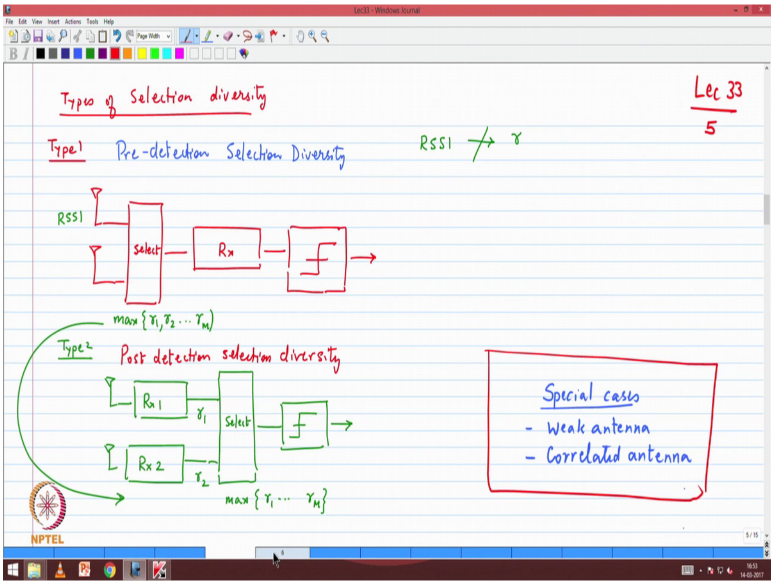

We said that there are two types of ways of exploiting selection diversity you can select and then, do the processing and then, do the selection and both of these are valid. There are you know tradeoffs which we have discussed in the last class in the context of selection diversity. There are two special cases that we always want to keep aware of. One is the weak antenna. What happens, what is the context of the weak antenna and the second one is what happens when there is a correlated antenna and again, there are special scenarios where these antennas may still be of value to us, ok.

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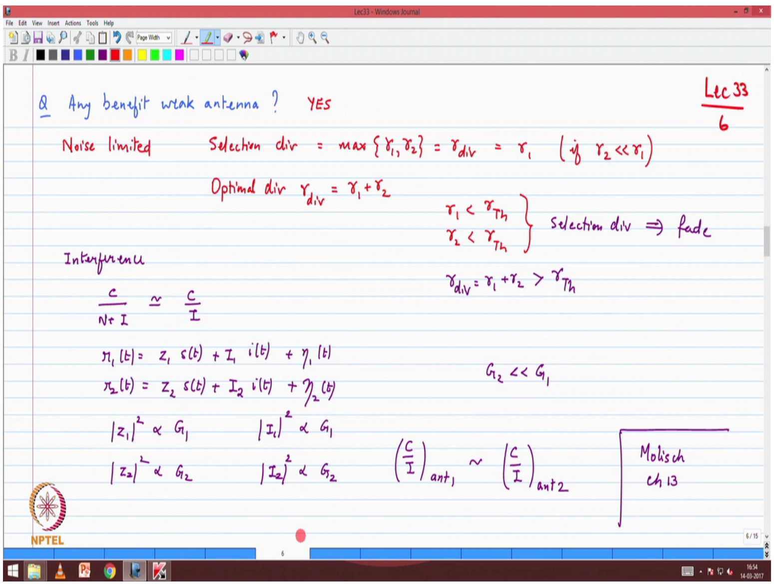

We ended the lecture by saying that in a noise limited environment if you had a weak antenna, it would basically mean that the second antenna is not selected. You will always be selecting the one that is stronger. However, in the case of an interference limited scenario, both the antennas are seeing approximately the same c over i. I thought that it would be beneficial for us to revisit this result just so that we can start today's lecture.

So, the key point is that selection diversity means picking the best of the antennas. Keep in mind that in the context of a noise limited scenario, it is signal to noise ratio. In the context of an interference limited scenario, it will be signal to interference ratio and the fact that a signal is an antenna strong or weak, we just want to make sure that you are comfortable with that.

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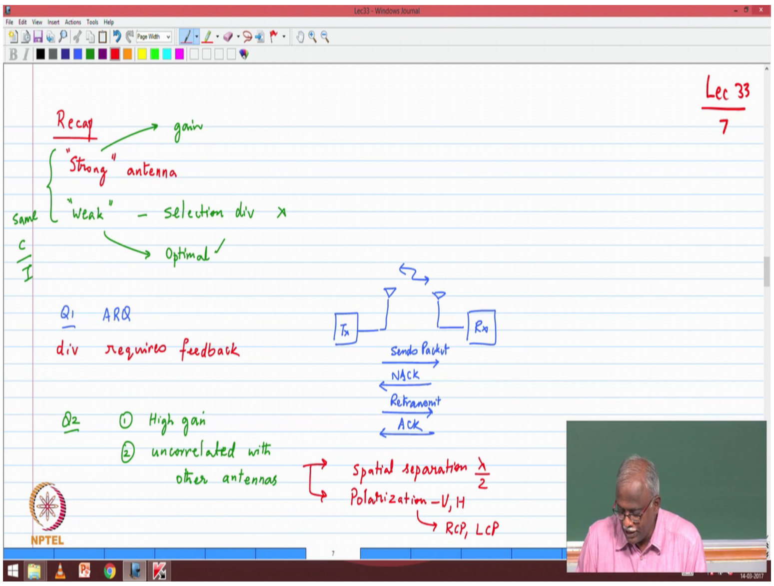

So, a couple of points before we move into today's lecture. So, here are some points may be for you to think about and note down. So, what do we mean by a strong antenna? What is the notion of, what is your understanding of a strong antenna? The strong actually refers to a high gain. So, antenna a stronger than antenna b is a same as saying the gain of antenna a is greater than gain.

So, basically this strong or strength that we are referring to is the gain of the antenna and when we have a weak antenna, that means I have a antenna that does not have much gain. What happens is, it has a limitation when it comes to the signal to noise ratio. We will pick up little bit later, but basically in the context of selection diversity, this antenna is not of much use. Selection diversity, this antenna will not be selected, basically the weak antenna would be, but in the context of optimal, we will show that there is a benefit that even the weak antenna will have some utilization for our purposes.

So, strong and weak both have same c over i. That means, if I can ignore the noise, then the key point to keep in mind is that the notion of what is strong, what is weak and which scenario is the one that we are talking about. Before we go into today's lecture, I would like you to think about couple of questions. This is just to think about diversity in its entirety before we start focusing a little bit more.

So, are everyone familiar with that? What is ARQ?

Student: Automatic Multi Request.

Is that a form of diversity? Answer is yes, it is a form of time diversity, but it has some constraint. So, if you have a transmitter and you have a receiver, so the transmitter sends the packet, sends packet does not receive or did not get it. So, again retransmit and this time the ack was received. Again this is very simplest form of ARQ.

Now, notice that the information was sent to times, it was sent with some spacing in time and hopefully that was more than the coherence time and therefore, this is a valid form of diversity. It is a form of diversity that requires feedback. So, it is a form of diversity that requires feedback and again that is a very powerful set of scenarios when you have feedback going from the receiver to the transmitter and we will study more about these feedback channels.

So, this is just keep in mind that you know they were talking about a very broad concept and this is one of those second questions. Again very different in terms of nature what are the two things that we look for in antenna diversity? One is that it would have high gain that would make it a strong antenna that is number 1. Second is uncorrelated with other antennas because even if it had high gain, if it ended up being correlated with the other antennas, it is not of much benefit uncorrelated with the other antennas. Those are two things that you would want for diversity benefit.

Now, what are the ways in which you get uncorrelatedness? One of them is by spatial separation that means distance. So, spatial separation you would need at least lambda by

2. Again this is for lower wavelength. This is little difficult for a handset. Of course, on a base station you can do it reasonably. Well, if you cannot do spatial separation, what are the ways do you get? You just said no way, nowhere I can get diversity on a mobile. Is there any other way that you can get diversity, anything that you are familiar with?

Student: Polarization.

Little louder. Polarization? Yes, polarization is a way of getting. So, you can have two antennas. One of them is vertically polarized and the other one is horizontally polarized. Vertical and horizontal or you can have right circular polarization, left circular polarization and there are theories to show that your vertical polarization signal and the horizontal polarization signal are uncorrelated with each other. Similarly, the right circular and left circular depending upon how you have transmitted your signal and your environment, you could be using circularly polarized antennas and those would also give you uncorrelated.

So, again something for you to think about in the broad context of diversity having understood this element, I would now like to spend a few minutes on notation and in the last class, we wrote this down, but we probably went a little bit fast. I just want to repeat that.

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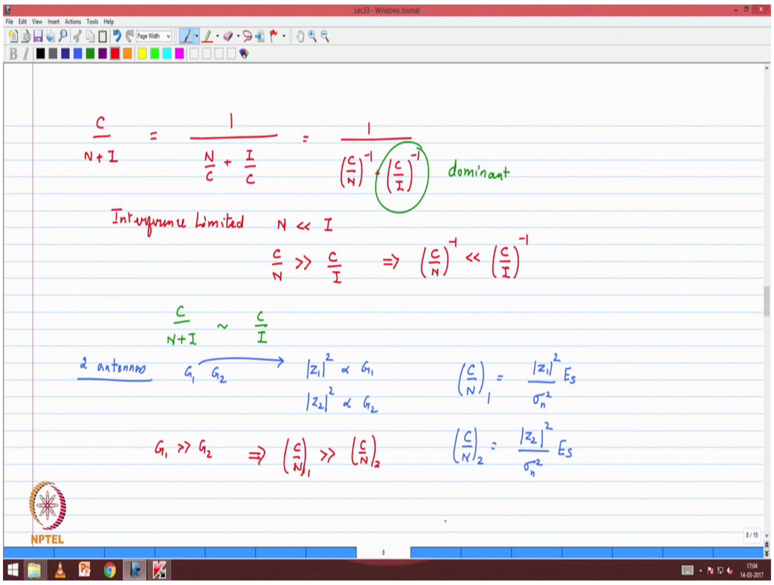

So, the matric that we are often looking at is C over N plus I and C over N plus I can be nicely written as divide numerator and denominator by C. It becomes I by C. Before you wonder why I did this becomes C by N inverse plus C by I inverse, ok.

So, this is a nice way to visualize that. So, if we say that we are interference limited, that means the noise power is much less than the interference power which means that C over N is much larger than C over I because I is much larger. This would also mean that C over N inverse is much less than C over I inverse. So, that means the dominant term, this is the dominant term and therefore, C over N plus I is approximately C over I. Of course, you could have said that because n is much less than I, I am going to ignore I, but this is a nice way to visualize saying we always talk in terms of carrier to nose ratio. We do not talk about just, we do not measure the noise power or the interference power separately. So, therefore, this is a good way for us to visualize that.

So, suppose I have two antennas. Now, the signal on with antennas gains G1 and G2. Now, what does G1 does? What does G1 influence? It influences mod Z1 square, right. What you transmitted is a same power, right. There is nothing that does not affect. So, what you eventually picked up what the channel coefficient and if your gain was higher that mod Z1 square is going to occur, so mod Z1 square is proportional to G1 mod Z2 square is proportional to G2, ok.

Now, carrier to noise of antenna 1 can be given by mod Z1 square by sigma N square into Es. Whatever was your signal power what you transmitted mod Z1 square is the gain because of the channel and sigma N square is the noise power. Again that is something that we do not have any control over. So, similarly C over N for antenna 2 is mod Z2 square divided by sigma N square Es and now, if I am told that G1 is much stronger than G2, now you can see why C over N 1, C over N for antenna 1 is going to be much larger than C over N for antenna 2. That is why antenna 1 will keep getting picked most of the time, ok.

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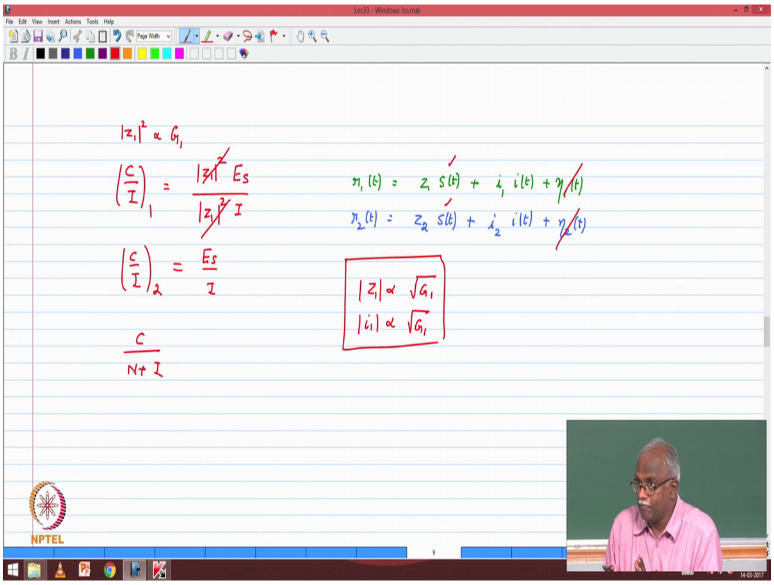

So, these is the noise limited scenario and again extend it to the interference limited scenario. The mod Z1 square is proportional to G1. Now, C over I for antenna 1, this is proportional to the signal that is from the desired signal and the interfering signal. So, basically mod Z1 square affects the desired signal Es. The interfering signal I1 mod Z2, sorry mod Z1 square again affects that also. So, basically the ratio of the two is what removes the dependence on the gain and likewise, you can see that for C over I for antenna 2 is also equal to E s by I 1. It should be the interfering signal power of the interfering signals. So, it is no I 1, I 2. So, it is only I. This is also going to be S over I, ok.

So, that is why even if G1 is much stronger than G2, you may still have some benefit in interference in the context of addressing the interference issue.

Student: Sir.

Yeah.

Student: Fundamentally the antenna should see different signals, right? Why are we considering the powers to be the same?

I am sorry, I did not get the question.

Student: Sir antenna 1 antenna 2 are supposed to see different signals. So, how are we considering the powers to be the same?

Good question. The question is what is the model underlying? Model let us clarify that. So, r 1 of t is Z1 times s of t plus some interfering interference coefficient. Let us call it as I 1 times I of t plus eta 1 of t, right. So, the second signal r 2 of t is z 2 s of t plus I 2 is that a coefficient times I of t plus eta 2 of t. Now, since we are talking about an interference limited scenario, I am just going to ignore the noise for the moment.

So, now the question that you are asking is, it is the same signal that is transmitted, then what different is how much of it ended up in my receiver and that is dependent on the gain that I will see and that in turn will will determine what Z1 is and gain of antenna 2 will determine what Z2 is. So, the same antenna is what is picking up the interference signal also. So, therefore, how much of the interference I pick up also depends on the antenna gain.

So, basically what we can say is Z1 is proportional to mod Z1 is proportional to square root of G1. It is one is the amplitude; other one is power So, similarly I1 is also proportional to root G1. So, when I take mod Z square by I1 square, the G 1 goes off and likewise for G2, but here the question what you have raised is actually valid because if you take C over N plus I, there is no scenario where you can completely omit the noise. If the gain is very weak, the noise starts to become the dominant factor.

So, even in such scenarios, weak antenna is slightly worse than the stronger antenna. So, it is not fair to say that you know it is exactly the same, but all the important thing to note is that this factor is something that we should not miss out because noise is not affected by the gain, but both signal and interference are affected by the gain of the antenna and therefore, we have to take that into account in our consideration; very good. Any other questions?

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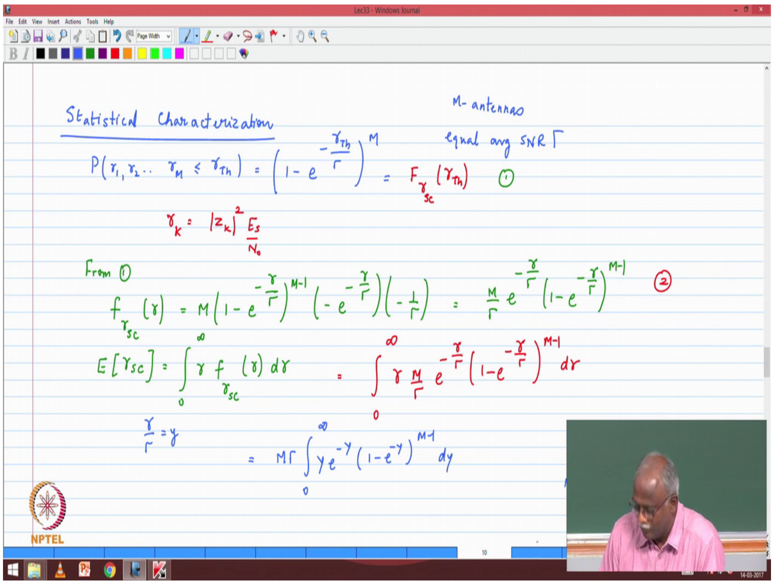

If there are not, then we will spend a few minutes on the statistical characterization of selection diversity and I believe you will find this very interesting and also, lot of intuition can be obtained from this result and like before in this mathematical derivation, there I will skip a few steps, but I hope you will fill it in and make sure that you are comfortable with the result. We start with the fundamental result of a selection diversity that is the probability of fade or probability. The cumulative distribution function is derived is defined as gamma 1 gamma 2 gamma M. I have M antennas and all of them less than or equal to some threshold and assume we are going to make the assumption that there are m antennas and all of them are identical. Identical means they have equal statistics, equal average SNR which is denoted by gamma average. SNR is always denoted upper case. So, this is expression that we have already derived. We will just write it down 1 minus e power minus gamma threshold by gamma raise to the power m. This is the cumulative distribution function and we will call it as the CDF of gamma subscript Sc, that is for selection diversity and the value is gamma threshold. So, this is the expression that we have.

Now, individual SNRS are obtained or can be written down as gamma case is equal to mod z k magnitude square Es by N naught. So, basically there is a signal to noise ratio that is when that would be the signal to noise ratio, in the absence of any fading and because fading is present, the instantaneous SNR is modified, is obtained as mods at k square times E s by N naught. Now, from the cumulative distribution function from this is equation 1.

From 1, I would like to get PDF of gamma SC. Instead of writing at gamma threshold, I will just write it as gamma which basically says I have to differentiate the CDF with respect to gamma. So, please do that. It is just one step in the difference process differentiation. So, the exponent I get 1 M into 1 minus e power gamma divided by gamma raise to the power M minus 1; then, the second term which will be minus e power minus gamma by gamma into minus 1 by gamma, ok.

So, combine them and you will get M by gamma e power minus gamma by gamma 1 minus e power gamma by gamma raise to the power M minus 1. So, actually we have obtained PDF or the probability distribution function. Now, you may say what is the use of PDF. Wait a minute. One of things that I would very much like to understand is what is the average SNR of selection diversity? So, one of the things that I would be very interested to know is e power gamma SC on average is selection diversity is going to give me benefit and how much of an advantage is it going to give me over the other antennas.

So, the first thing we would immediately like to calculate is expected value and expected value is integral 0 to infinity gamma f gamma Sc of gamma d gamma, right. So, you can substitute from equation 2 into the expression that will give us 0 to infinity gamma m by gamma e power minus gamma by gamma 1 minus m minus 1 d gamma 1 substitution and then, we will be able to work with the result. The substitution that we will use is let gamma by gamma be equal to y. I am sure you can do the substitution. You can please verify that this integral now becomes m gamma times 0 to infinity y e power minus y 1 minus e power minus y raise to the power m minus 1 d y.

Again just mix it a little simpler because we need to do some algebra with this. So, this is the general case and one of the non-trivial cases is m equal to 3. So, we will take m equal to 3 as a special case and see for insight because it is always good to get insight first.

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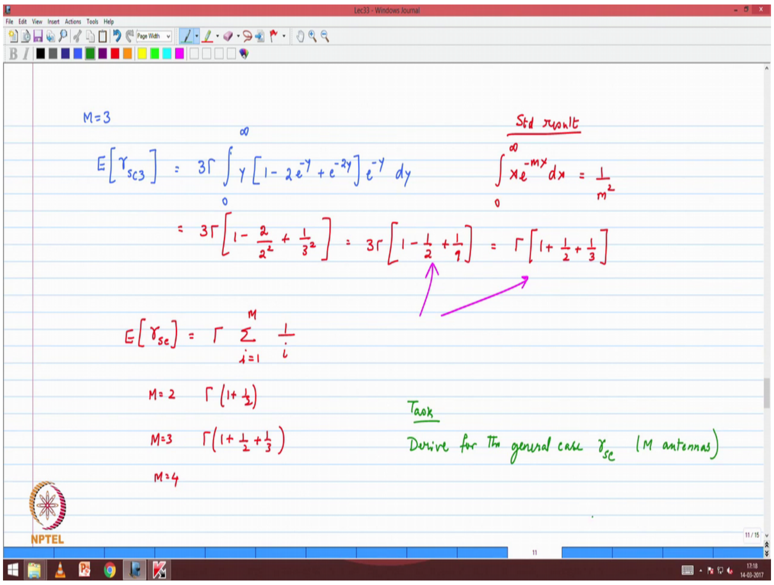

So, if I substitute M equal to 3 in the previous equation, expected value of gamma SC3 that 3 stands for 3 antennas. M equal to 3 is equal to 3 times gamma integral 0 to infinity y into that is a square term 1 minus 2 e power minus y plus e power minus 2 y into e power minus y d y. So, basically you can multiply out that term and you will basically get like integrand with three terms.

Now, there is always all you perfectly at liberty to use standard results. Very convenient standard result is available to us. The standard result says integral 0 to infinity x e power minus m x d x is 1 by m square and that will make life much easier because notice that each of the three terms is of this form. So, the answer now is 3 times gamma 1 minus 2 by 2 square plus 1 by 3 square, that is 3 gamma 1 minus one-half plus 1 by 9, that is equal to gamma 1 plus one-half plus 1 by 3.

Now, you may ask what is inside the bracket. Here can be interpreted in many ways. Why did you interpret it like this not for convenience because in general case, the integral is actually much harder for us to do, but basically you would have to do a binomial expansion of the term with the exponent and then, use the standard result and what you will get for the general case is expected value of gamma selection combining is equal to gamma times summation I equal 1 over M 1 divided by i, ok.

So, if it is two antennas m equal to 2, it is gamma into 1 plus half m equal to 3. It is gamma is equal to 1 plus one-half plus one-third m equal to 4. You can expand it very interesting result, very satisfying result which says that yes I do get a benefit, but the benefit is reducing as the number of antennas increases, but you know it has made the assumption that the antennas are equally strong. If the antennas are not equally strong, you know you can only, this result is applicable only in that context, ok.

So, again once you have the standard result, I am sure you will able to do that. So, let me just give you the task derived for the general case and in moodle, we will give you the hints because I believe it is not straight forward. Even after trying the special case for m equal to 3, you will find that there are certain substitutions that you will need. So, derive for the general case gamma Sc for M antennas, but the important thing is to know the result and appreciate that if they have certain benefits, ok.

We move on. So, what have we said so far about selection diversity is, selection diversity is very easy to do pick the strong antenna and you will get benefit in terms of overcoming the fades. It is definitely better than a single antenna. How much benefit you get we were not sure, but we have now been able to quantify saying that the average SNR is actually going to be if you have two antennas, it is going to be 1.5 times on average antenna and SNR of a single antenna and if you have a third antenna and fourth antenna, you are going to see corresponding benefits and I think this is good enough understanding of the benefits of selection diversity.

So, now we move into other more deeper assessment of the asked problem on the study of diversity and of course, students always are curious. So, the question raised or the question posed before us is what happens if I just added the two antennas? Why do you take so much of trouble finding out which is the better SNR and what if I added the two, would I have not get the same benefit? Just add 2 r 1 plus r 2, what do you get? You get z 1 plus z 2, right. So, you if one is in a fade, the other one will pick you up. Your answer is that question is clear. The answer is I have r 1 which is z 1 times s 1 r 2 is z 2 times s 2. Just add the two signals. No problem, right. When one is in a fade, the other one will come up. So, you get a benefit.

So, to answer that question and answer it in a correct manner, let us take a look at the correct way of addressing that particular problem. So, it is actually a very interesting problem. It is a very important problem. So, we will make sure that we address it very carefully, ok.

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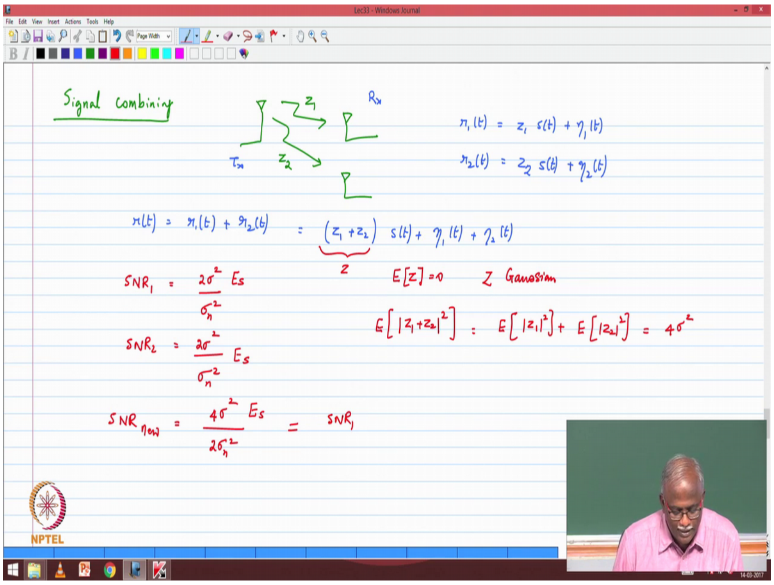

So, the question that we have before us is signal combining, that means you are going to do something and add or combine the two signals in the simplest form. You just add the two together. So, the framework same as before I have one transmit to receive the channel gain to antenna 1 is z 1 to antenna 2 is z 2. This is the transmit side, this is the received side, so that we are complete r 1 of t. The received signal is z 1 times s of t plus eta 1 of t r 2 of t is z 2 times s of t plus eta 2 of t. What can you tell me about eta 1 and eta 2? Both are samples of AWGN which are uncorrelated because they are coming from two different antennas, experiencing two different you know noise environments, but they are just sampling a random process with the same variance. So, they are uncorrelated with each other, but they have the same variance, ok.

So, now the question is what if I did r of t is equal to r 1 of t plus r 2 of t looks like a good way to do it because what comes out is z 1 plus z 2 times s of t plus eta 1 of t plus eta 2 t. Now, that itself should give you a little bit of warning signal. You added the noise terms that mean you have twice the noise. You should have at least gained twice in the signal to noise ratio, otherwise you actually have you lose SNR. So, it is always you have to be careful. So, of course, we do not want to make a guess. So, if I were to call this as a random variable z, now tell me the statistics of z expected value of z. Expected value of z is 0, ok.

Now, if you have z 1 and z 2 Gaussian and you add them together, what do you get z as z n is also z is also Gaussian. So, z is also Gaussian, ok and we know that z 1 and z 2 are independent of each other. So, if I was interested in expected value of mod z 1 plus z 2 whole square, the modulus square, this would be expected value of mod z 1 square plus expected value of mod z 2 square and that would be 4 sigma square, each of them where 2 sigma square, but now you have got something with us slightly larger variance, ok.

Now, let us go back and answer it from a statistical view point. You have a new channel coefficient which is z which is zero mean and the real and the imaginary parts are Gaussian. Forget the variance. What is its envelop?

Student: Rayleigh.

Rayleigh did you gain anything? No, the level crossing rate everything is going to be more or less the same. Now, you said oh wait a minute. I gained may be something in the numerator, right because I did z 1 plus z 2. So, let us see if we actually gained something. So, SNR 1 if I had only one antenna, what would it have been? It would have been 2 sigma square times E s signal divided sigma N square, right. 2 sigma square is the mod z 1 square expected value of mod z 1 square. So, SNR 2 is 2 sigma square by sigma n square e s. Again SNR of the combined signal new is what? It is 4 sigma square times E s, energy of the transmitted signal, but have two noise terms. So, you have to add 2 times sigma n square in the denominator which is the same as SNR1. On average your SNR is also not any different. You did not gain anything by this process. So, therefore, it is as good as waste of time. So, your BER is not going to change. All of it is going to be pretty much where you were, ok.

So, little bit disappointed because we thought it was a good idea, but you know let us see if there is a better way and the answer turns out to be is that yes, there is much better way and how do we work on that and obtain the results that is associated with that. So, here is a way that we would like to do it.

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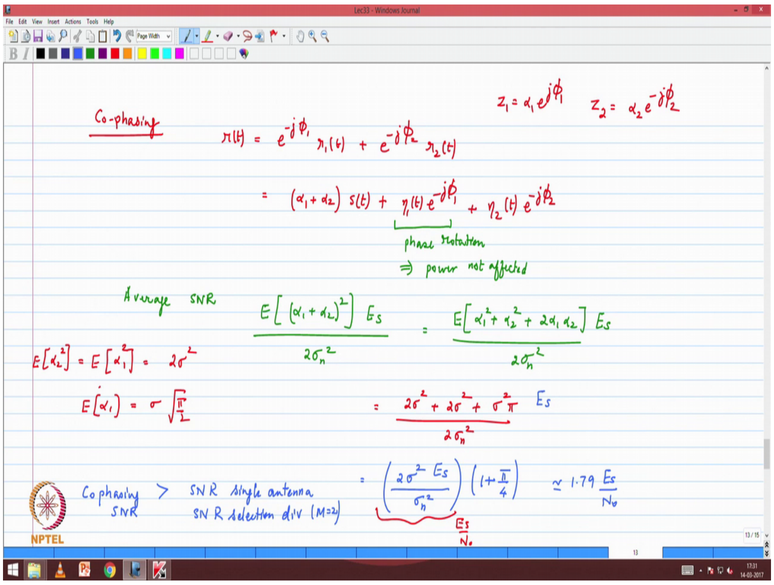

So, here is the proposed signal. We are going to call it cophasing and the answer and the reason for that name will become apparent in a minute. So, r of t is equal to e power minus phi 1 of phi 1 times r 1 of t plus e power minus j phi 2 r 2 of t, where z 1 is equal to alpha 1 e power j phi 1 and z 2 equal to alpha 2 e power minus j phi 2. Those are the complex coefficients.

So, what did you do? You did a counter rotation of the received signal. For the first antenna, the channel gave it a rotation of phi 1. You gave it minus phi 1 in the receiver n f. Correspondingly you do that. So, basically now if you write down the expression, what do you get is alpha 1 plus alpha 2. They are already cophase because alpha 1 and alpha 2 are real valued into s of t, yes like before I do have two noise terms eta 1 of t e power minus j phi 1 plus eta 2 of t e power minus j phi 2. It is a very important observation. This is nothing, but a phase rotation of the nose sample. The noise sample is a complex sample you just need a phase rotation.

So, this is nothing, but a phase rotation of the noise sample, right. So, that means, the power is not affected power in the noise is not affected. You just basically rotate it. You take real square plus magnitude imaginary square, you will still get the same value. The power is not affected. You did not affect the noise, but you did something very advantageous to the signal. So, let us see if we can assess what the benefit that you have obtained is. So, what is the average SNR? Average SNR says expected value of alpha 1 plus alpha 2 whole square into E s divided by I have two noise sources. So, therefore, I have to write 2 sigma n square correct.

So, this can be written as expected value of alpha 1 square plus alpha 2 square plus 2 alpha 1 alpha 2 e s divided by 2 sigma n square. Couple of results that we already know I just want to write it down, so that it will be easy to get to the final answer. Expected value of alpha 1 square is 2 sigma square. This is same as expected value of alpha 2 square. What is the expected value of alpha 1? Do not say 0. It is equal to sigma root pi over 2. So, please substitute into this result. So, you should get 2 sigma square plus 2 sigma square plus sigma square pi divided by 2 sigma n square. So, if we were to rewrite it with just a simple level of manipulation, do not forget E s in the numerator. It should be 2 times sigma square E s divided by sigma N square. That would be SNR of one antenna into 1 plus pi divided by 4 which is approximately 1.7 times E s by N naught whatever was if you call this as Es by N naught, ok.

So, the important thing to note is that when I just added the two, I did not gain anything in terms of SNR, but when I just did the phase rotation, it seems like to have gotten a huge benefit. So, almost 1.8 times the antenna SNR of single antenna. So, cophasing is much better than selection diversity cophasing. I will just write greater than just stands for better cophasing is greater than SNR of single antenna. It is also better than SNR of selection diversity. We have just shown that a selection diversity m equal to 2, but what is the price that you had to pay for this? What is it you have to do? You must know phi 1 very accurately because if you did not know phi 1, what will happen is, you will rotate basically what are you doing, you are taking the received signal one rotating it in a particular direction, so that it does not have any the signal along the real axis, right.

Basically you have taken out complex rotation part and likewise you took the second also. So, that is why they got aligned on the real axis, but if you did not do that, they will still have some angle. Then, you will lose some of the the benefit that you would get. Does it affect the noise? No. It is just another rotation. It does not matter, noise does not get affected, but your signal gain will get affected.

So, the accurate estimation of the phis is going to make a very important difference. So, let us quickly write down what are some of the differences that you would see between the two of those selections versus.

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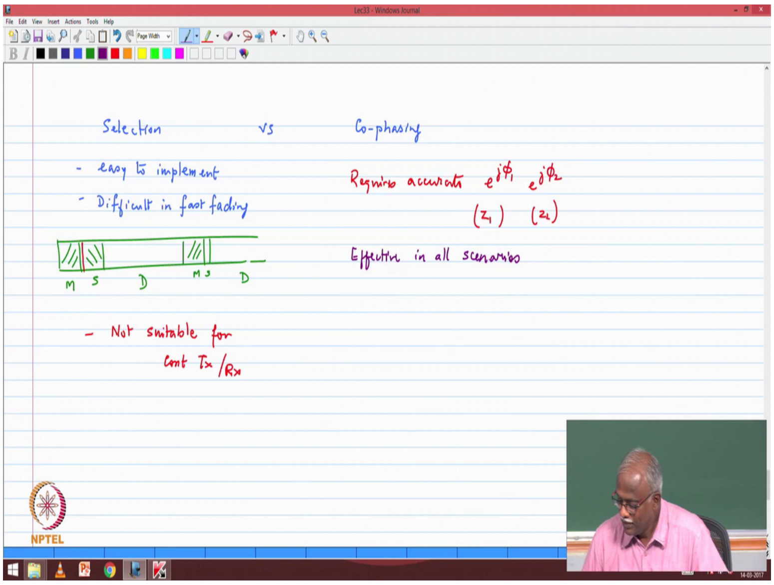

Selection versus cophasing, this is easy to implement. You just have to pick one of the two antennas. You do not have to measure. Of course, you have to measure the SNR of the two antennas, but you do not have to get the complex channel gain; one thing that is little difficult in fast fading because the antennas will keep switching with the one that is better fast fading, ok.

So, because the antennas will keep switching and it is hard to keep track of that. How is it implemented? The antenna selection typically is you measure, then you select, right. You have to pick the two and then, you detect, measure, select, detect. So, obviously there has to be a time for you to measure even if you say that you know I am going to my selection is going to be almost instantaneous, the minute I measure I am going to do that, but still you have to do the measurement and you have a reliable measure of which one is the stronger antenna. So, there is a time when you have not done measurement of either antenna 1 or antenna 2, you are still deciding. Of course, after some period of time, you will repeat the process again. You will do measure, select and then, detect again.

So, the important point is, this is not suitable for continuous transmission. If you have to detect continuously, then when do you do the measurement. So, suitable selection diversity is not suitable if you have to detect continuously for continuous transmission reception. So, again those are some, but on the other hand cophasing it requires accurate estimate of e power j phi 1 and e power j phi 2. In other words, this is the same as saying z 1 and z 2. The way you would do it is, you would estimate z 1 and z 2 and take the phase of that.

The good news is that it is effective in all scenarios, even if you have continuous transmission, not a problem. You have to keep tracking it, you have to estimate it, but effective in all scenarios. So, of course it has got slightly better performance than selection diversity to begin with. So, this is a good result for us to keep in mind. Any questions on what we said so far? So, as always we ask the question yes, it is good that we know that it is better than selection exist cophasing, but we want to know what is the optimal, what is the best that we can do in this environment.

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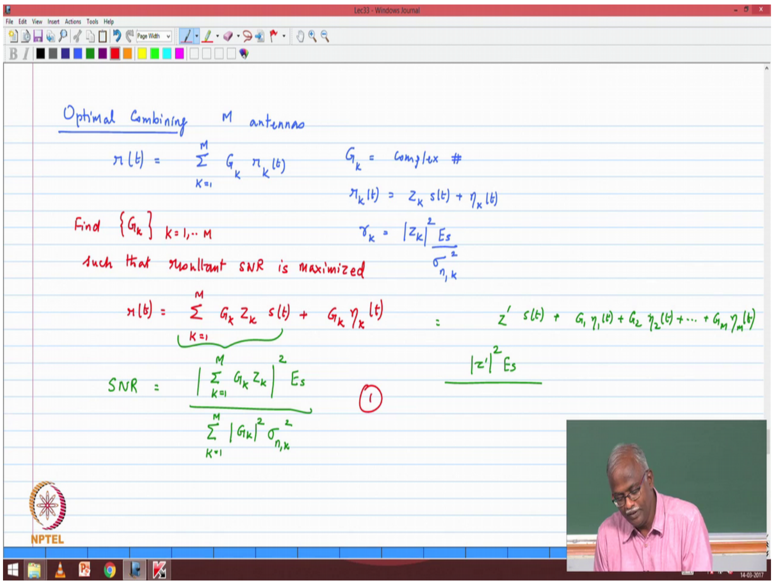

So, we will now address the question of optimal combining and the result will come out to be, it will be such a intuitively satisfying result that you know the whole thing about diversity is sort of starts to really become fascinating. So, here is the problem statement. So, optimal combining we have m antennas and I am allowed to apply a complex rotation to each of the antennas as antenna signals. So, k equal to 1 through M. G k is not gain. It is a complex rotation. If it is confusing with the gain term, please replace it with some c k or d k, something else which does not have any other ambiguous notation. I apologize because I did not think of the gain part r k of t.

So, g k is a complex number and you are allowed to choose it to be anything. The r case are given to be r k of t is a single tap which means it is a non-frequency selective fading or frequency flat fading channel z k times s of t plus eta k of t. That is your received signal. The instantaneous SNR gamma k is given by mod z k square E s by noise in the nth antenna. Again that depends on the gain or you know it depends on the electronics. So, it could be that the antennas on the different antennas, the noise on the different antennas can be different. I mean if it is connected to low noise electronics in its receiver, it could be different.

So, this is the most general case. Normally we would have said it is all the noise terms are the same, but in this we are even allowing provision for the noise power of the antenna to be different. So, if it is different than it will be the instantaneous noise would be given by this. So, the problem statement is find the optimum set of G k, the set of G k, where k equal to 1 through m, such that the resultant SNR of the combined signal resultant SNR is maximized. So, I have to pose it as a problem of estimating the SNR and then, I have to apply the process of maximization and then, show what is the choice of G k that will achieve that maximum that we are interested in, and it turns out that is actually a much simpler than what it sounds. So, let us write it down. It is just a two step answer, but let us just formulate it carefully.

So, r of t if I substitute the result, the expression k equal to 1 through m g k z k s of t plus g k eta k of t. Do not forget the g k on the noise term as well. By now we have computed SNR enough number of times. So, I am going to trust to help me compute the SNR for this expression. There is the signal part, there is the noise part. Help me. So, the signal part if I take it will be magnitude summation k equal to 1. It will be like a one complex coefficient representing that summation k equal to 1 through m g k z k magnitudes square times e s.

That will be the signal component every one with that basically the complex gain, that is multiplying the signal. You take the magnitude square of it in this time. In this case it happens to be that it is not one term, it is several terms that are adding together to give you that complex gain noise. It is easier for us to compute because there are uncorrelated sources. So, therefore, you just have to take the variances of each of those.

So, the denominator will be summation k equal to 1 through M mod G k square sigma n, k whole square. I just want to make sure that everyone is comfortable with this. Basically we have written down expression, where you think of it like this. Think of it as some z prime times s of t plus some G k times eta k of t plus the summation of the terms would be g 1 eta 1 of t plus G 2 eta 2 of t plus G n eta m of t. So, basically I am taking mod z dash square times Es as my numerator and the variances of the denominator is all the other terms. So, basically that is what is happening here and here. Again some very useful results from vector algebra will help us in the process of you do nott even have to do the differentiation. So, we have a result known as the Cauchy Schwartz in equality.

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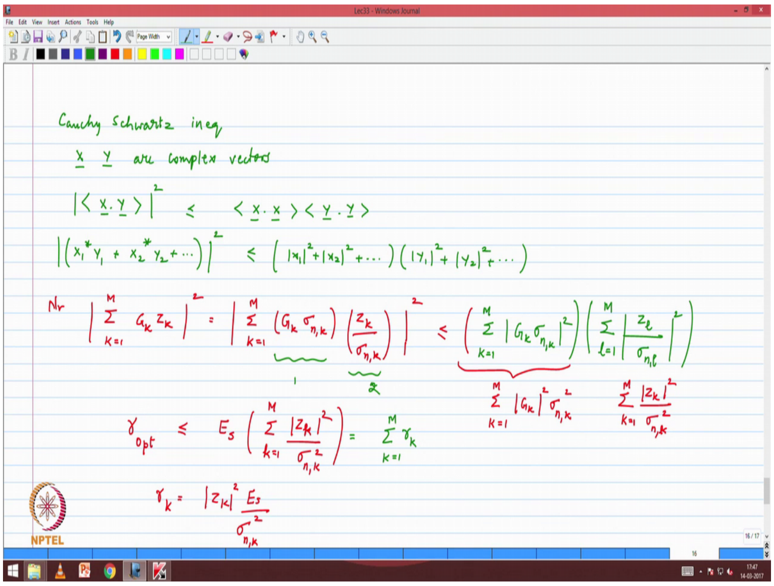

Cauchy Schwartz inequality, it turns out to be very useful for us. In this particular context, Cauchy Schwartz in equality says that if x and y are complex vectors, are complex valued vectors, then the following result is valid. The inner product of x dot y inner product of two vectors means you take the transponse conjugates of 1 and then, the other one. So, basically inner product is a scalar value. So, you take x transpose conjugate times y and then, you will get the inner product.

So, the inner product is of the magnitude square is less than or equal to the inner product of x times the inner product of y. So, if you were to rewrite this, this is basically saying x 1 star y 1 plus x 2 star y 2 plus dot dot dot the number of vectors that you have the magnitude square of that is less than or equal to magnitude x 1 square plus magnitude x 2 square dot dot dot magnitude y 1 square plus magnitude y 2 square dot dot dot. So, basically this is the Cauchy Schwartz inequality.

Now, I would like to use the Cauchy Schwartz inequality to the numerator. The numerator is summation k equal to 1 through m g k z k magnitude square. Notice that it looks very similar to the expressions that we have or very similar to form that we are interested in, but I will do it with the slight manipulation that will help us in the final result. So, I am going to multiply and divide by the same quantity. So, it is not going to change the expression, but it will help us summation k equal to 1 through m g k sigma n, k. I am going to multiply by sigma n, k. I am going to divide z k by sigma n, k. So, actually I did not do anything, but just re-did this and then, magnitude square.

So, this is the left hand side of the Cauchy Schwartz inequality. So, this is always less than or equal to the inner product of those vectors by themselves, so the inner product of the first vector. For me vector number 1 is corresponding to this one. This is vector 1; this is vector 2; the term corresponding to vector 1 summation k equal to 1 through m mod g k sigma n, k magnitude square correct.

The second inner product comes out to be summation. I will just use a different variable for the summation l is equal to 1 through m mod z l square divided by sigma n k. Well, I need to take the modulus over the whole thing z z l by sigma n, l magnitude whole square may seem like may be not very clear as to where we are even heading with the whole, but probably just in one step, we can explain this term sigma is a real valued number. So, I do not need to keep it inside the modulus.

So, this can be rewritten as summation k equal to 1 through m mod G k whole square sigma n, k whole square. Am I right? The reason for doing it is the same term is in the denominator. So, that will cancel of the term in the denominator and what is left on second term, this is summation. I will go back to the k as my variable k is equal to 1 through m z k magnitude square by sigma n, k whole square, ok.

So, my gamma mrc or gamma optimal, I will not even call it MRC yet gamma optimal which is what I am trying to achieve or through this combination process is less than or equal to the signal power. The gain term on the numerator, cancel the gain term on the denominator and what is left is summation k equal to q through m z z k magnitude square divided by sigma n, k whole square. What is the definition of gamma k? Gamma k is mod z k whole square e s divided by sigma n, k whole square.

So, by that this right hand side is nothing, but summation k equal to 1 through m gamma k. So, the result is that if I try to do optimal combining by allowing each of the gain terms to be complex, the best that I can do is the sum of SNRS and of course, if I can find a choice of G k that will achieve this sum of SNRS then. So, gamma opt is less than or equal to sum of SNRS is what we have shown.

Now, let me just give you the following as a task for you to do, but it is very important that you actually try that.

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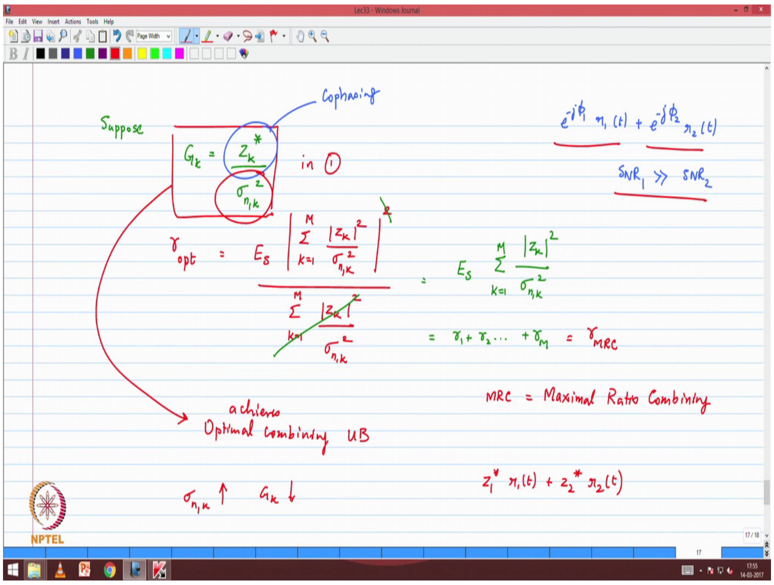

Suppose you choose the following gk is equal to z k star divided sigma n, k whole square. Even before we substitute and see it along the lines of cophasing is this along the lines of cophasing. Yes because when you have z k star, what will it do is, it will rotate in the correct direction, but here you do not take only the phase, you are taking the complex gain also into account and not only that, we seem to be taking the noise variance also into account, ok.

So, again we have to come back to justify what is it that we are doing . So, if you substitute this in the expression, then what you will please I would like you to verify is that the gamma optimal if you, but you will have to go back and recompute the expression, but basically what we will find is the following gamma optimum is given by E s times summation k equal to 1 through m z k magnitude square by sigma n, k whole square, where this mod z k come from because z k conjugate multiplied by z k and then, I have to add all of them, all the different ones together and then, square that to get the gain of the signal component. Then, I have also affected to the noise component by the same G k. So, I have to take their variances into account.

So, basically this would be summation k equal to 1 through m z k magnitude square. I would get sigma power sigma n k power 4 in the denominator multiplied by sigma n square, but you can simplify it to get sigma n, k whole square. You would have to check it, but please do that. It is just one step of substituting this G k in the earlier expression to get it.

So, basically you have to substitute this one, where 1 is given by this expression. So, please substitute G k as given by that expression and verify what happens. Now, this term denominator will cancel one of the terms in the exponent. So, what you get is e s times summation mod z k whole square divided by sigma n, k whole square k is equal to 1 through m. This is equal to gamma 1 plus gamma 2 plus gamma m. So, we have found a waiting term waiting coefficience, set of coefficience that will achieve the upper bound for the optimal combining.

So, this is a form of optimal combining upper bound. It achieves the upper bound. So, this is very useful result. This particular form of diversity combining optimal, it achieves the SNR bound that the sum of the the resultant SNR. So, this is equal to gamma MRC and MRC stands for Maximal Ratio Combining. It is the best form of diversity combining that we know maximal ratio combining because it achieves the maximum SNR benefit for us. Why is it called maximal ratio combining? What is the intuition behind it? First notice that this is the cophasing part, ok.

Now, supposing you had two antennas. Suppose I had just two antennas and you did e power minus j phi 1 times r 1 of t plus e power minus j phi 2 r 2 of t. Suppose this is what you were doing in the cophasing, but I told you that SNR of antenna 1 is much larger than SNR of antenna 2. So, what have you ended up doing? You have taken this one all had a low noise to component to begin with because it had only eta 1. What do you do? You took and add to eta 2 which was much worse than eta and you actually could end up hurting the SNR of antenna 1 because of this scenario, ok.

So, cophasing like this when the two SNRS are very different is not a good idea. Obviously, that would not be the optimum way of combining. So, what is the optimum combining tell you that if the SNR or if the noise variance is very large on a particular antenna, you have to give it lower weightage. You can cophase it no problem, but you have to give it lower weightage. How do you do that? By dividing by sigma n square because if your noise variance is large on a particular antenna, notice that g k will become small.

So, if sigma n, k is large g k will go down which is the right thing to do. You took cophasing, but you do weighted cophasing. So, that is why it is called maximal ratio combining. You combine it with the certain ratio and that ratio is dependent on the SNRS or it depends on the noise variances.

So, last question to close todays lecture for cophasing, I needed to compute the phases. What do I need to do for maximal ratio combining? I need to do channel estimation which means z 1 and z alpha 1 e power j phi 1 alpha 2. That is all.

I need to estimate the noise variances. So, that is a non-trivial task, but if you are willing to put in the effort to do the noise variances, then you will get the optimal combining or you can make the assumption saying well all the noise variances are the same in which case what should you have done in the cophasing, what could you have done in the cophasing.

You could have done z 1 star r 1 of t plus z 2 star r 2 of t is. That correct that would have been. That is pretty much because you assume sigma n 1 and sigma n 2 are the same.

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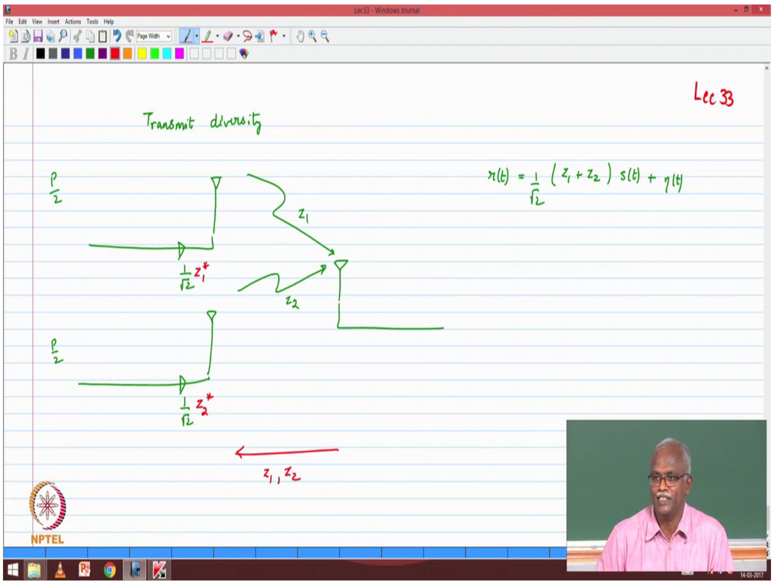

So, basically then that brings me to my last figure for today. If I cannot do diversity in the receiver, I do something called transmit diversity and I will stop with this. So, the transmit diversity scheme says I will now put the diversity component at the transmit side, ok.

So, now we have to be a little bit careful, so that we make the, obviously, there is a channel gain from here. This is z1. Let me call this as z 2, but in order to be a fair comparison, I have to transmit the same total power. That means, I must transmit power p by 2 from here, p by 2 from here and the way to do that p 1 by 2 is to make sure that I do 1 by root 2 here, 1 by root 2 here correct. That is only then it is a fair comparison because this should be the same as what I do in the case.

So, I have divided my power into p by 2 p by 2. I am transmitting from these antennas. So, what will I get at the receiver r of t will be z 1 plus z 2 times s of t, right. How many noise terms?

Student: 1.

One noise term because there is only one antenna at the receiver, right. Now, you may say well you know what actually I have done better than you know a diversity combining at the receiver. No, wait a minute. There is a 1 by 1 by root 2 sitting in the front which will scale the advantage that you got by having only one noise term actually went away, but more importantly what did you do, you did z 1 plus z 2 which is going to give you no advantage at all. So, what should you have done? So, what we should have done is 1 by 2 z 1 star 1 by 2 z 2 star and then, transited it. Then, you would get optimal combining at the receiver and this would be a very good mechanism except that you need a feedback channel which tells the transmitter what z 1 and z 2 are, ok.

So, if you have feedback from receiver to the transmitter, then you can actually move the diversity part on to the transmit side and get some additional benefit or basically get the same benefit. So, if you say that my receiver is low complexity, simple receiver can do something at the base station side, yes provided you give me feedback. Think about it. We will build on this in the next lecture.

Thank you.