# Introduction to Wireless and Cellular Communication

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## Lecture - 32
## Computer simulation of Rayleigh fading, Antenna Diversity
## Jakes' Method Properties

Today's focus is to complete our discussion on Jakes Method and also to introduce the notions of Antenna Diversity and the benefits of Antenna Diversity. We are going back and forth between simulation theory concept intuitions. So, Antenna diversity is a concept which you will eventually implement using computer simulation for which you will use the Jakes method. So, basically Jakes method is our tool that we will use for any form of Rayleigh Fading Simulations.

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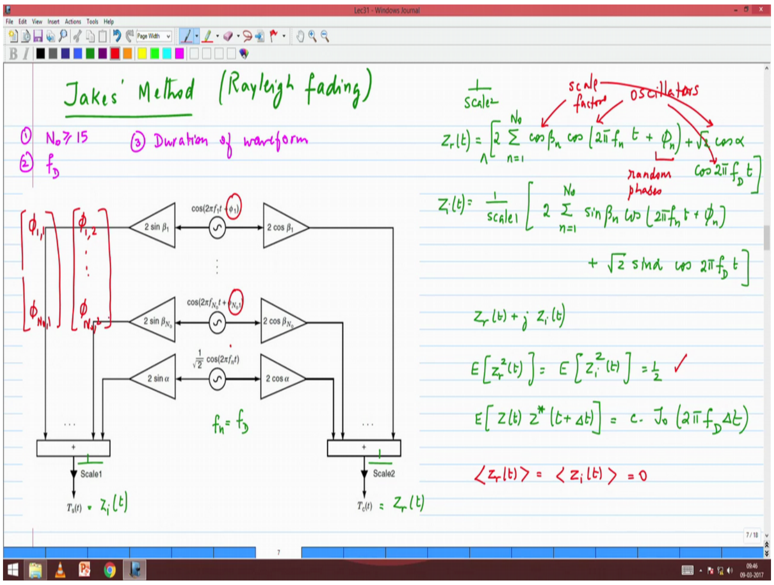


So, before we get into that let us just review the concept of Jakes method. Jakes method uses a set of n, n naught plus 1 oscillator.

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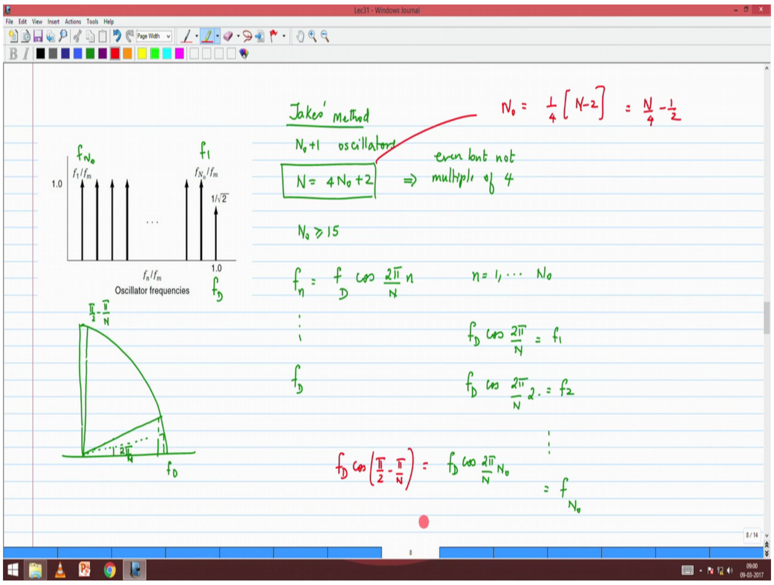


The relationship between n naught and n is given n is a even number, but not a multiple of 4. You got a set of n naught plus 1 oscillator and the frequencies of the different oscillators are given. There is a very deterministic set, but when we go into the implementation, we find that we have some degrees of freedom which is primarily in the form of these multiplication coefficients and also, the phases that are selected for the oscillators.

So, based on frame work was what we had developed in the last class. So, let me pick it up from here. So, here are the oscillators and this represents the oscillators. This is also an oscillator and then, the scale factors, there is also a scale factor here cosine alpha and then, we have random phases. These are random phases and you have them both in the real part and imaginary part and this is the frame work that we have developed. What we had verified in the last class was that Z r of t if I took, the time average is zero mean and so is the imaginary part which is obtained by these two.

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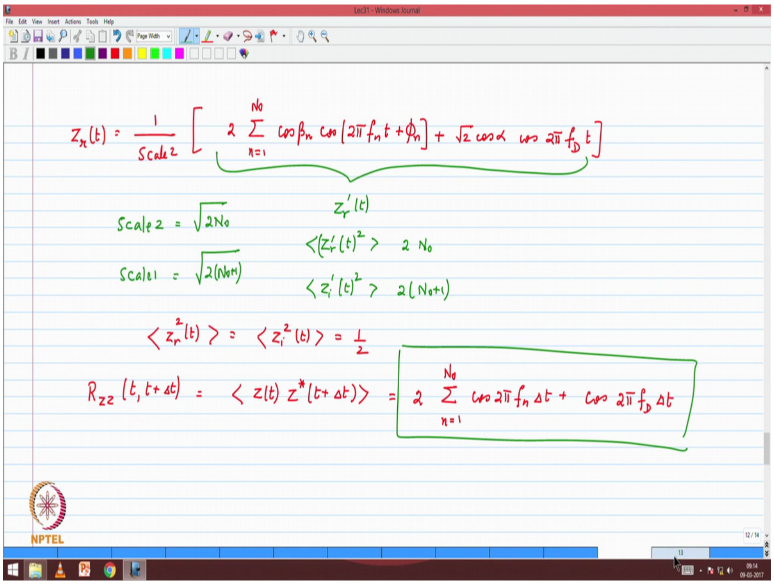

So, we also had reached the point of, so Z r of t just it will be helpful for you to write it one more time. Z r of t is 1 by scale 2. There are N naught oscillators scale factor of 2 N is equal to 1 through N naught, the multiplication terms cosine beta n of the scale factors cosine 2 pi f n t plus phi N. Those are the random phases plus the multiplicative factor for the last term root 2 cosine alpha cosine 2 pi f D. Now, if you were to take this portion of this expression, actually it is not 1 by scale 2, it is actually we wrote it as scale 2, right. It is multiplication by scale 2, 1 by scale 2, 1 by scale 1, ok.

So, expected value of, if I took this portion of it, let me call that as Z r prime of t. Just minus the scale factor the Z r prime of t whole square time average that was what we had computed yesterday. I think that comes out to be 2 times N naught and similarly, if we were to compute Z i prime of t whole square average, it comes out to be 2 times n naught plus 1. So, basically scale 2 has to be chosen, such that scale 2 has to be equal to square root of 2 n naught, scale 1 has to be chosen to be square root of 2 n naught plus 1. So, that will get the variance is equal to one-half.

So, this will ensure that the following conditions are satisfied Z r square of t is equal to Z i square of t that is equal to one-half and I think that was, please make sure that there is no confusion that we have changed the figure as well as our expressions. I apologize. I should have written it up differently. The point at which we stopped in the last lecture was to ask the verification of the following expression, the autocorrelation of the complex channel coefficient z of t t plus delta t and you are asked to show a result. I hope you had a chance that this would be expected value. Let me just write it in terms of the time averages. The time average would be the time average of z of t z star of t plus delta t. You are asked to show that this was equal to quantity. I believe I have made a mistake in the scale factor. It should have been 2 times summation n is equal to 1 through n naught cosine 2 pi f N delta t plus cosine 2 pi f D delta t. I think I had 4 and 2. I had overall multiplication factor of 2. That was not correct. This should be the correct value. Can I assume that everybody is comfortable deriving this or will need to give at least a few hints to get to this point? Everyone is comfortable deriving this result?

Student: A few hints.

A few hints again if it is not needed, I do not want to take time, but if it is needed, I do not mind at all.

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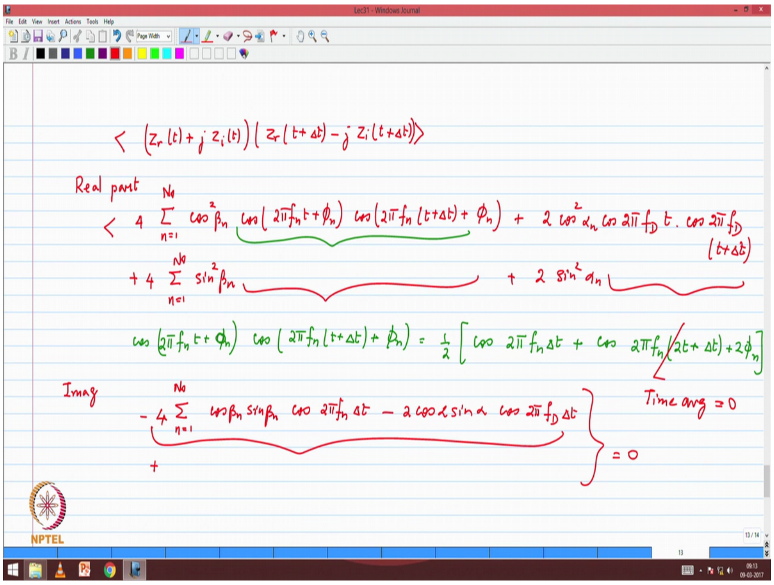


So, basically what you would have to do is to do the time average of the following Z r of t plus j times Z i of t, this multiplied by Z r of t plus delta t minus j of Z i of t plus delta t. This is what you would close bracket time average again like the previous derivations that we showed. Any time you have a multiplication of two different cosines of two different frequencies, the time averages will go to 0 because it will come out as sum of 2 cosines and therefore, it will go to 0. What you will be left with is terms that are the belonging to the same frequency.

So, if you were to of course, the product is going to have a real part and imaginary part. Let me just write down a hint for the real part. The real part will contain terms of the following type, 4 times summation n equal to 1 through n naught basically frequencies of the cosines of the same frequency. So, it is cosine square beta n cosine 2 pi f n t plus phi n cosine of the same frequency. So, the second term will be cosine 2 pi f n t plus delta t plus phi n and then, there is the last term f d term that will give you 2 times cosine square alpha n cosine 2 pi f D times t times cosine times 2 pi f D t plus delta t, ok.

So, you see basically cosines of the same frequencies, those are the only terms that remain. You will also find that there are terms of the following from, you will also find that there will be 4 time summation n equal to 1 through n naught, you will get sin square beta n. Basically, this term plus 2 times sin square alpha and this term, ok.

So, these cosine square plus sin squares will add up. What you will be left with is terms of this form. I will just give you a hint for that cosine 2 pi f n t plus phi n times cosine 2 pi f n t plus delta t. This is only the real part by the way plus phi n. So, product of 2 cosines, you should get the cosine of the sum and difference frequencies. This can be written as half of cosine 2 pi f n delta t. That will be one term and that is a difference term. The sum term will be cosine 2 pi f n 2 t plus delta t plus 2 phi n, does not matter. The phase terms are basically you will get a cosine of some frequency; if I take the time average that will vanish to 0.

So, this has time average equal to 0 because it is a cosine function. So, what you will be left with is basically the terms that are of the form cosine 2 pi f delta t which if you go back and look at the final expression that is that is what is present. So, these cosines 2 pi f n delta t. So, the real part has given you this. What happened to the imaginary part? Imaginary part actually completely gets cancelled. Let me just write that one step and then, we will close the imaginary part. Again apply the same simplification. What you will find is that you will find terms that of this form minus 4 times summation n is equal to 1 through n naught cosine beta n sin beta n cosine 2 pi f n delta t minus. That will be part of the summation. I think I need the brackets. Then, it is minus 2 times cosine alpha sin alpha. Basically the scale factors are now cosine and sin instead of cos square and sin square. This will be sin alpha cosine 2 pi f D delta t plus, you will get exactly the same quantity, so plus the same quantity. Therefore, the imaginary part completely cancels and it becomes equal to 0. So, it is plus exactly the same quantity, ok.

So, you please do verify again. It is not difficult, but just need you to be careful with the expressions. So, the final result that we have is the following that the autocorrelation function is of the form that is given in this by this expression. We are one step away from the answer. So, let us complete the discussion that we need for today. Now, we are trying to show that the autocorrelation is a Bessel function, right and that is our goal.

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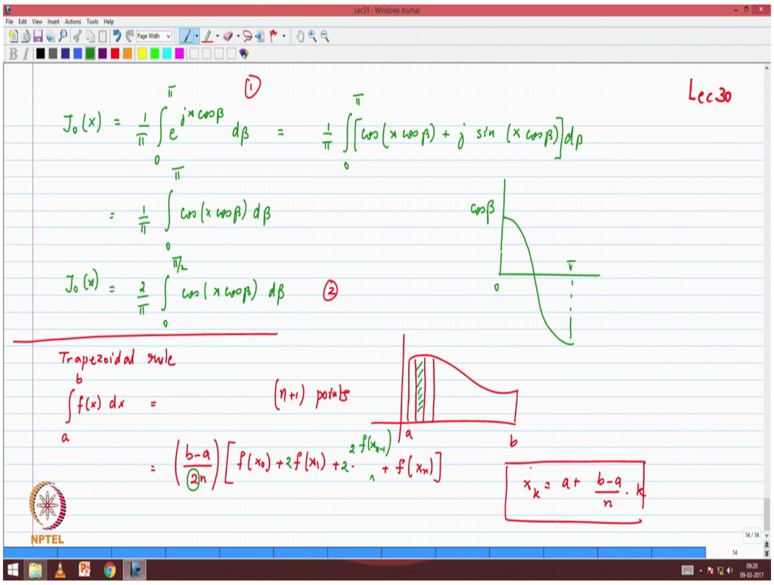


So, here is a result that is very useful for us. J 0 of x is what is the Bessel function, a expression for the Bessel function. If you look up Haiken or Molisch, any of those you will find the following 0 to pi e power J x cos beta d beta. That is one of the expressions for the Bessel function, the J 0. We will simplify it for our purposes. Again this is for general, but we are interested in, we would like to write it as 1 over pi integral 0 2 pi cosine of x cos beta plus j times sin of x cos beta d beta. That is rewriting the integrand and the variable of integrations is beta going from 0 to pi and the only form of the variable that occurs is cos beta. So, cos beta is of this type from 0 to pi. This is cos beta. Now, the actual integrand is sin of that argument. So, since this is a symmetric integrand, the imagery part contributed by, imaginary part will go to 0 because the sin of this portion will be positive. Sin of the other portion will be negative and when I do the integration that term goes to 0. So, what we are left with is 1 over pi integral 0 to pi cosine of x cos beta and I think some books already give you the direct expression in this form and again applying the symmetries that are present, you do not have to integrate from 0 to pi. Integral 0 to pi by 2 is sufficient and you can multiply by 2 because you are taking cosine of that cosine of a cosine of minus thetas cos theta.

So, this will be 2 by pi integral 0 to pi by 2 cosine of x cos beta d beta. So, Bessel function approximated not approximated expression, rewritten in this following fashion. So, again this you can more or less take it as a standard result. Some textbooks may actually give it to you in this form, but you know the more general form is one that is given in equation 1. Couple of steps of simplification brings it to equation 2 and this is the form that we are interested in.

Now, from your study of calculus integrals can be approximated by a rule called the Trapezoidal and let me refresh your memory in case some of you probably are very familiar with it integral a to b f of x d x. This is the very general rule of real integrals. This can be approximated by b minus a by 2 n. Suppose I have n points. So, basically if it is a function that I am trying to integrate between a and b, I am going to divide it into different values and I am going to have n plus 1 points total including a and b the end points. So, n plus 1, total number of points in that case the trapezoidal rule says that the integral can be approximated as b minus a divided by 2 n. Think of that as a scale factor into f of f of i. I will just in a minute x naught x 1, all the way to f of x n, there are total of n n plus 1 points, where x k is equal to a plus b minus a divided by n into k.

So, it is a linear interpolation of the range between a and b and you are sampling your function at each of those values, the trapezoidal rule often that will be that you would probably be familiar with is the first and last terms have got slightly different weighting than the middle terms. The middle terms all have a scale factor of 2. So, basically it will all the way up to 2 times f of x n minus 1 and then, f of x naught and f of x n have got a scale factor of 1, but basically it is a and this 2. Basically offsets the scale factor of 2 that is happening there. So, the b minus a by n is the area of your area of those trapezoids, ok.

So, basically the trapezoidal rule says you have to add the areas of those trapezoids. You may give slightly different weightages for the first and last trapezoids. It again depends on how you interpret the trapezoidal rule and how many sampling points that you get, but basically this is a form of the trapezoidal rule.

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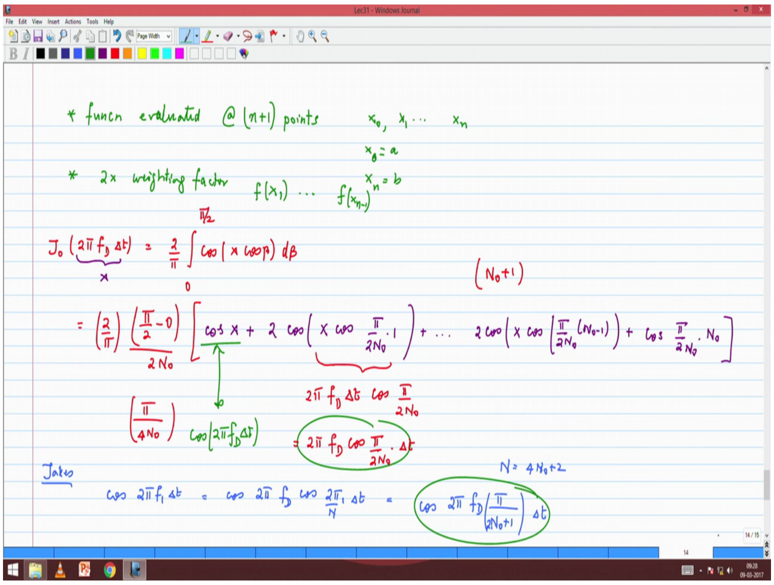


So, the observation is the following that you have to evaluate the function. The function evaluated at n plus 1 points and you get to decide you want to have a more tighter approximation. You would take larger number of values n plus 1 points and they are denoted as x naught x 1 and x n. X naught is equal to a, the starting point of the integral. X n is the ending point of the integral and basically, it is linearly interpolated in that range and we also note that you have 2 x waiting factor for the following points which is x 1 all the way to f of x n minus 1, ok.

So, except the first and last, you get slightly higher weightage or a factor of 2, but the important point is that you preserve the area of the trapezoids and do the integration . So, now comes the important part and then, you appreciate the insight and the practical engineering mind of Jakes says I have to eventually approximate J 0 which is 2 pi f d times delta t, and this we have shown a to be equal to 2 by pi integral 0 to pi by 2 cosine x cos beta d beta. Now, if I were to tell you please apply the trapezoidal rule to this Bessel, this integral that approximates the Bessel function, then we will leave this 2 by pi alone, that is sale factor does not affect upper limit is pi by 2 lower limit is 0. I am going to divide into N naught plus 1 point. I think you will start to see where the similarities are coming N naught plus point is what I am going to do in my approximation.

So, then it becomes pi by 2 minus 0 divided by 2 N naught. This is the same as pi by 4 N naught. That is another scale factor. The first one is that and if I call this as x, let me call this as x. So, the first term in the trapezoidal rule will be cosine x because b beta will be 0 cos beta is 1 cosine x. The second term in the trapezoidal rule will be basically beta uniformly separated between 0 to pi by 2 and you are going to increment it. So, the next term is going to be scale factor of 2 cosine x times cosine pi by 2 n naught into 1. Am I right? That is your pi by 2 n naught is the spread and into 1 because this is the first component of that. Second one will be pi by 2 N naught into 2 pi by 2 pi by 2 N naught into 3 so on and so forth and the last one will be pi by 2 n naught into N naught, ok.

So, please fill in the terms, but one term will be 2 times cosine x times cosine pi by 2 N naught into N naught minus 1 plus. The last term will be cosine pi by 2 N naught into N naught. That is trapezoidal rule applied to the integral. That approximates the Bessel function. You just take a minute to you know just make sure that you are comfortable with that because what the trapezoidal rule says is the variable of integration gets uniformly divided in the range of the integral and then, you apply it to the function and that is exactly what we have done and obtained this expression, ok.

Now, we are just one step from the answer. Can you please see what this expression is, 2 pi f D times delta t into cosine pi by 2 N naught. Is that correct? So, if you rewrite this as this is equal to 2 pi f D cosine pi by 2 N naught into delta t starts to look like the oscillator that Jakes was using and if you go back and verify, it is very close. It is not exact. So, what was the expression that was used for f 1? The first oscillator basically in the expression what you would have got in the Jakes model, this is what you would have obtained in Jakes model.

The first one would have been cosine 2 pi f 1 delta t. Let say if you wanted to approximate that, that would have been equal to cosine 2 pi f D. If you go back and look at the expression, it will be cosine 2 pi by N. Go back and look at the frequencies into n is equal to 1 and then, delta t. This can be rewritten, where n is equal to 4 N naught plus 2. We use that relationship and rewrite this in the following way. We write this as cosine 2 pi f D into pi divided by 2 N naught plus 1 times delta t and basically, the oscillator frequency is slightly different. Basically you will have to compare this term and this term and of course, cosine of that and you find that the terms are matching.

Now, the term cos x actually matches precisely, because that corresponds to cosine of 2 pi f D times delta t; so this matches perfectly. All those oscillators are very closely approximated. It is in the trapezoidal approximation. You will get 2 times n naught in what Jakes has used; we get 2 times N naught plus 1. Again why did he choose that n naught plus 1? Probably some other reasons, we do not know. This is what how he chosen it and we find that it is pretty good approximation anyway by the trapezoidal rule.

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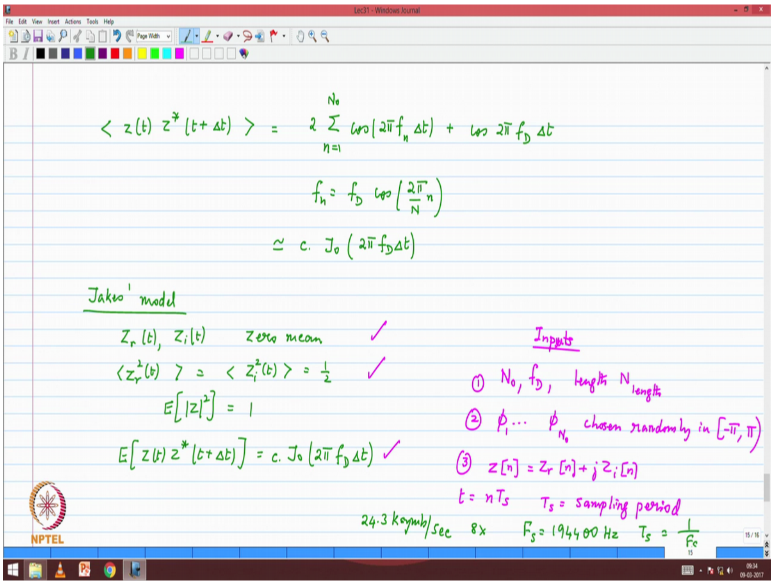


So, we go back and revisit the result that Jakes expression gave us summation or a time average of z of t z star of t plus delta t which comes out to be 2 times which we have verified to be equal to n naught equal to n naught cosine 2 pi f D f n times delta t plus cosine 2 pi f D delta t, where f n is given by 2 pi divided by n into n. So, basically let me make sure that is a correct expression that we have. Is that correct? Are the expressions for the oscillators correct? Yes, 2 pi by N into N 2 pi by N into n. So, this we can say is a approximation, fairly good approximation and the approximation gets better as N naught or N becomes large and this is approximately some constant into j naught times 2 pi f D times a. This is wait, I have written something wrong here. This is f D times cosine 2 pi by N into N, right. Please catch me if I am making such mistakes 2 pi f D times delta t, ok.

So, Jakes model does the following for us. It establishes that you get z r of t z, the time averages are zero mean. Let me just write it like this z r of t z i of t are zero mean. They also have the variances equal to one-half. So, z r square of t time average is equal to the time average of z i square of t equal to one-half. So, if I write down expected value of mod z square, this comes out to be equal to 1. So, it has got the correct variance that we want. It also has the correct expected value of z of t z star of t plus delta t is a stationary process which gives out some constant times j naught 2 pi f D times delta t.

So, it satisfies the important things of 0 mean proper variance and the appropriate time correlation that we need. So, how do we use the Jakes model? Jakes model asks for three inputs. The first one, it ask you is how many oscillators you have to specify and the thumb rule is n naught greater than or equal to 15. You can take 20-25. The higher the number, approximation becomes better. Now, what is the drawback of choosing a large number? Remember these oscillators have to keep running. So, in order to get the next sampling, the next value of z r of t or z i of t, you must get those cosine components.

So, basically you will have to keep incrementing those oscillators, but it is just a simple operation. When you are doing it on a computer, you have to first specify n naught and second, you will have to specify what is your maximum Doppler frequency and then, you will have to specify the duration of the wave form that you need duration of the wave form and basically, this is what we need to specify inputs to Jakes model. Inputs would be the maximum Doppler, the number of oscillators n naught and the length of the fading wave form, let us call that as n length of the fading wave form.

Now, we also have to make another choice, the random phases of these oscillators phi 1 to phi N naught. These are chosen to be random, uniformly distributed random variables chosen randomly in the range minus pi to pi. So, basically some random phases are sufficient, and then we start running Jakes model and generating the samples. The samples are taken as z of n to be equal to z r of n plus j times z i of n. These are the samples at that specific value, what we have done? We have taken t and set it equal to n times t s, where t s is equal to the sampling period that you are trying to simulate and as an example in the case that we have considered if I had 24.3 kilo symbols per second system and I was doing 8 x over sampling, then my sampling frequency is 194400 hertz and t sampling will be equal to 1 over f s, ok.

That is what you would increment because ultimately these oscillators are going to ask you what the value of t is. Even you start at some point and keep incrementing each time, you increment by the sampling period and depends on what over sampling factor you want and notice that everything else is in this picture is frozen, alpha is fixed, all the betas are fixed, the phases you have chosen each time, you just supply the value of t. It will give you the cosine values.

You multiply and add with the appropriate scaling and then, that becomes your fading coefficient and it turns out that this is a very effective way of generating Rayleigh fading on a computer which satisfies the statistics almost exactly as you would see in the real world. In terms of the fading statistics that we would observe, any questions? Definitely, please take the time to work through the numerical parts, so that you can feel comfortable with the results that are presented, ok.

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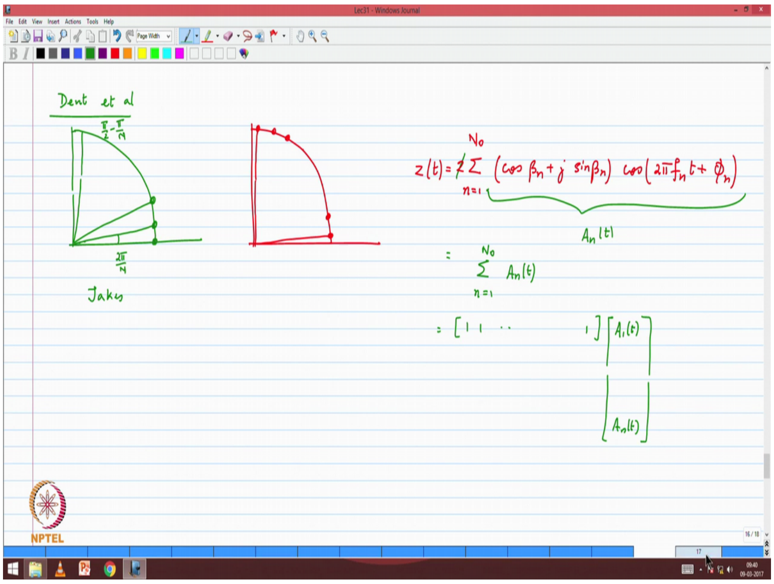


Now, there is a paper by researches Dent et al which I would ask you to read. I will upload it on the website and just of what they are proposing is the following. This is Jakes model, the oscillators where chosen to be 1 at f D and then, equally spaced with the spacing of 2 pi by N, right. The spacing was 2 pi by N and the last one was before pi by 2. So, the last 1 was at pi by 2 minus pi over N. Now, this is Jakes Dent et al did a slightly different approximation which they said, what they said was, the frequency is that were chosen, where starting from the f D itself.

What they said was see anyway you are trying to do an approximation to the integral, so it does not make a difference if your first oscillator is not at f d, but slightly offset and the last one is also offset. So, basically it is a different sampling grid of the oscillators, but notice that neither f D nor pi by 2 are present. You are basically in between same number of oscillators that are again the spacing is the same slightly different grid that is used, ok.

Now, the difference is that the approximation that they have proposed to the Bessel function is that you can now write z of t to be equal to n is equal to 1 through n naught cosine beta n plus j sin beta n into cosine 2 pi f n t plus phi n. So, if you want to put a factor of 2, it does not matter. So, basically all the oscillators have got same scale factors, same weighting. There is no one particular. One does not get any special treatment and they have shown that this is also a good trapezoidal approximation with the n naught being sufficiently large.

So, the key point of the method is one is the claim. This works better than the original Jakes model. We are not as much interested may be that is a useful result for us, but a more important result for those of you who are already have worked in wireless communication, you will appreciate this. Let me call this as n of t, the part which is in the green bracket. So, this is nothing, but I am going to for a moment, I am just going to ignore the two summation N equal to 1 through N naught N of t. This is how you would write z of t.

Now, why is this important and what are the benefits? Basically this paper by Dent et al, why do we even want to read it if it is just another approximation? You know why not just accept it. We will just take this, but there is a very important off shoot of this result. The result is as follows. This can be written as 1 1 1 in vector form times a naught, sorry a 1 of t a 1 of t all the way to a n of t.

Now, you may still be wondering what is there? There is nothing new that we have gained in this whole process. Just wait for a result that needs to be embedded here. There is a series of matrices that we encounter in the context of CDMA.

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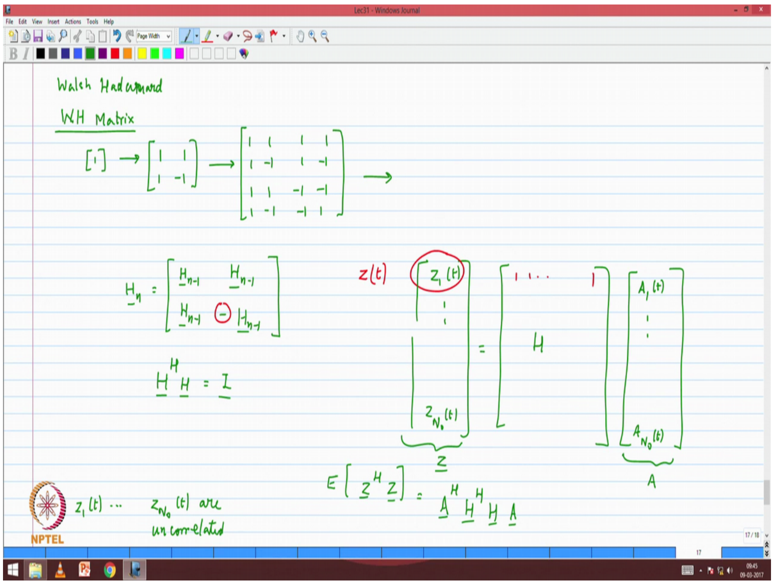


It is very useful in this context as well call the Walsh Hadamard matrices. Walsh Hadamard matrices is a family of matrices which is derived from a basic matrix at the lowest level. The matrix is just 1 by 1, then it becomes 2 by 2, it becomes 1 1 1 minus 1. So, the matrix is repeated three times without a sin reversal and the fourth time, it is repeated with the sin reversal.

The next level of the family, it becomes 1 1 1 minus 1. That is a first repeat 1 1 1 minus 1 1 1 1 minus 1 third repeat and the fourth one has to come with the sin reversal minus 1 minus 1 minus 1 plus 1. Now, you seen the pattern. You can work on it. The general formula is the next higher level in the Walsh Hadamard matrices is given by take the lower level matrix, repeat it three times and the fourth time you take it with the minus sign. It is very important that you do not miss this and that is the Walsh Hadamard matrices. The beauty of the Walsh Hadamard matrices is that hermitian H is equal to i. Any of these Walsh Hadamard matrices if you take its transpose and multiply the real matrices, hermitian is a same as transpose, conjugate is same as transpose. So, H hermitian H you will find that basically these rows and columns are orthogonal to each other. It is a very useful property that is present here, ok.

Now, if I now write the following result z 1 of t all the way to z n naught of t to be equal to Walsh Hadamard matrix of the appropriate size times a naught of t all the way to N naught of t. So, what I have done from the previous line, I took the same functions A naught to A1 and rewrote it and wrote in terms of Walsh Hadamard matrices. So, this first element is the same as z of t that I was trying to generate, correct. That is basically the first row is all ones.

So, the first row is all ones, but the beauty of it is, you now have another function which is basically derived from the original set A naught to A, sorry it should start with A1 to A n A naught, but the beauty of it is its expected value. If I call this as some z matrix z hermitian z will mean a hermitian. Let us call this as a matrix. This will be a hermitian H times A H hermitian A is diagonal matrix. So, then what do you find? You find that all these z z 1 through z n naught are what type of variables. There is no correlation between them. They are all uncorrelated variables because the correlation matrix is a diagonal matrix. That means, they have only correlation with themselves. They are not correlated to anybody else, any of the other functions. So, this z naught through z z 1, this basically means that z 1 of t z n naught of t are all uncorrelated with each other, ok.

Now, the beauty of it is, this is exactly what we will want to assume when it comes to diversity and that is why this modified model is a useful model which is proposed by Dent et al and since you are all PG students, I would like you to get the feel for you know this is how research progresses, somebody proposes. He has some basic structure, somebody else thinks you know I cannot extend it using Walsh Hadamard matrices because this one has got a difference scale factor. Can I do a different approximation? Yes, then I can represent it in terms of this form, then link it to Walsh Hadamardcy matrix and then, get uncorrelated sequences.

Now, did not Jakes know that you had to get uncorrelated waveforms for fading. Answer was he knew and his way was how do I get uncorrelated fading out of this one. So, he said well the only thing that I can ensure that is is all the betas and alphas have been frozen, right. So, what you really did not fix was phis. So, what he said was you generate one set of random phases let call them as phi 1 1 all the way to phi n naught, 1. I mean he said generate another set of random phases that will be phi 1, 2 phi n naught, 2 and let us hope that the way forms are uncorrelated, but he really had no way of telling you know I can guarantee you that things are uncorrelated because they actually tried this and found that in some cases, it worked very well uncorrelated way form, but some cases it is not working very well. There is a residual correlation whereas, the method proposed by Dent et al says very clean. There is no issue of residual correlation. This will assure you that the correlation is exactly as you wanted to be, ok.

So, again part of the course is to expose you to some of the milestones that happened in terms of research and this is a very useful one, that is present. I would like to just take a few minutes to add and introduce you to diversity.

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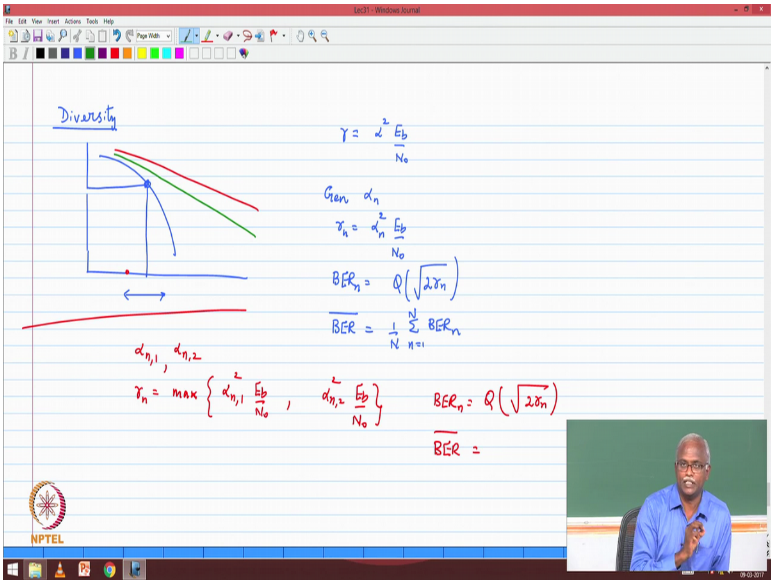


Before we end todays lecture and our starting point for diversity, we will do an intuitive experiment and if time permitting, I would like you to try it out on the computer as well.

So, this is the experiment that you have already done AWGN, you have a nominal SNR and you generate instantaneous SNR as alpha squared E b by N naught. So, basically there will be some perturbation of SNR around the nominal value and you calculate the BRS. You took the average and then, you obtain the result that is. So, the steps that you followed was you generated different alpha ns, right. Instantaneously you generated, from that you will calculate gamma n which is alpha n squared E b by N naught, then you calculated BER of that particular trial as q of square root of 2 times gamma n and then, you finally average the BER and said that this is BER expression. Let me call 1 over n n is equal to 1 through n BER n, right. Am I right? This is what you have done in your computer experiment and you were able to show that the graph is something of this type.

Now, I want you to try this following experiment. For each of those trials instead of, generating one random variable, I want you to generate two random variables alpha n, 1 alpha n, 2 independent and SNR is going to be the maximum of the following alpha n, 1 square e b by n naught, alpha n, 2 square e b by n naught. Each of these random variables is going to give you an instantaneous SNR. I give you the benefit saying you get to pick the better of the two and ber n is the same as before q of square root of 2 gamma n and ber average also that you have obtained like before will the graph. Will the graph be different?

See this is the problem, clear. I generate instantaneous values of Rayleigh fading and then, I square it to get the instantaneous SNR. It may turn out that my instantaneous SNR at this point was here. That was when I did only one trial, but when the second experiment that I want you to try, you are going to generate each time two Rayleigh random variables and I am going to pick the better of the two. Is this graph going to change; BER graph going to change? Yes and it will change for the better.

So, what you would expect is that it is going to be somewhere lower, right. It can be worst; it can be the same as red, but it will hopefully be lower because you get to choose the better of the two. It you may end up with a lower BER and therefore, your overall average is also going to go down. That is the starting point of our intuition regarding diversity.

If you have a choice of more than one copy of the signal and if they are uncorrelated, it is very likely that one of them has got a better SNR. You should be able to get a better BER than if you had only one copy of the signal. So, with that as the starting point, we are going to build our intuition on the whole diversity is a very vast field, but what we would like to do is the essence of the concept of diversity, how do we exploit diversity and how is it exploited or how is it used to our advantage in the fourth generation and going forward into the systems.

So, that is our starting point, we will pick it up from here in tomorrow’s class.

Thank you.