Introduction to Wireless and Cellular Communication
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Lecture – 31
Computer Simulation of Rayleigh Fading, Antenna Diversity
Rayleigh Fading Simulation – Clark and Gans Method, Jake's Method

Good morning, welcome to lecture 30, today's lecture is about computer simulation of fading channels. So, if you look at the material that we have covered. So, far our goal has been to understand the nature of the wireless channel we have understood that the there is the phenomenon of multipath then we also understand that because of the phenomenon of multipath you have the resultant signal fluctuation which we refer to as fading. So, in real life you will encounter channels that are wireless channels which are channels which have fading and which will have time dispersion.

Now, we have been able to characterize them through the wide sense stationary uncorrelated scattering model we have the coherence time coherence bandwidth we have got the entire panorama of understanding the wireless channel the goal of this of the end of last lecture. And today's lecture is to show how do we mimic these channels on a computer so that we can then assess the performance of the different systems.

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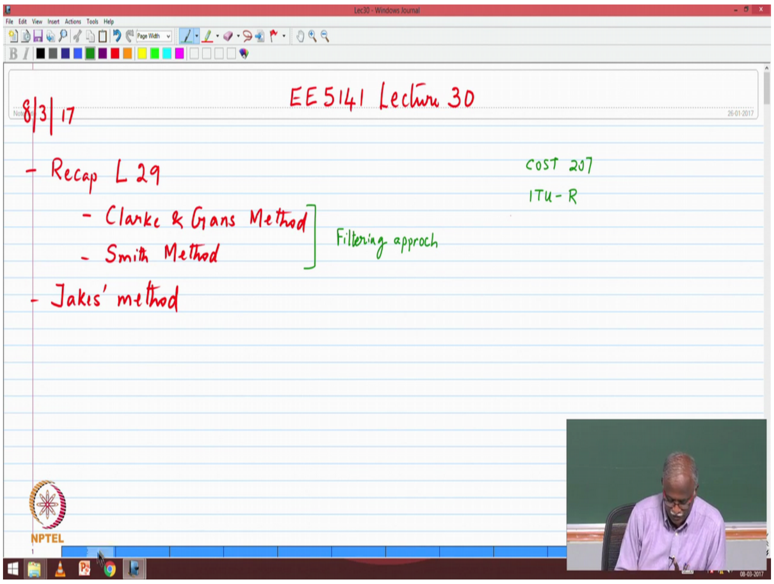

So, in this context if you remember we said that there are some practical channel models that are cost 207 the ITU-R these are all models that are used in the industry first in standards to mimic these wireless channels now yes we understand what the cost model is how do I actually implement at it on the computer, which says that there are many taps.

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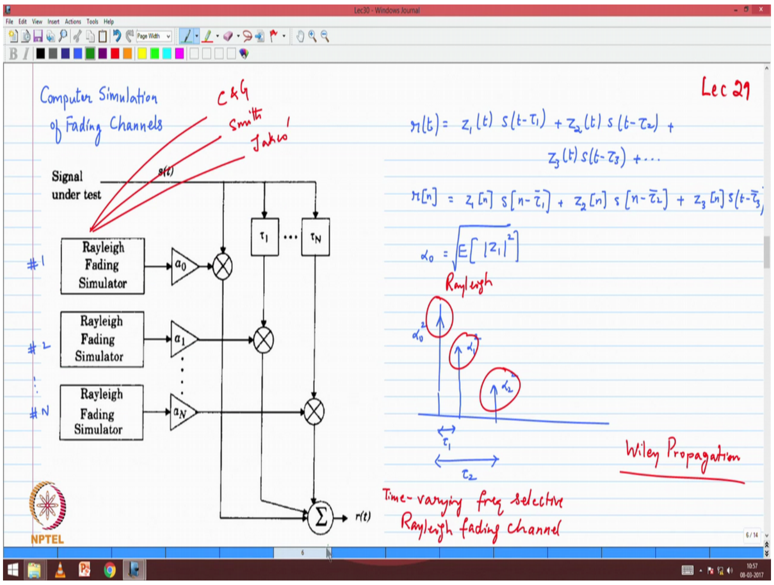

Now, these taps have to be generated using a fading process multiple number of taps each of them with the different power levels different delays which I have to generate.

Now, the block that we finally, have to worry about is how do I actually generate a Rayleigh fading in a computer because it is a random process how do I the only things that I can do on a computer are you know are limited in terms of. So, how do I manipulate the tools that I have on a computer to achieve the goals that I to make a fading way form that looks very similar to what you would see in a fading channel.

So, that is the goal of today's lecture as we saw in the last lecture there are 2 methods that. In fact, I would say it is more like one method the filtering method. So, if you could combine these 2 I would say that these 2 are based on the implementation of a filtering approach you take white noise you pass it through a filter which has got a certain spectral shape this spectral shape will give a coloration to the noise and coloration is the same as correlation in the time domain and the correlation is the Bessel function that we are interested in.

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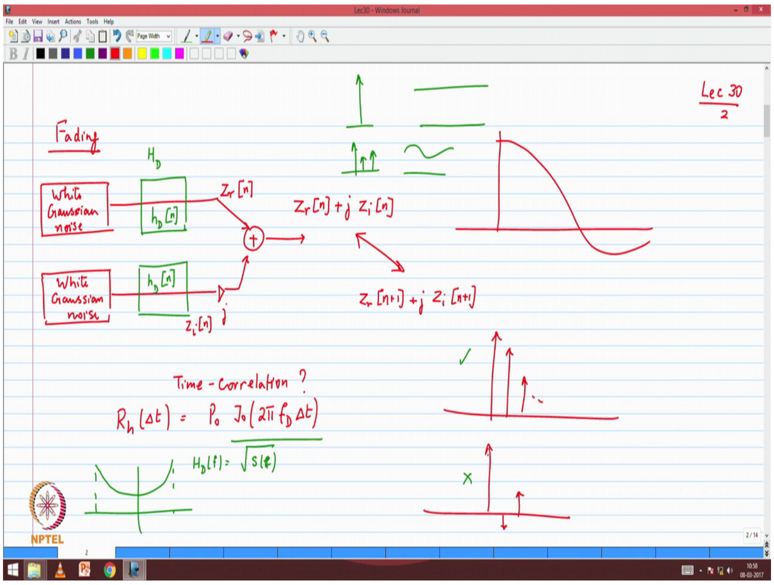

So, basically the filtering approaches are the Clark and Gans method and the smith method is an efficient implementation of a Clark and Gans. So, to just summarize the Clark and Gans method we are generating white noise 2 independent white noise sources uncorrelated white noise basically the success of sample are correlated we will pass them through a filter who which will introduce for us the appropriate coloration of the noise coloration in the in the spectral domain is the same as correlation in the time domain. So, therefore, the output of these 2 sequences have got the Bessel correlation, but again the Bessel correlation is not what you do not; you actually need it for the complex channel.

So, basically we need to worry about how to establish it for the complex channel, but (Refer Time: 03:54) what we have is the upper branch I treat it as Z r the lower branch I treat it as Z i multiplied by j add them together that gives me a complex a channel gain now these have got have been appropriately correlated in their the real part with itself imaginary part with itself. Now we need to confirm that the overall correlation is obtained as expected and it is not surprising that because they are independent you are you what you will be left with when you compete the correlation will be the correlation of the real part and the correlation of the imaginary part and there therefore, things will work out, but we have to verify that today’s lecture has got a lot to do with the verification aspects.

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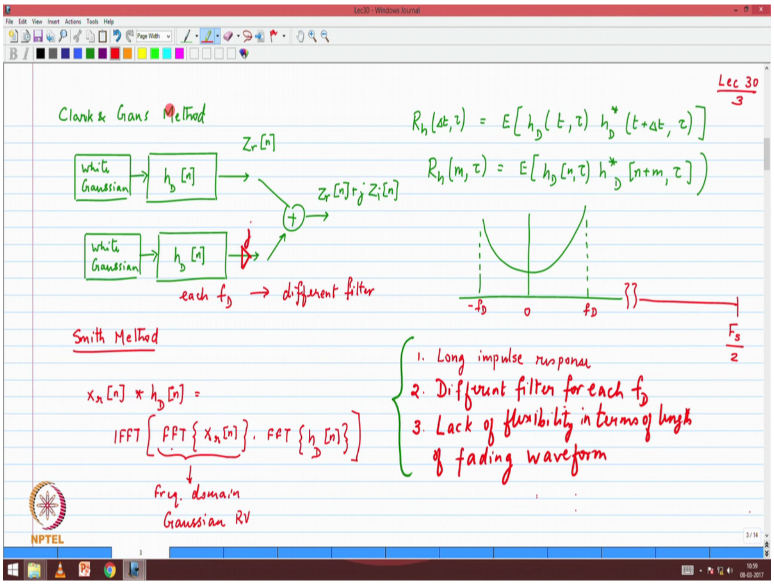

So, the Clark and Gans method basically has 2 sources each of them appropriately filtered multiplied by j now the some of the challenges that we observed with the Clark and Gans method. In general, the filtering approaches is that this is a sharp filter a sharp filter will give you a long impulse response which means that you will have to do a lot of competition to compute a small segment of data you have to do a lot of competition the other challenge is that Doppler is not may not be known ahead of times. So, we cannot pre-compute these filters you may have to computer them in real time.

So, basically different filters will be needed and of course, we also saw that once you have these long filters you do not have too much flexibility in terms of the length of the fading way form that you are generating you will be constrained to generate a minimum set. So, that is Clark and Gans; Clark and Gans basically does a time domain convolution the smith method is the same approach. But implemented in the FFT domain you take the Fourier transform of a block of data X r of n take a the filter take its FFT multiply the 2 and get the inverse, but we also made the observation that I Do not have to take the FFT off the Gaussian samples.

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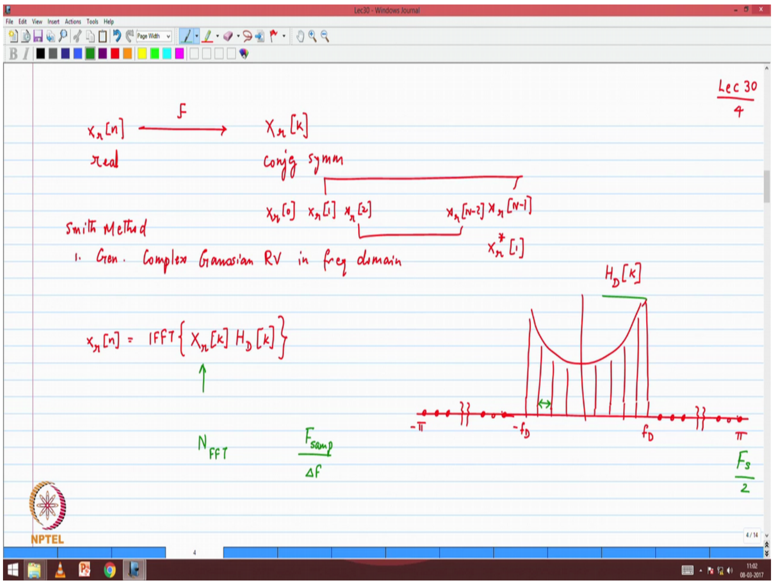

I can generate them directly in the Gaussian domain provided I retain the symmetry. So, that I get the appropriate real values in the time domain.

So, basically I directly generate X r of k h D of k i, I have generated the samples of that. So, I Do not generate an impulse response I just sample the spectra the power spectral density. So, this is what gives me h D of k. So, it s a very simple process I generate m Gaussian random variables I have sampled the spectrum of the power spectral density and then I multiply take the inverse FFT that gives me a sequence of length.

Now, we one of the constraints that we ran into was the size of the FFT the size of the FFT depends on what is the spacing that you want within the window of interest basically from minus f D to f D and the other one that is constraining you is f sample is a sampling frequency divided by 2 that corresponds to pi. So, basically the size of the FFT is f sampling divided by delta f and that is more that is going to determine the size of FFT that is also going to decide the length of the way form that you will generate through this process.

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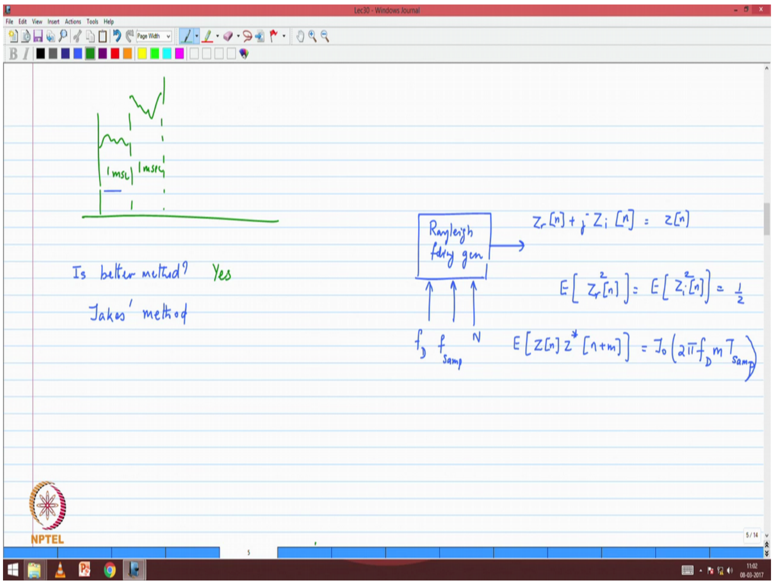

So, we made the observation that very often in simulation we are not interested in very long fading sequences we are looking at independent sequence or fading forms of shorter duration.

So, the question is there a better method is and the answer turns out to be yes and that is based on the Jake's model and that is what we will focus on today. So, is the context clear we are trying to implement fading in a computer we basically have only a limited set of tools that we can use on the computer, but we want to be able to generate the time domain sequence with the appropriate time domain correlation with the appropriate spectral shape with the flexibility of generating any Doppler any length of sequence that is the goal that we want to do today.

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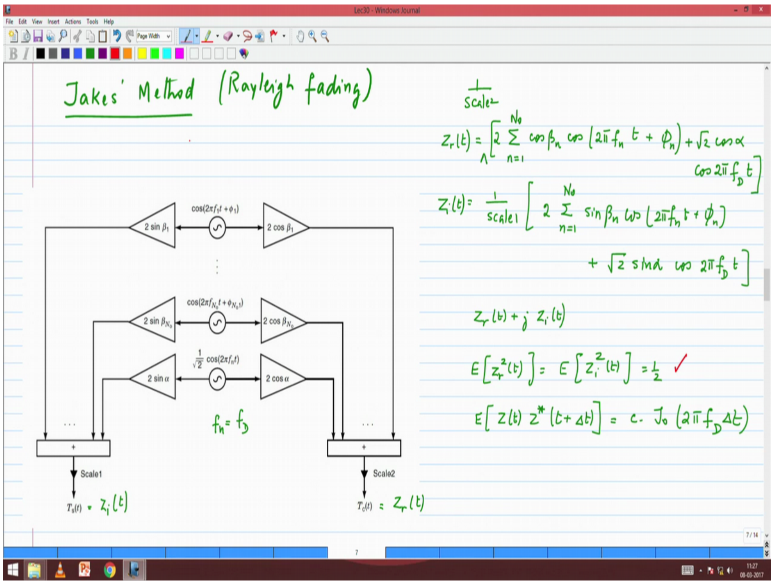

So, the task before us is to look at the Jake's model as I mentioned to you William Jake's was one of the pioneers in terms of the wireless communication his book on microwave wireless microwave communications is considered one of the best references for the early work that is done in this field. So, let me capture for you the essence of Jake's model it consists of a number of oscillators; oscillators are deterministic once you set the frequency the oscillators are a very predictable way form, but what makes it somewhat random is notice that there are some phase terms each of these oscillators has got a phase term except one of them and these phases will be chosen in a appropriate manner to in and ensure randomness. And there are some gain terms which are applied to the oscillators and one set of gain terms are added to get the imaginary part, the another set of gain terms are added to get the real part.

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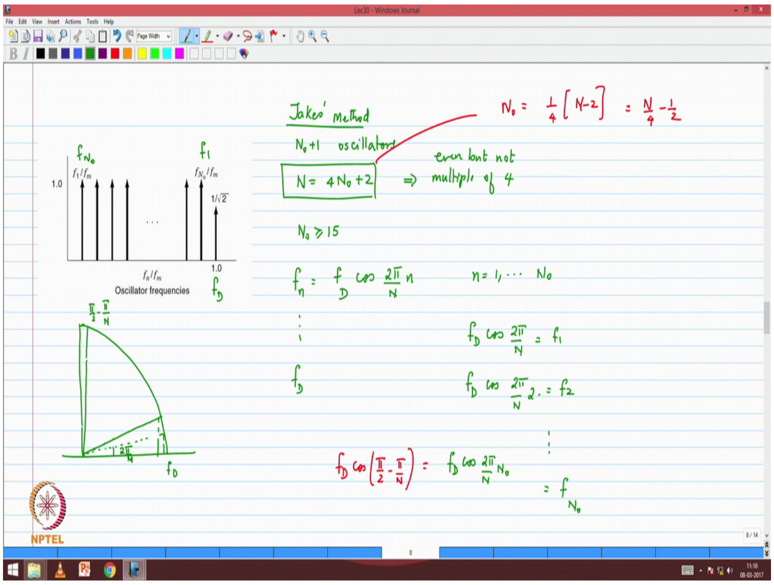

So, basically the goal is to see whether this structure achieves the goal for us and it turns out it achieves it in a very nice manner, but today's lecture is that. So, what are those oscillators? So, basically Jake's method let me just start by writing the following Jake's method consists of a number of oscillators. So, we first specify a number of oscillators. So, there a total of N naught plus 1 oscillators now N naught has got a specific property N is equal to 4 N naught plus 2 where that basically means that N is a multiple is a multiple of 4 it is it is even, but not multiple of 4. So, it is this is an even number, but not a multiple of 4 that is that is what this means now there is a very specific reason why it is chosen that way. So, N naught happens to be an a number and you know for any integer number would satisfy this relationship N naught greater than or equal to 15 turns out to be the thumb rule for good modeling of the fading channels, but again the interesting part is how the entire frame work is derived.

So, now these oscillators there are N naught plus 1 of them the different oscillators are given in the or defined in the following fashion the maximum Doppler times cosine 2 pi by N into N; N is equal to 1 through N naught and then there is a final oscillator which is at the Doppler frequency there as N naught of these and then the final one has got a its basically at f f Doppler maximum Doppler. So, that is what is represented for us the amplitudes of those oscillators are what is given again an interesting observation this corresponds to the maximum Doppler all of them that one has got a slightly lower amplitude than the others and interesting for us to visualize how that will that will play a part in the overall picture, but a more intuitive way to understand the oscillator frequencies can be seen in the in the following form, but first lets interpret what are these different oscillators.

So, basically you will have f D cosine 2 pi over N that will be equal to f 1 f D cosine 2 pi over N into 2 will be f 2 dot dot dot I want to know what is the last oscillator f of N naught that will be f D cosine 2 pi over N times N naught I would like to rewrite or simplify this in a little bit better manner. So, you can rewrite this as N naught can be written as 1 by 4 into N minus 2 am I right or it can be written as N by 4 minus one-half. So, basically if I can if I can write it in this form I can show that this last oscillator is actually equal to f D cosine pi by 2 minus pi over N pi by 2 minus pi over N that is ok.

So, the mathematical expressions can be nicely understood if you were to visualize it as one quadrant of a circle this corresponds to Doppler f D; the full length the radius of the, and then you are going to take at different angles. So, the angles are at 2 pi over N 4 pi over N that is your second one and then the final one happens to be at pi by 2 minus pi over N almost at pi by 2 pi by 2 minus pi over N basically you take their projections that will be the oscillator frequency.

So, the highest frequency is f D then you have this is f 1 then the last one is f of N naught right in our notation. So, they then one with smallest frequency will be f of N naught. So, basically Jake’s method requires us to use a set of oscillators the frequencies have been understood there is of oscillator at f D and then there are several which correspond to uniformly spaced in the angular domain and then their projections on to the real axis the that length gives you the actually frequency of the oscillator is that clear that basically what the oscillator frequencies are.

So, now lest go back and plug in to the system the oscillators. So, basically its cosine 2 pi f 1 T times 5 1 and similarly all the N naught oscillators and then the last one consists of a oscillator at the Doppler frequency. So, what I would like you to right down or if you call this as a let us call this as Z r of T this as Z i of T and. So, therefore, what I would like you to write down is an expression for Z r of T Z r of T each of these multipliers has got a scale factor of 2. So, 2 I take out as a common term summation N is equal to 1 through N naught representing the N naught oscillators it is cosine beta N cosine 2 pi f N times T plus 5 N that is that captures the N naught oscillators plus the last term has got square root of 2 cosine alpha cosine 2 pi f D T in the figure I have used f N f N is the same as f D the notation that we are using.

So, this is the expression for Z r of T of course, there is a scale there is a scale factor we will we will maybe we should write that down 1 over scale 2 times this one we will now worry about the scale factor at the very end. So, write down the expression for Z i of T it is 1 over scale 1 expression very similar 2 times summation N is equal to 1 2 N naught now it is sin beta N oscillators are the same cosine 2 pi f N T plus 5 N plus root 2 sin alpha cosine 2 pi f D T.

So, now the set up is more or less complete the claim is that if you combine Z r of T plus j times Z i of T you will get the properties that that we are interested in what are the properties that we are interested in expected value of Z r square of T is equal to expected value of Z i square of T must be equal to one-half expected value of z of T z star of T plus delta T this is equal to some constant times j naught 2 pi f D times delta T it is a wide sense stationary process that it, this will be satisfied and probably not clear at all why this should satisfy why this should even look like a random process and the whole thing hanged together.

So, to summarize what we have said. So, far there a set of oscillators whose frequencies have been very carefully chosen as uniformly space samples along the first quadrant of the circle they are integrated into a structure where phase terms have been added and then multipliers have been added and then you are adding them together to get the way forms that are of interest. So, let us quickly verify some of the parameters of interest and build from there on.

So, the first task that we have is to answer this question is it 0 mean. So, expected value of Z r of T and as in many of the stochastic systems we will replace the expected value with a time average.

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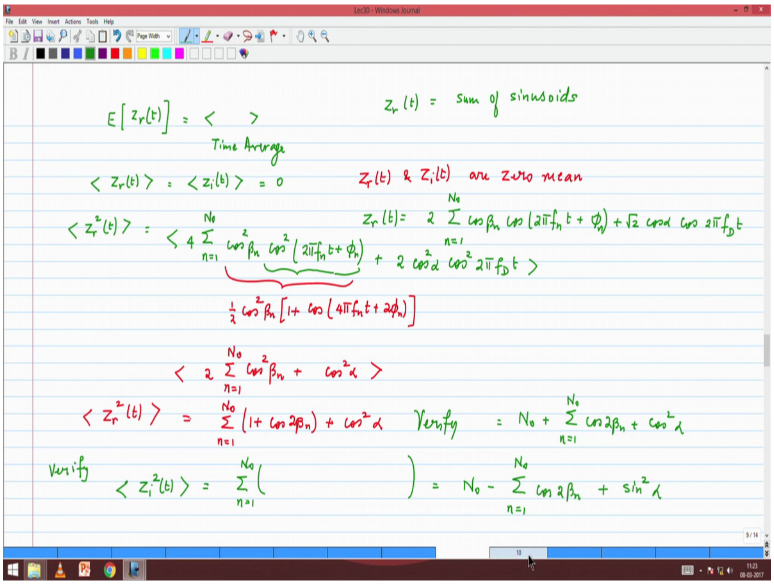

So, if you can show that the time average is satisfies the property then the process that you are working with satisfies the property. So, basically I will use this as this is denote time average. So, the first task for us is to look at the time average. So, time average of Z r of T does that go to does that go to 0. So, basically if you were to look at the time averages you will find that it is a sum of sinusoids right if you have a sum of sinusoids and you take its time average it will go to 0 guaranteed right. So, it does not matter the random phases Rayleigh. Do not matter the scale factors really do not matter if you have a sum of sinusoid. So, basically Z r of T is equal to sum of sinusoids. So, one part of the property sort of is very obvious for us to verify.

Similarly, Z i of T you can by the same argument both of these will be 0. So, therefore, the processes that we are working with are 0 mean Z r of T and Z i of T are 0 mean the second property that we need to verify is that they have variance equal to one-half. So, now, this is this is a non trivial verification that we need to do. So, and this is where you can see Jake’s gave himself several degrees of freedom to satisfy this property because it is non-trivial for us to satisfy. So, if you were to go back and look at the model where are degrees of freedom the degrees of freedom once your frequencies are fixed the degrees of freedom are in the phases that you have and the scale factors that you have those are the degrees of freedom and of course, the scale factors, but by and large the property hinges on the on. So, let us verify the and again it is a very in instructive exercise I will skip some steps, but I would request you to definitely validate that.

So, I want to now look at the time average of Z r square of T we need to have the expressions for Z r of T and Z i of T readily available. So, let me just write down Z r of T once more this is 2 times summation n equal to 1 through n naught cosine beta n cosine 2 pi f n T plus 5 n plus root 2 cosine alpha cosine 2 pi f D T correct, that is the expression again you can write down for Z i m all the explanations I will do using Z r as my reference.

Now, comes the sort of the careful task of squaring Z r square squaring z r, which means that there is a summation term which you will have to square. So, you will get lots of cross terms and then there is one extra term which will multiply with all the terms in the summation and then it will get squared itself. So, basically you will get a large number of terms some of them are square terms some of them are cross products first observation all those terms which have got a multiplication of 2 cosine terms of different frequencies product of 2 cosines will give you sum of 2 cosines which means when I do time average those will go. So, what I am left with is only those terms which are products of self terms basically square terms.

So, what we have is I would now have to take the time average square of each of the terms in the first part will be 4 times summation n equal to 1 through n naught cosine square beta n cosine square 2 pi f n T plus 5 n plus 2 times cosine square alpha cosine square 2 pi f D times T I have to take the time average of that again just make sure that you are comfortable with the terms that are that are being written down.

So, cosine square 2 pi f n T plus 5 n I am going to simplify this term in the following manner I am going to write it as 1 plus cos 2 theta by 2. So, this term can be this term can be written as one-half of cosine square beta n keep that I am not touching that the other 1 becomes 1 plus cosine 4 pi f n T plus 2 5 n correct notice that this is also a cosine term when I take the time average that is going to vanish. So, this entire term becomes a time average of 2 summation times summation of n equal to 1 to n naught cosine square beta n that is that is basically the first term that is coming out plus cosine square alpha wait I think I made some mistake oh then its correct 2 times cosine square beta n plus cosine square alpha.

So, this can I need to take the time average, but basically there is no time dependence on this; this is the expression that we have as the time average of Z r square of T that is equal to summation n is equal to 1 through n naught 2 cosine square beta n I am going to write it as 1 plus cosine 2 beta n plus cosine square alpha. So, this is the expression that we have for Z r square please verify corresponding result please verify this first also verify the following Z i square of T time average you get something very similar summation n is equal to 1 through n naught the corresponding expression I will write it for you in a minute you please derive the result for expected the time average of Z i square plus n naught.

Now, let me quickly do the next step of or maybe we can write it down here n naught minus n naught minus summation n is equal 1 through n naught cosine to beta basically it will be 1 minus cosine 2 beta n cosine 2 beta n and then the outside term is sign square alpha. So, basically the difference the this is equal to n naught plus summation n is equal to 1 through n naught cosine 2 beta n plus cos square alpha that is the first term please verify that you get these 2 terms.

Now, for the I hope you have been able to get up to this point next is a very crucial step this is where the beauty of the entire model lies the choice of the degrees of freedom.

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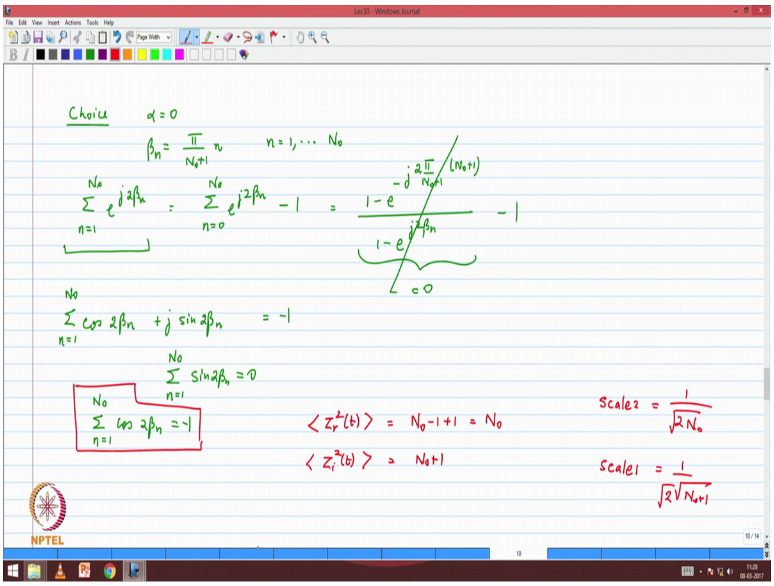

Choice alpha is chosen to be equal to 0 beta n is chosen to be pi divided by n naught plus 1 times n where n is equal to 1 through n naught. So, basically I would now show a result which will be very helpful for us to simplify the earlier expression the result is if I do summation n equal to 1 through n naught e power j 2 beta n notice it is in the form of a geometric series.

So, the summation is as quite easy except that the starting term is not one. So, to compensate for that I am going to stay n equal to 0 through n naught e power j to beta n I introduced n equal to 0 which means that the I have subtract the first term minus 1. So, basically these 2 are equivalent write down the result of the geometric series there are n naught plus 1 terms this comes out to be 1 minus e power j 2 times pi over n naught plus 1 into n naught plus 1 denominator 1 minus e power j 2 beta n. The important thing to remember is that this numerator term is this is equal to 0 there is a minus 1 sitting there on the right side the reason for this is what is this one, this is cosine 2 beta n summation n equal to 1 through n naught plus j times sin 2 beta n this is equal to minus 1. So, which means that the expressions that we have summation n is equal to 1 to n naught sin 2 beta n is equal to 0 summation n is equal to 1 through n naught cosine 2 beta n is equal to minus 1, but this is a result that we are interested in because this shows up in our expressions that we have derived earlier. So, this cosine 2 beta n is present in those 2 terms.

So, basically what this says is that mode Z r square of T if I choose my alpha and betas in the following fashion can be shown to be n naught minus 1 plus 1 that basically will be equal to n naught mod Z i square of T will be equal to n naught plus 1. So, if I choose scale 2 to be equal to 1 over root n naught and scale 1 to be equal to 1 over root of n naught plus 1 go back to the Jake’s model scale 1 and scale 2 have been appropriately chosen then we can put a tick mark that you will satisfy because the time averages satisfy the value equal to one-half I made a small mistake you should make it 1 by 2.

So, normally the variance would have come out to be n naught if I do a scale factor of 1 by root n naught that will make it unit variance, but I want variance of 1 by root 2 1 by 2 n naught square root and this also will be square root so that that will give me the appropriate variance that I need and will satisfy the second property.

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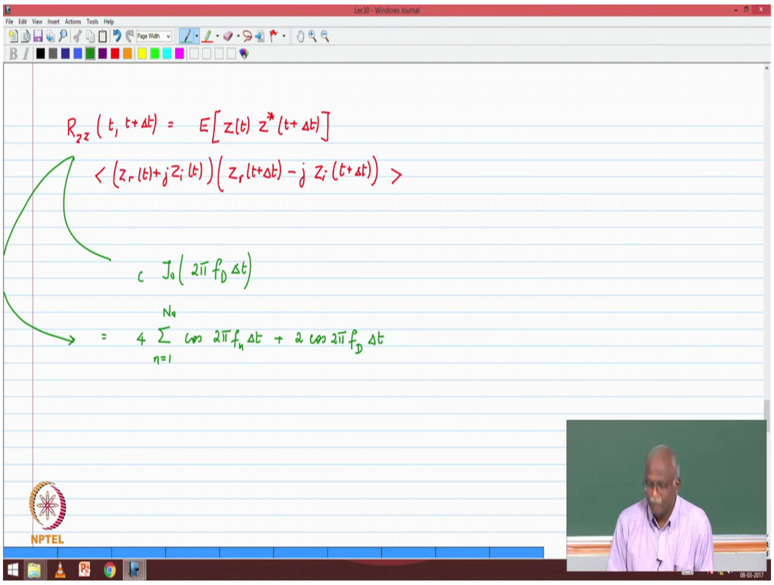

Now, the most crucial in our in our entire discussion is going to come in terms of our verification of the third property which is the probably the most non trivial of that. So, if I take the auto correlation of z with itself at T and T plus delta T the reason I am writing this is I do not have a guarantee that the way form that I have generated is wide sense stationary, I cannot straight away write it as delta t. So, I am going to measure the correlation between at time T and T plus delta T this is going to be expected value of z of T z star of T plus delta T I am going to approximate this with the time average of Z r of T plus j times Z i of T that is the first term second term Z r of T plus delta T because of the conjugate sign it becomes minus j Z i of T plus delta T time average of that.

Now, you have the expression for Z r you have the expression for Z i. Please plug in the terms and use the property that we have used before basically if you get a cosine of 2 different frequencies it will become sum of 2 the, it will become a sum of 2 cosines with the sum in different frequency time averages will send them to 0. What will be left are those terms which are the same frequency present we need to simplify those and then come up to the final verification to see whether this gives us some constant times j 0 2 pi f D times delta T it is an absolutely brilliant model as you derive it you will start to see you said my goodness this guy must have been a genius to figure this out.

But it is one of those things where only if you go through that that is why I felt it was important that we should take the time to derive it. So, I would leave it at this point request you to please try out and will give you sufficient hints I will assume that you have been able to get the expression for R z z the final step of showing that it is equal to the Bessel function will be our first task for tomorrow. So, please do make an attempt to get then expression for this and at least once you have; once you come up to that point we can then we can show that it is very easy; it is easy for us to develop it in into the into the final step.

So, yeah just; so, that you can verify R z z of T (Refer Time: 35:30) T comma T plus delta T will come out to be the following expression 4 times summation n is equal to 1 through n naught cosine 2 pi f n delta T plus 2 times cosine 2 pi f D times delta T its actually a very compact expression it is very close to you start to see the relationship to the to the to the Bessel function, but the actual proof of showing that this is an approximation to the Bessel. Bessel function will be our last step, but please do show that R z z of T actually is obtained if in case you are having some difficulty no problem. We will just give; I will give you a few hints at the start of tomorrows lecture and then go on to the final discussion. I will have to end the lecture today; I will pick it from here in tomorrow’s class.

Thank you.