Introduction to Wireless and Cellular Communication
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Lecture - 33
Computer Simulation of Rayleigh Fading, Antenna Diversity
Introduction to Diversity, Antenna selection diversity

Good morning. We begin lecture 32 with the quick summary of lecture number 31, and then in today's lecture hope to cover a lot of very interesting aspects of antenna diversity from the simplest schemes to the more complex schemes and then finally on to the optimal scheme.

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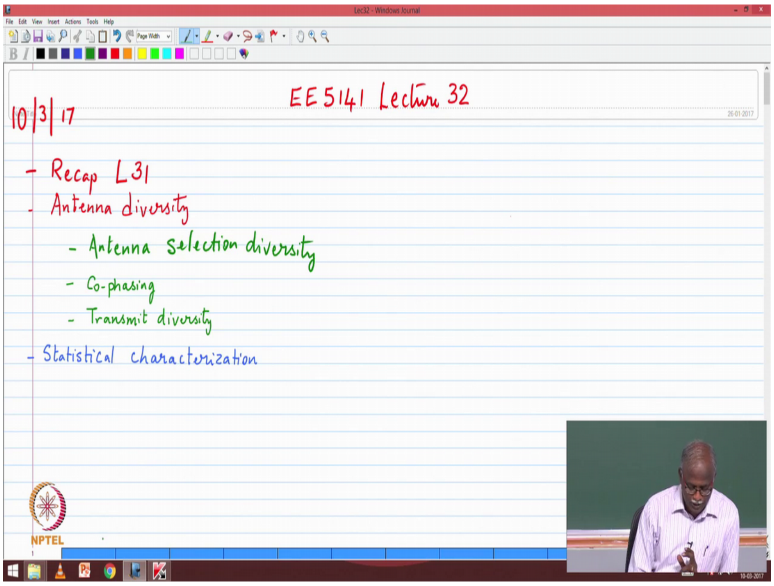

So, again all of this hopefully will build up intuition will also build up a strong analytical framework and help us to work with the concepts of antenna diversity that are very important in the context of a wireless channel particularly, if you are experiencing severe fading such as Rayleigh fading.

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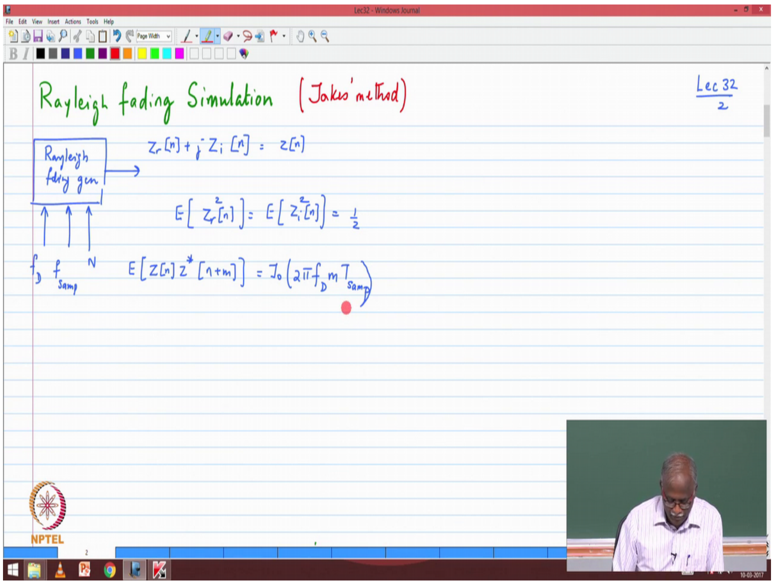

A quick summary of what we have discussed in the last class. We have through the various steps have developed a model for which by which we can generate Rayleigh fading in a computer which satisfies all of the properties that we are interested in. This particular time domain model is referred to as the jakes model, people do not even call it jakes Rayleigh fading model they just say jakes model everybody assumes that you know what they are talking about it is the Rayleigh fading simulation or Rayleigh fading wave form that is generating using the jakes method.

So, jakes model is very widely used and this is something that I would like you to be very, very familiar with comfortable with and actually be able to program and used because the model itself is quite simple it is very versatile in terms of it is application. So, typical jakes model a block that implements jakes model will ask for 3 inputs, one would be the Doppler frequency the rate of fading depends on the Doppler. How much correlation is there between successive samples? That depends on the Doppler it also depends on how finely you are spacing it in time. So, the sampling frequency is also very important and the length of the wave form that you are asking the, the fading block to generate is also an important parameter.

So, what comes out of it is a discrete time sequence complex sequence satisfies the properties that the real and imaginary parts are Gaussian. Both have equal variance equal to half and then the autocorrelation satisfies a the Bessel function as we have studied it in the in the in the analytical model. Most often the practical realization does not require you to have just one tap, but will have required you to have multiple taps.

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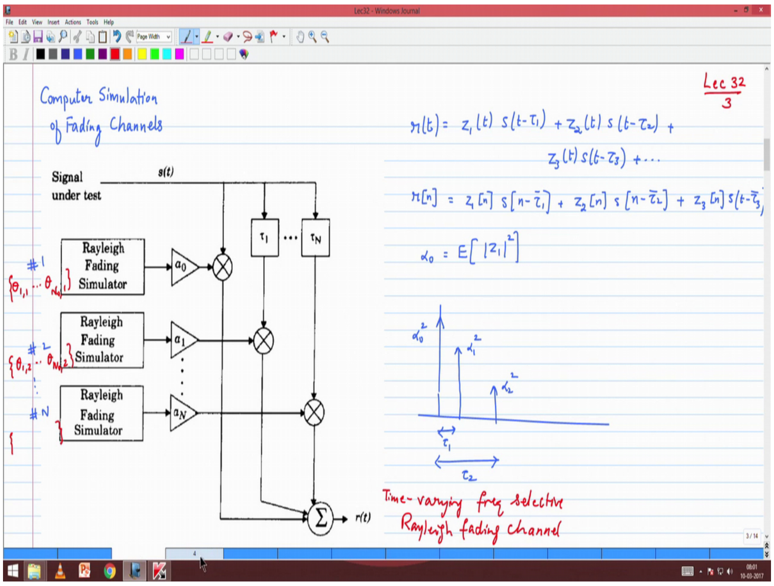

That means, you will have to call the Rayleigh fading simulator multiple times if you just call them and not specify the random phases. So, what is important is that you would have to set different random phases for each of them.

So, for example, this one you would have to set random phases theta 1, 1 to theta N not comma 1. So, you would have to specify this one you would specify theta 1 comma 2 theta N not comma 2. Because if you did not specify these random phases both of them will generate the same way form, because there is nothing else that is different between the 2 all of them have the same frequencies all of them have got the same scale factors the So. So this is very important that depending upon how many Rayleigh fading coefficients you want to simulate you must generate that many random phases.

Now, typically you would want these to be uncorrelated with each other basically that they are in the fading is independent on each of the taps. Now this as we talked about yesterday does not guarantee you that the fading wave forms are uncorrelated. The only way to guarantee that there is uncorrelated is to use the dent variation of the jakes model. So, again that is something that I hope you will be able to read and understand and appreciate and also implement.

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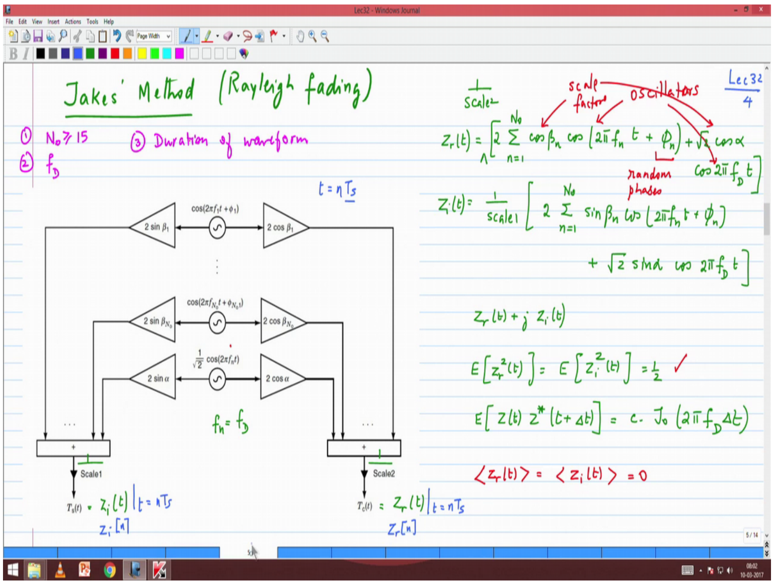

The basic model of jakes just, So that it is always fresh in your memory there are n plus 1 oscillators with scale factors which are then combined with a appropriate scale factor the scale factor is to ensure that the variance is equal to 1 half. So, you get the real part and the real part and the imaginary part and you can then combine them to get a complex coefficient. In practice this is these are not continuous variables we use the discrete time. So, t is equal to nT s you choose the sampling frequency and then you increment it by sampling frequency and then compute the model. So, what you get out of this model is actually a discrete time sequence. So, this would be Z r of n this would be Z i of n with t equal to nTs ok.

That is the sampled version of the jakes model which is what we will need for our simulation purposes. Again just to refresh your memory

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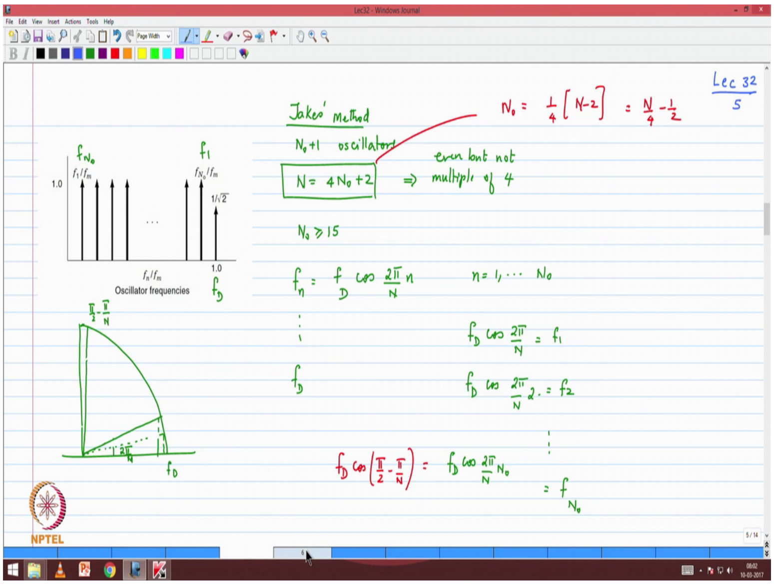

There are n plus 1 oscillators all of them have got equal weightage except the one at the Doppler maximum Doppler frequency. So, if you think of it as a quadrant of a circle then the one that corresponds to the x axis corresponds to the maximum Doppler and then all of the other oscillators are evenly spaced in that region.

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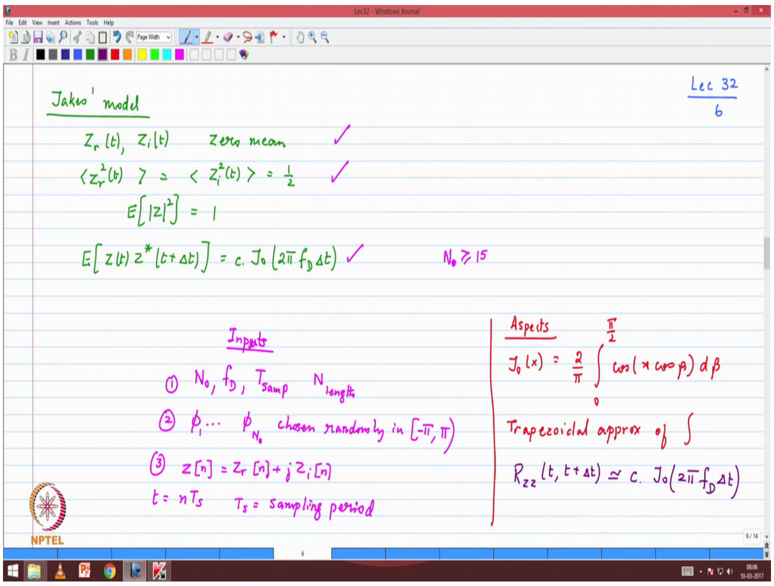

The modification, so in a sense jakes model satisfies the properties of 0 mean the complex Gaussian random variable having unit variance and the appropriate autocorrelation function. What we specify is the Doppler frequency length of the wave form that we want to generate and also the number of the number of oscillators that we are going to use.

So, I know this is not length this is sampling frequency. So, this is the sampling frequency. So, N not is also needs to be specified because N not is a variable typically we take N not greater than or equal to 15. That is more than sufficient to get the correct statistical behavior, but you can choose a larger value because a larger value gives you better approximation of the trapezoidal approximation ok.

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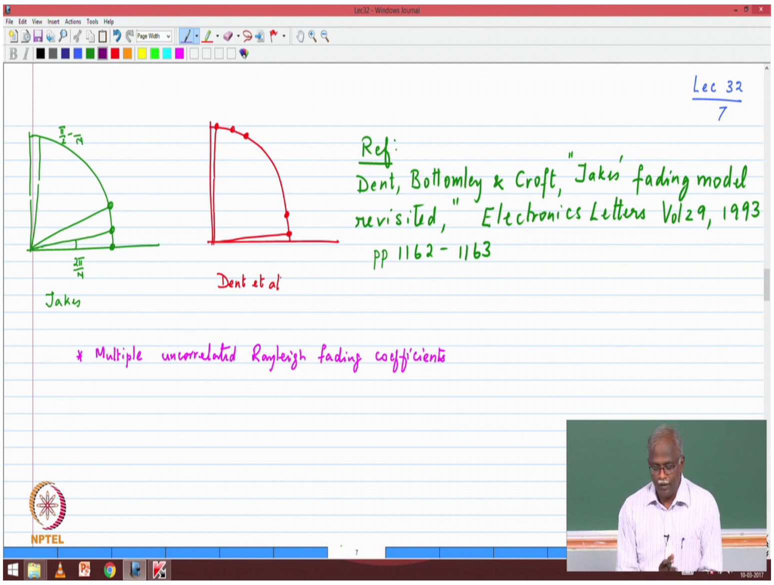

So, 5 1 to 5 N not are chosen randomly and this is the wave form that is obtained. A quick observation very often we want to generate multiple uncorrelated Rayleigh fading, multiple uncorrelated Rayleigh fading coefficients. And we will see today in a in a few minutes why these multiple Rayleigh coefficients are being generated. Multiple uncorrelated Rayleigh fading wave form coefficients both are wave form is what is actually happening coefficients are when you observe it at a instant of time, you get a sample of the fading wave form.

So, if you this is the goal that you want generate multiple uncorrelated ones, the jakes model gives you a way to do that. You have to generate it using our randomization of the thetas, but a guaranteed or an improved method is given by dent et al where which says shift the oscillators slightly, it is anyway up trapezoidal approximations. So, a between this and this there is not a big difference. This model allows you to interpret all the oscillators have got the same weighting function which then interprets it using the Walsh Hadamard, the first row of the Walsh Hadamard matrix and then that tells you that the subsequent rows of the Walsh Hadamard matrix, can be used to generate any uncorrelated wave form wave form that is uncorrelated with the first one ok.

So, may be to summarize the in the jakes model I will just add a couple of comments to the to the right hand side. We have used the following results in our derivation, J 0 of x we have used the expression 2 by pi 0 to pi by 2 cosine of x cos beta d beta, that is the expression for the J not. And we have use the trapezoidal approximation trapezoidal approximation of the integral and that is what we have used to show that the wave form or the auto correlation R zz of t plus delta t for the wave form that we have generated is approximately some constant times J 0 2 pi f d times delta t, satisfies the correlation that we are looking for, the time correlation ok.

So, that is a summary of jakes model we will definitely encourage you to try out will attest give you 2 computer exercises where this will be utilized and of course, welcome to try more because you know as you can see there are several things that can be verified or validated using simple computer experiments. So, we will not implement the dent model as a computer exercise, but you can see that changing it once you have the jakes model implemented modifying it to implement the dent method then using that to generate multiple uncorrelated wave forms is something that I would definitely encourage you to try, ok.

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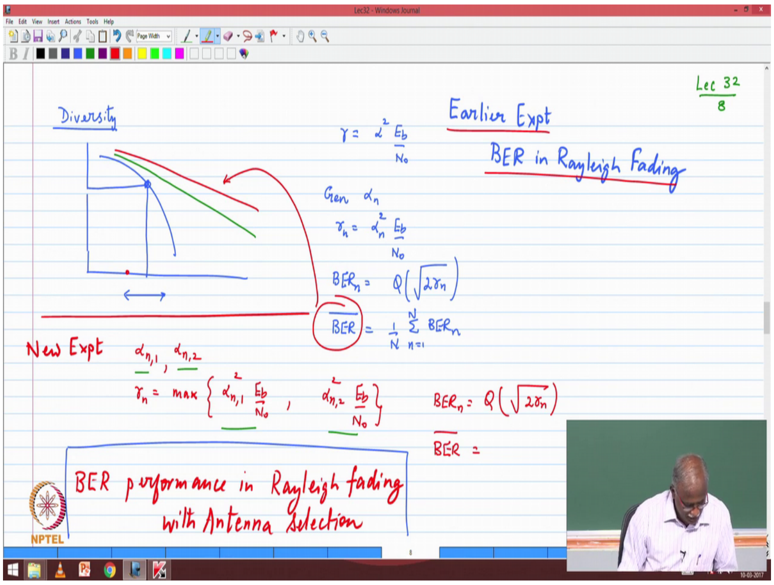

The last point that where we ended last lecture was the notion of diversity; so, the first experiment that we had done as a computer experiment was a to look at the effect of b r in Rayleigh fading. So, you took the AWNG graph generated a Rayleigh coefficient which then basically perturbed the SNR, instantaneous SNR you computed the corresponding BER for the for the instantaneous SNR value. And then you repeated this experiment multiple times and then took the average. So, this average is what we the this average BER is what we saw was reflected by this graph. I hope you are comfortable you see the relationships.

Now, the whole notion of diversity at an intuitive level says that is it is along the following lines of thought just, just too sort of explore the concept of diversity. In this above experimental scenario if you had the option of trying to random variables basically. Let us assume that you had for every instant of time you had 2 Rayleigh coefficients, they will give you instantaneous SNRs alpha 1 square E b by N not alpha 2 square E b by N not.

Previously you had only alpha previous you had only the only the 1, 1 term now, you have got 2 and you are allowed to choose the maximum of the 2. There is no reason why would choose the minimum because that is that is the idea is that you want to get a better SNR. So, you would chose the maximum of the 2 and continue the rest of the experiment as if pretend as if nothing else happened.

So, all of all you are doing is the same experiment except that the instantaneous SNR is not chosen from just one random variable, but it is chosen as the maximum of 2 identical Rayleigh random variables. And what you will find is that you cannot do worse than the previous case you will always do either as good or a better sometimes much better than the previous case. And that is why you will see the green line is, it possible that I will reach the blue line in this method.

Student: (Refer Time: 12:19).

Pardon.

Student: Increase the diversity.

If you keep increasing the order of diversity than may be eventually you will get to something that is closer to the blue line. But again what you will find in this case is that there is something called the law of diminishing returns. As you keep trying this experiment you know over large number of variables you will find that the benefit increasing from one to 2 is very significant 2 to 3 is less 3 to 4 is even less. So, basically it is a law of diminishing returns, but our goal as we have always said is that it will be nice if we can approach the blue graph AWNG graph.

So, this experiment I would describe it as BER in performance in Rayleigh fading. We have not actually generated the Rayleigh fading, but we have an intuitive experiment with antenna selection. And the antenna selection part actually happens here where you are choosing between the 2 antennas and that is an interesting observation that that we will pick up.

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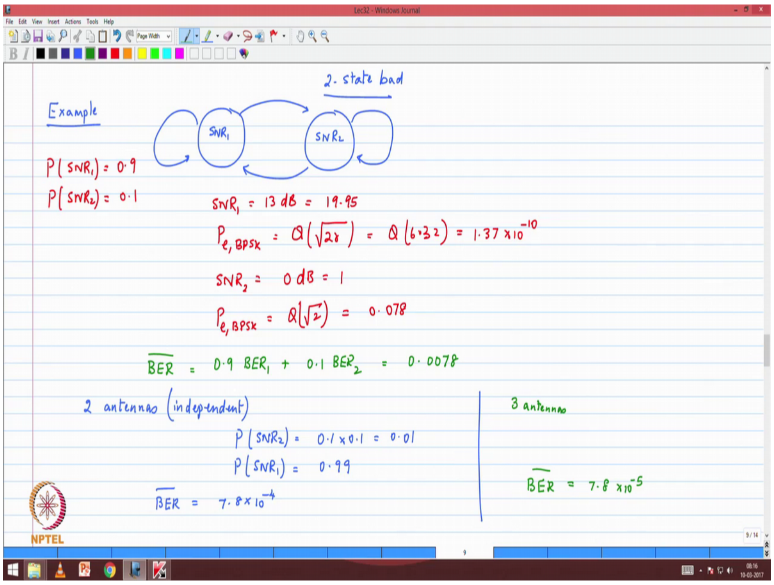

So, let me start with an example, an example that will tell us why antenna diversity or antenna selection diversity is a very good thing. So, let us assume that you have a system which has the following characteristics. It has got a 2 states basically it is not all SNRs are not possible it has got 2 states: one is called SNR 1, the other one is SNR 2, this is like a finite state system. So, you can remain in state 2 or you can transition to state 2 and then you can transition back or you can remain in SNR state 2 that is very simple system it is a 2 state system.

So you can, you can take into account the transition probabilities you can say what is the probability that you are in SNR 1 what is the probability that you are in SNR 2 and let us assume that all of that has been done and the final result has been given to us. So, this is a 2 state model, a good state and a bad state. And there are several interesting scenarios of analysis based on this, but this is more for us to explain the concept of diversity.

So, based on the analysis of this finite state machine we are told the probability that you are in state 2 SNR 1 is 0.9. And probability that you are in SNR 2 is 0.1, and the good news is that SNR 1 is 13 dB that in linear scale would correspond to 19.95. So, if you are looking at a BPSK system, just So that you get a feel for the numbers this would be Q of root 2 gamma where gamma is 19.95, you can please do the calculation this would be Q of 6.32, 6.32 this comes out to be 1.37 into 10 power minus 10.

For a wireless channel this is amazing and your very, very happy that one of your states which is their 0.9 percent of the time right I mean it is it is a highly probable state. And of course, interested to look at what is the other system the SNR 2 unfortunately is not such a good one it is 0 dB. 0 dB means the SNR is equal to on the linear scale it will be equal to 1. So, the probability of error not so good this will be Q of root 2, which comes to be 0.078 does not look too bad, but anyway compared to 10 power minus 10 this is not so good.

So, now we want to calculate the average BER, BER average. Just like you did in the previous experiment, there they were huge number of SNRs possible because you were generating them instantaneously it is a simpler version. Where we say that the average probability is 0.9 times BER 1 plus 0.1 times BER 2, BER 2 just do the calculation and you will find that it is 0.0078. What happened if no where is your what happened to your 10 power minus 10, because that you know really did not contribute to the bit error rate what was actually dominant even though it was only at 0.1 percent of the time 0.10 percent of the time, is that it was point is the bad SNR state ok.

Now, here comes the crucial observation. Supposing I told you that you had 2 antennas both of which are in this environment, but they are independent of each other 2 antennas which are independent. For each of those antennas the BER 1 probability is 0.9 the sorry, the SNR 1 probability is 0.9 SNR 2 is 0.1. Now tell me, what is the probability of SNR 2? When I have 2 antennas and both of them have got identical statistics, what is the probability?

Student: 0.1.

0.01, because both antennas have to be in the bad state; so only then you will be stuck with in even if one of the antennas is at the good state then you will you will get the benefit of the. So, this is 0.1 into 0.1, 0.01 which tells me that the corresponding probability of SNR 1 under this assumption will be 0.99. Now compute the BER improved it goes up to 7.8 into 10 power minus 4, just as a interesting exercise try 3 antennas and you will find that it is even better. The average bit error rate goes up to 10 power minus 5, 7.8 into 10 power minus 5.

So, you starting to see the benefit is of the antennas selection diversity. So, something that you know gave us not so good performance with a just by adding a second antenna or third antenna you start to see the huge benefit that is coming from the diversity process. So, this is very important element and this is what we want to exploit we want to understand we want to make sure that we can get the maximum benefit.

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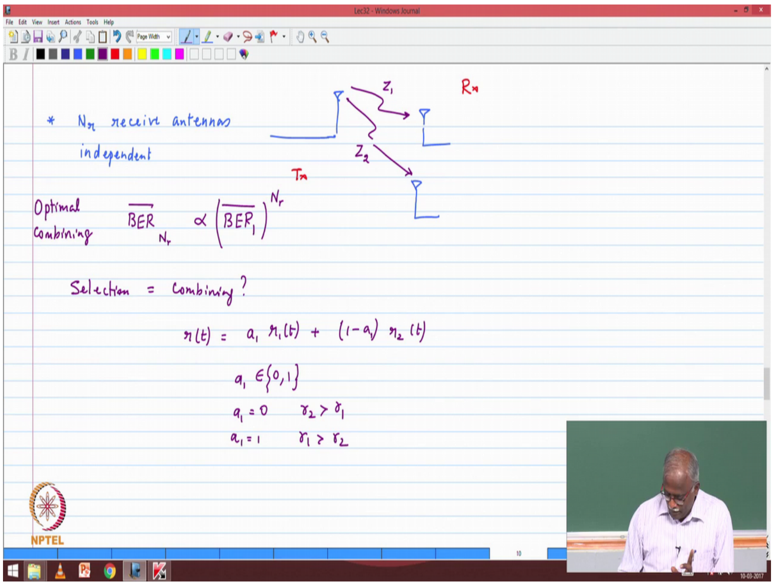

So, the general frame work says that if I have N r receive antennas r stands for receive, because we will also talk about multiple transmit antennas where we will say the number of transmit antennas is N t So N r receive antennas. And if they are independent that is each of them experiences independent fading. Now what do we mean by that? I have one transmitter I have multiple receivers.

So, this is the transmit side, this is the receive side. The channel from only one signal is transmitted, only one signal is transmitted, but is picked up by the 2, 2 receive antennas. And what we are saying is this one experiences a Rayleigh coefficient Z 1 this one experiences a Rayleigh coefficient Z 2, Z 1 and Z 2 are uncorrelated or basically they are they are independent in uncorrelated with each other, ok.

So, each of these antennas are independent means that the Rayleigh coefficients that they experience are independent of the others. So, under this condition what we can what we will show is that in the best of scenarios the average BER, I will write a subscript N r should be of the order of BER 1 raise to the power of N r. Raise to the power of N r it is So is it is a exponent, exponential benefit that you will get if you do the this is under the optimal combining ok.

So, that is why we are So interested. So, in this is such an important concept for us in terms of the antenna and the whole concept of diversity. Now we use the word combining, but what we have actually done is selection. So, is selection a form of combining is it the answer turns out to be yes, because the received signal with diversity we say is equal to a 1 into r one of t plus 1 minus a into r 2 a 1 times r 2 of t and a 1 is element of 0 or 1, a 1 is equal to 0 if gamma 2 is greater than gamma 1 a 1 equal to 1 if gamma 1 is greater than gamma 2. In other words you are going to just set binary weighting functions.

So, it is it is trivially a form of combing, but selection diversity also falls into the frame work it clearly may not be the optimal one, there may be better versions of it and the goal would be is to find out what those best or most optimum versions are. But first since we already know that selection diversity gives us a good benefit we would like to first understand and establish a frame work for the selection diversity, any questions on what we have said so far. As far as the concept of diversity the benefit is of diversity that of the forms of diversity and the benefit and the impact that it will have in terms of our performance, any questions? Ok.

Let me draw a figure for you which I hope will be very helpful in understanding the concept and also reinforcing whatever we have said so far.

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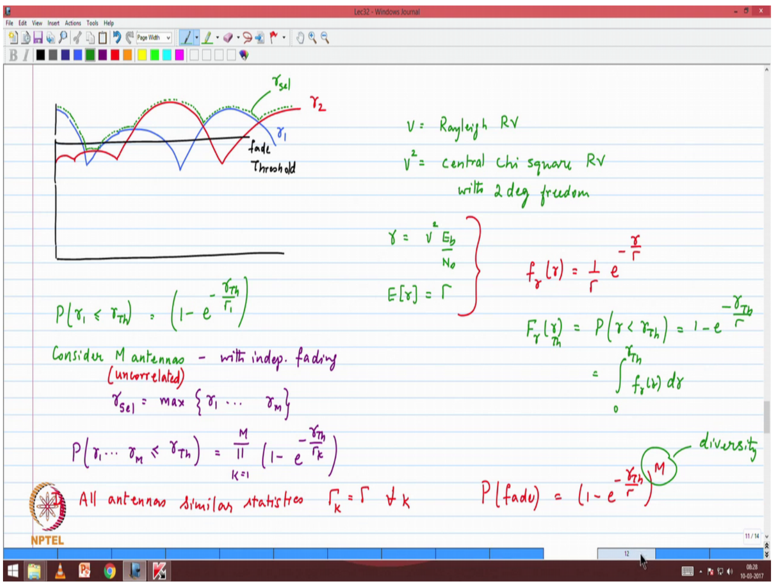

Let us assume that this is the fade threshold. And I am going to have 2, 2 wave forms. So, the first antenna has a signal that that is following this pattern. The second antenna has the following pattern. So, this is the fade threshold, this is the fade threshold the blue lines are gamma 1 this is gamma 2; that means, the SNR of antenna 1 and antenna 2. Now independently both of them experience fading, but you can see from the 2 graph that their fade the times when they go into fade are not necessarily related to each other when one is weak the other one may be strong so, but there are times when both of them are in trouble because you know you can see region here where they both in trouble.

Now, the notion of selection diversity says pick the stronger of the 2 antennas. So, the selection diversity basically follows the following dotted line. It says pick the blue up until this point it is gone into a fade both of them are in a fade, but red is slightly better you follow the pick the red at those points. Then blue comes out of a fade you take advantage of that you follow blue, then red becomes stronger you take red, and then when red is going in to a fade fortunately blue is strong.

So, you completely avoid that fade when blue seems to be dipping once more red is picking up. So, you can see that this would be the gamma selection diversity if I did it correctly at every point in time I pick the stronger of the 2 antennas. So, keep this picture in mind again it hopefully solidifies whatever we have been saying so far.

So, let us quickly establish the analytical framework for selection diversity. So, our basic premise is that the envelop of the fading wave form is a Rayleigh random variable. Again this is just a recap Rayleigh random variable. If I look at random variable v square the random variable v square this is a central chi square, central chi square random variable with 2 degrees of freedom, with 2 degrees of freedom. Again we have already I mean we have seen this earlier 2 degrees of freedom. And the reason we work with this one is because the instantaneous SNR can be written as a the envelop square times E b by n naught.

And expected value of gamma is denoted by upper case gamma and all of this tied to a PDF, which says that the probability density function of the SNR probability distribution of SNR is given by one over upper case gamma E power minus gamma by gamma again this is a form that we have already developed. So, that is that is our reference point you can probably look it up in your notes or rewrite it just for reference.

Now, comes the frame work that we are interested in developing probability that SNR 1 is less than or equal to the gamma threshold. What is that? Basically or may be just write one more step the CDF, CDF of f gamma of gamma is a probability or gamma threshold, is a probability that gamma is less than gamma threshold which is given by 1 minus E power minus gamma threshold by gamma. We have already derived that result as well. This is nothing but accumulative density function 0 to gamma threshold f of gamma f gamma of gamma d gamma. That is the expression that we have received.

So, what I am what you are asking is basically the cumulative distribution function for gamma 1. This would be given by 1 minus E power minus gamma, gamma threshold let us write be consistent T h, divided by gamma 1 where gamma 1 represent the average SNR of antenna 1. Now if we were to extend it 2 M antennas, M independently fading antennas. So, consider M antennas you can call them as uncorrelated antennas.

So, I will write both you can you can specify them as uncorrelated or you can also specify saying that these are M antennas with independent fading; both are the same for us, under this assumption and the fact that you are doing selection diversity. So, selection diversity basically says gamma selection is maximum of gamma 1 through gamma M. You are going to choose over all of these M antennas and the probability that your gamma selection is less than threshold; that means, even under gamma selection you are in a fade.

So, probability that gamma 1 to gamma M all of them are less than or equal to gamma threshold. That is like saying the maximum is less than the threshold that is a equivalent to saying all of the antennas are less than the gamma threshold. Because they are independent becomes product pi of K equal to 1 through M. 1 minus E power minus gamma threshold by gamma K, where gamma K is the average SNR of that particular antenna. Just one more step and we start to get some very, very interesting results.

The interesting results start to emerge here with the first assumption where we say that if all antennas are equally strong or they have similar statistics. If all antennas have similar statistics, another of way of saying it is that equally strong, similar statistics that is a same as saying gamma K is equal to gamma for all values of k; that means, all of them are seeing the same average SNR then the probability of a fade that is all of them being below the threshold will be equal to 1 minus E power minus gamma threshold by gamma raise to the power M. Very good sign because your starting to see the benefit of the diversity factor it is in the exponent it is not we have not shown it in the context of BER, but you already started to show that the benefit is starting to come as a exponential form in terms of the order of diversity ok.

Now, a couple of very interesting offshoots of this particular result: let me share that with you because that will be very helpful for us in terms of our understanding.

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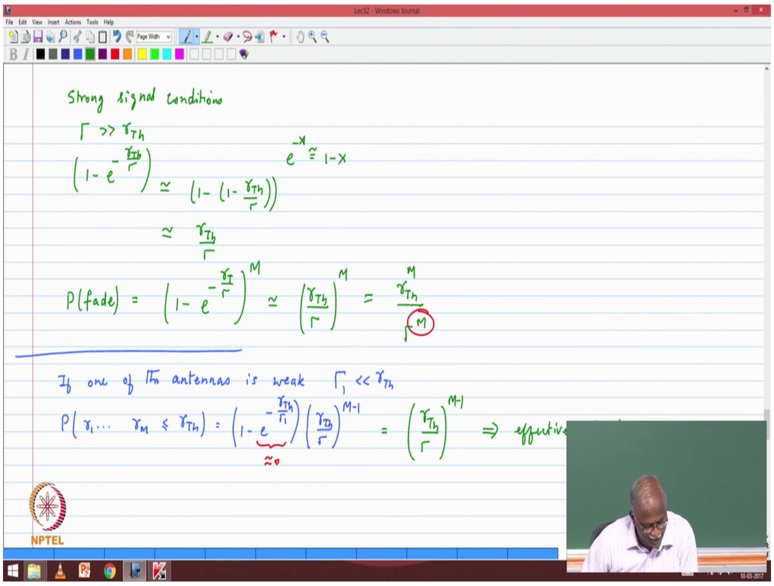

So, first one is what happens under strong signal conditions? Strong signal conditions, this is a very useful result. So, under strong signal conditions says that the average SNR is much greater than the fading threshold, right average SNR is much stronger.

Now, under this assumption what we would like to use is the following 1 minus E power gamma threshold by gamma. Notice that the exponent is a very small a very small. So, you can use the following approximation E power minus x is approximately 1 minus x that is approximation when x is small your that is a very good approximation. So, this can be approximated as 1 minus 1 minus gamma threshold by gamma.

So, this is nothing but gamma threshold by gamma. And the probability of, probability of a fade probability of a fade or probability of outage when there are M antennas we said was 1 minus E power minus gamma T by gamma raise to the power M you can see the directly the direct exponent form this can be approximated by gamma threshold by gamma raise to the power M or in other words this is gamma threshold raise to the power M by gamma raise to the power m.

So, the SNR comes very directly in terms of the benefit of diversity. So, the key thing is these are going to have a huge impact, because if you had only one antenna it will be gamma threshold by gamma. If you have 2 it becomes by gamma square then gamma cube. So, you starting to see the benefit of diversity under strong signal conditions that is a useful result for us to for us to keep ok.

Now, comes a very useful practical result. Supposing one of the M antennas is weak supposing one of them is weak, now when can that happen if you have a handset one of them is an antenna at the top of the handset one of them is in the body of the handset and your hand is around it. Then what will happen the antenna at the top sees a better SNR then the one because your hand is blocking part of the signal. So, you cannot we cannot always assume that all of them have got similar statistics one of them may be weaker. So, if one of the antennas is weak, if one of the M antennas so; that means, M minus 1 are strong M one antenna is weak, if one of the antennas is weak, what happens is weak. So, without loss of generality we will say that gamma 1 is that is the antenna 1 is the one that is the weak antenna.

So, we will make the following extreme condition that gamma 1 the average SNR of antenna 1 is actually less than gamma threshold. Just to make the extreme go. All of the others are much above the threshold. So, now, we are interested in the probability that gamma 1 through gamma M is less than or equal to gamma threshold, that is the probability of a fade. Like before we will write down the cumulative distribution functions for all of them. 1 minus E power minus gamma T by gamma 1 in to the others are all strong antennas. So, I can write them as gamma threshold by gamma raise to the power of M minus 1, am I correct? The reason I could not make the approximation for the first term is because, that approximation is valid only when the exponent is very small.

Now, if a if gamma 1 is much less than the threshold the exponent is actually much larger than 1. Now very interestingly if gamma 1 is much less than gamma threshold this exponent becomes negative of a very large number so which means that this term is almost 0. So, what is the net result a very, very interesting result? So, the net result is approximately gamma threshold by gamma raise to the power M minus 1. So, it looks like there are only M minus 1 antennas it is a very satisfying result effectively only M minus 1 antennas from a selection diversity point of you that is precisely they say situation one of them is very weak you are not never going to choose that one. So, you might as well you can discard that one antenna and pretend that they were only M minus 1 antennas that were present effectively M minus 1 antennas ok.

So, the notions of diversity with intuition with the analytical framework are very, very important for us. So, that is part of the reasons why we are trying to establish this aspect now I would also like to ask you the following situation, ok.

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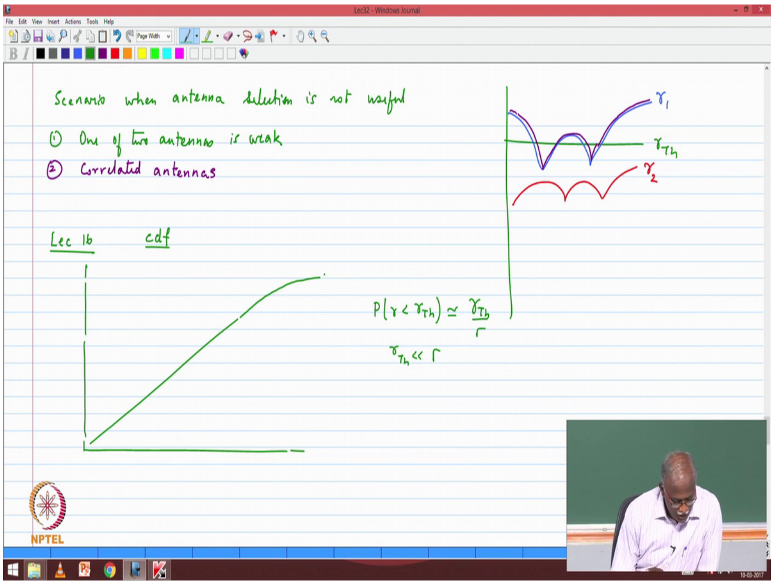

So, this scenarios when antenna selection is not useful, scenarios scenario when antenna selection, when antenna selection is not useful, is not useful, is if one of the antennas basically if you let us when the case of 2 antennas, one of the one of the 2 antennas, one of the 2 antennas is weak right if they are only have only 2 antennas and you have to choose between them if one of them is weak then this it is not going help us. So, here is the pictorial representation of that.

So, this is the gamma threshold, SNR 1 first antenna has got some SNR, So this is gamma 1 gamma 2 is this, notice it is always below the fading threshold. So, it really does not contribute to the SNR benefit right; so selection really. So, in this is a case where it is no use. Do you see any other scenario when SNR selection will not be beneficial at all? Large scale fading well basically means that your having some obstruction with respect to the transmit that is this scenario that is one of the antennas is seeing more obstruction of signal compared to the other.

So, this would be that scenario. So, so the large scale fading would probably be captured here any other scenario where selection has no benefit what is so ever.

Student: Both antennas are equally strong.

Both antennas are equally strong, but if they are independent fading there is actually benefit right, but they are not independently faded. So, basically the scenario where you will not see much benefit if the second antenna also had the same behavior. If this one also was having the same SNR variation instantaneous SNR variation then it does not add to the benefit. So, that is a second scenario the antennas are correlated antennas this happens to be a case where the correlation factor is actually equal to 1. So, whenever the original one antenna is in a fade the other one is also in a fade. Therefore, there is no benefit for this particular element, ok.

Now, little bit of homework for you if you, go back and look at our notes from lecture 16 we plotted a CDF. A CDF, the CDF basically we what we plotted was to show the behavior of the envelop of a Rayleigh random variable. And if you plotted it for a the SNR you would find that the plot will look very, very similar basically what we are saying is the probability that gamma is less than a gamma threshold, which we have just now shown to be equal to gamma threshold by gamma. And when whenever the this value is small and or in other words when gamma threshold is much less than gamma. And basically what we find is that the CDF is more or less linear when you look at, when you plot it on a logs scale you will find that it is linear and the and then eventually it will saturate ok.

So, please do look at that graph that for your just for as a reference I have I have shown it, but it is more important that you actually go back and refresh yourself. That was done with the envelop, this is with the SNR envelop and SNR are just related by a square. So, when you take the dB there is no difference there the plots on a dB scale as exactly the same. So, this is a CDF and basically the CDFs it is a normalized SNR. So, what is normalized SNR? Probability of gamma divided by the average SNR less than, what is on the x axis less than abscissa, ok.

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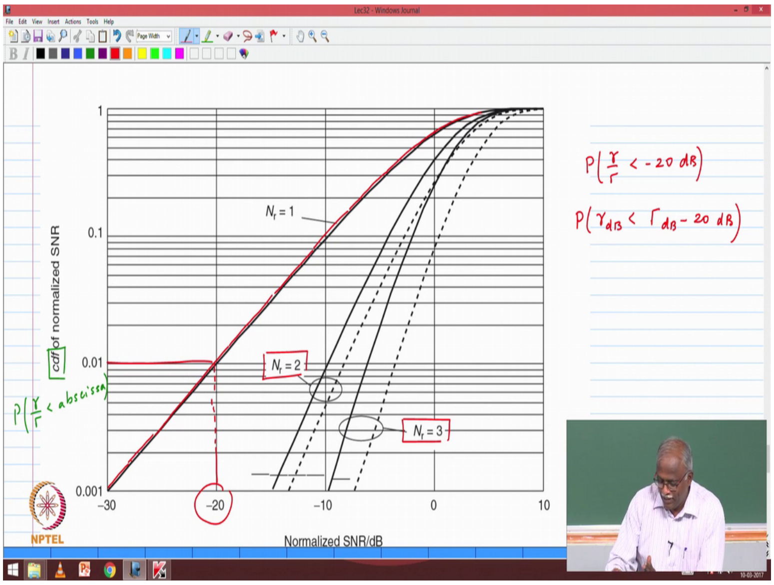

So, this is exactly what we have drawn before, what we had drawn before was this dark line. The line that is here is what we showed previously you can verify that for Rayleigh fading that is the case. And confirm that even for the SNR you will get exactly the same cumulative distribution function. Now when I have the option of selecting between 2 antennas it becomes the first function basically the high SNR approximation becomes this whole square. So, which means that now there is a different slope to the graphs. So, n equal to 2 you can see for a 2 antennas selection and then you can see for n equal to 3 the antenna selection basically look at all the solid lines.

So, you start to see that the CDF is changing and what exactly is the CDF what does it mean. So, so for example, let us take this point 20 dB what does that that tell me probability that gamma by gamma is less than minus 20 dB, right. That the probability, this is the same as saying probability of the instantaneous SNR expressed in dB is less than the average SNR in dB minus 20 dB. The probability that the instantaneous SNR falls 20 dB below the average SNR that is exactly what the CDF is trying to tell us.

When you had only one antenna you had a fairly high probability that you could hit that bad point, that you are instantaneous SNR is 20 dB below the average SNR, but notice when you have 2 antennas independently faded that scenario is almost taken out of the probability is actually very, very low for that. And when you take 3 antennas it becomes even less probable. So, what this is saying is the probability of those bad SNR conditions which is what is cause the causes the fading is actually very beneficially removed in the context of selection diversity.

So, actually by looking at these CDFs and by spending some time with it you can actually get a lot of insights into the benefit do not worry about that dash lines that is for optimal diversity. So, there is optimal diversity which will do better than the selection diversity and this is just to show that what we eventually we will get to that point as well.

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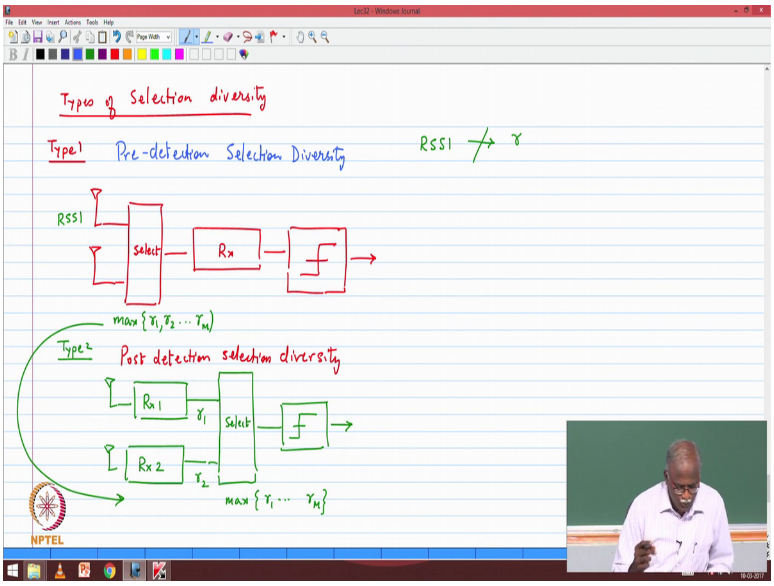

So, now comes the question of what are types of selection diversity, types of selection diversity. And it turns out there are 2 very important types of selection diversity I will just quickly mention it to them we would not spend too much time on those the type 1 says that you have 2 antennas, I have I am going to draw the figures for 2 antennas you can extend it for any number. I will select which of them is the stronger antenna. And then pass it through a single receive chain and then pass it through the decision block, and then I get the output decisions.

There is a type 2 where I take antenna 1 antenna 2. I pass this through R x 1 pass this through R x 2 that is I am I am not yet trying to decide which of them is the better signal. But having demodulated the signals then I decide, select which one you want to do. And the decisions from that particular branch are then are my final decisions, ok.

So, basically what is the selection happening? The selection will be maximum of gamma 1 comma gamma 2 if you had more than M 2 antennas, basically it will be and that is the same for this one as well. Your trying to fix find out which one is the best option for us. Now the differences between these 2 is in the one case you determine a priory which is the stronger of the 2 signals. And that you can do based on RSSI received signal strength basically you take the signal strength from antenna 1 you measure it received signal strength from antenna two.

Now, you know that not always RSSI is a not always a good indicator of the SNR. Sometimes it is not a good indicator especially if there is lot of noise present in the system you cannot differentiate between the two. So, what in this case you are doing is selecting on the basis of antenna hopping that it is a same as maximum SNR. In this case you can actually estimate what is SNR 1 and SNR 2 and then you can do the implementation more accurately saying that I want to choose the best of these SNR. So, the second one is called post detection diversity this is called post detection; that means, you detect which a both signals and then you decide which one you want to choose, post detection selection diversity. And if this is post detection the other one has to be p detection ok.

So, that is the difference pre detection pre detection simplifies the process because only one receiver is involved, but you have to decide ahead of time you have to decide it based on RSSI which may not be a very good indicator on the other hand this one the second one is a better option, but you notice that you have more complexity because you have to demodulate both those signals. So, pre detection selection diversity, just make a note of that it is interesting for us to keep that just mental note of how you would actually implement this in practice, alright.

So, now we quickly want to move in to some of the other aspects of the diversity discussion. And I want you to leave with you something’s for you to think about. We made the following statement that a weak antenna did not really help us much. So, that is a correct statement, but it has to be qualified. And here are some aspects that I want you to sort of think about and then we pick it up, from there in the next lecture. So, the question is there any benefit from a weak antenna, ok

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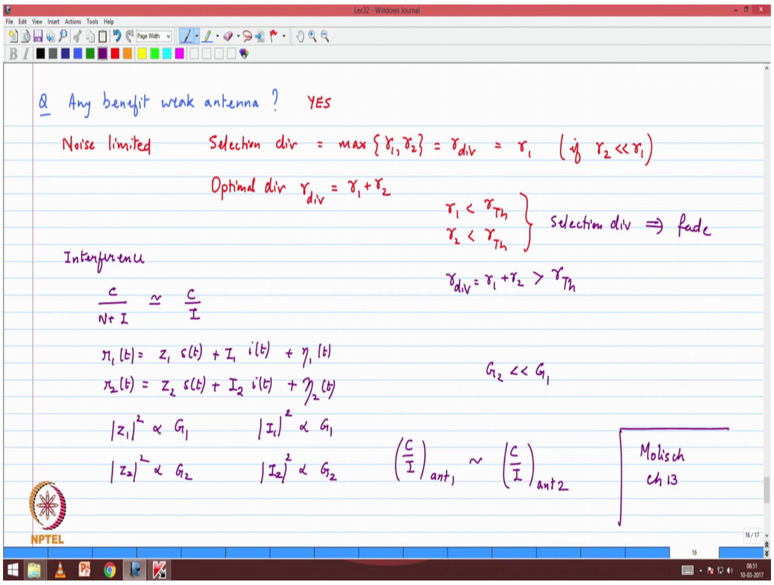

You we saw that selection diversity is sort of ruled out, but is there any benefit at all from a weak antenna. And it turns out that the answer is actually yes, and it is very important for us to know why the answer is yes and how come there is a benefit ok.

So, we will look at 2 particular scenarios. One of them is a noise limited scenario noise limited scenario and the other one is a interference limited scenario. So, in an noise limited scenario selection diversity, selection diversity basically says choose the maximum of gamma 1 comma gamma 2. And this is the gamma diversity the problem as this will always be equal to gamma 1 if gamma 2 is much less than gamma 1, because that is the weak antennas scenario. On the other hand when we talk about optimal diversity, we will show that the signal that comes out from optimal diversity actually comes out to be gamma 1 plus gamma 2. That is one of the results of optimal diversity combining.

Now, this has a huge implication because let us consider the scenario that gamma 1 was less than gamma threshold gamma 2 less than gamma threshold, what would have selection diversity I have told you? You have no hope right both antennas are out you are in a fade. So, selection diversity says, selection diversity says, you are in a fade you are in a fade no chance nothing to do, but optimal diversity combining if I do gamma 1 plus gamma 2 I may be greater than gamma threshold right. So, which means that first of all even if I have gamma 2 is very weak I would not throw it away because under some scenario it may just add a little bit more to the SNR of gamma 1 and make me get some benefit out of that.

So, that is one aspect that we need to keep in mind. The second aspect is when you are interference limited. Interference limited means that c over n plus I, carrier to noise plus interference is approximately c over I. That is the approximation and we will look at is a little bit more. So, the model of the signal is r 1 of t is equal to Z 1 times s of t plus I 1 times i of t I is the interfering signal I 1 is the Rayleigh coefficient that is affecting the interfering signal plus eta of t r 2 of t is equal to Z 2 of s of t plus I 2 of i of t plus eta 2 of t ok.

I am going to write down a statement which I want you to think about it. So, the signal strength Z 1 square is proportional to the gain of antenna 1. Z 2 square is proportional to the gain of antenna 2 the assumption that we have made is gain 2 is much less than gain one. So, we are the question is there any benefit. The scenario that we are looking at is the interfering signal also went through the same went through the same a gain antenna.

So, I one square sorry, mod I one square is also proportional to g 1 I 2 square is also proportional to g 2 which means the carrier to interference ratio of antenna 1 is almost the same as carrier to interference ratio of antenna 2. So, under interference limited scenarios I would never throw away a weak antenna because the antenna it is it is also seeing the same carrier to interference ratio.

Now, how come for SNR conditions it is bad for interference conditions is not so bad. In fact, it may be reasonably good option I want you to think about it we will pick it up from there and build on it. So, basically this is all that we have discussed today is in Molisch chapter 13, and corresponding chapters on antenna diversity in goldsmith and in rapaport.

So, please feel free to look at it Molisch chapter 13 will give you a good frame work for what we have discussed today.

Thank you. We will see you in the next class.