

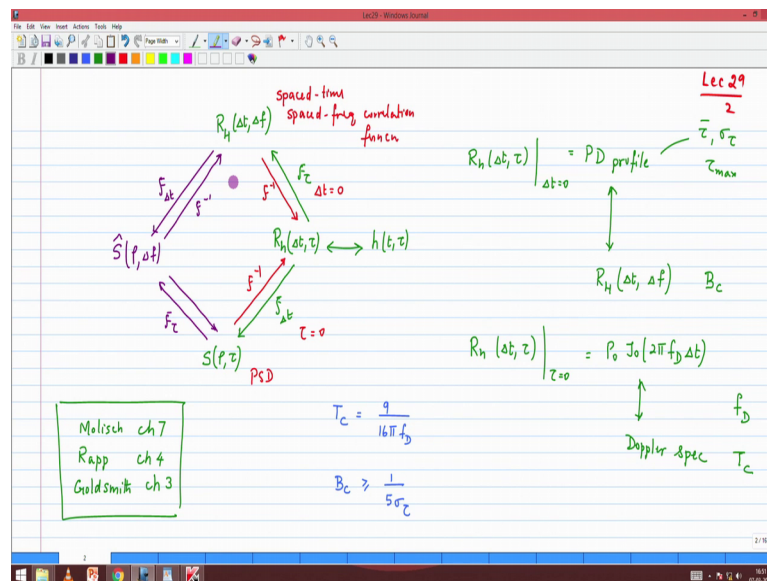
Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 30

Practical Channel Models (ITU, COST), Computer generation of Rayleigh Fading

We begin lecture 29, today we will bring to a closure or conclusion our discussion on the wide sense stationary uncorrelated scattering models, I hope you had a chance to sort of put the pieces together and see how each of them adds value. The classification of channels was something that we had discussed in the last class will briefly review that, but before that this picture is something that you will find in lot of the classical books (Refer Time: 00:43) on Heikan, in Molisch, Goldsmith all of them will give you this picture because this is the starting point for us to completely capture the wide sense stationary uncorrelated scattering model.

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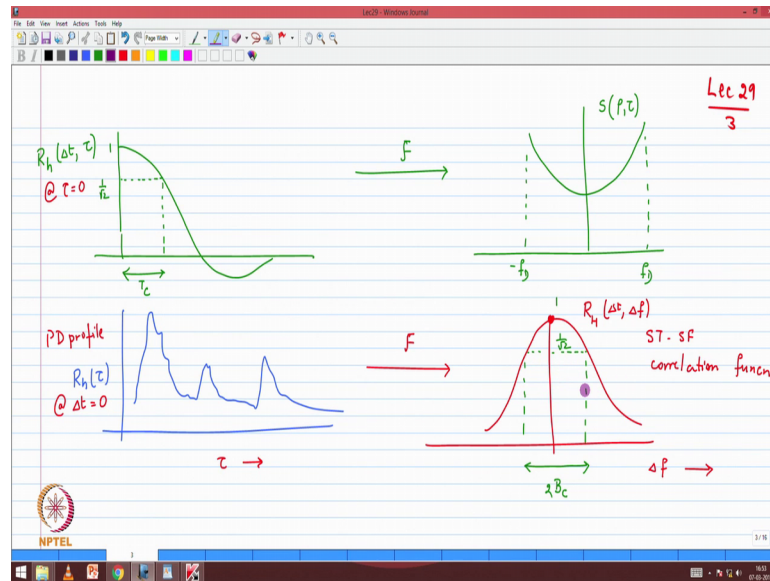


So, starting with the time varying impulse response you compute the autocorrelation function and then its 2 transforms and then the interpretations that that come from there. These are the reference points I would like you to look at and make sure that you are comfortable with that.

There 2 parameters that are very useful for us is the one is coherence time; other one is the coherence bandwidth coherence time tells you how frequently you should estimate

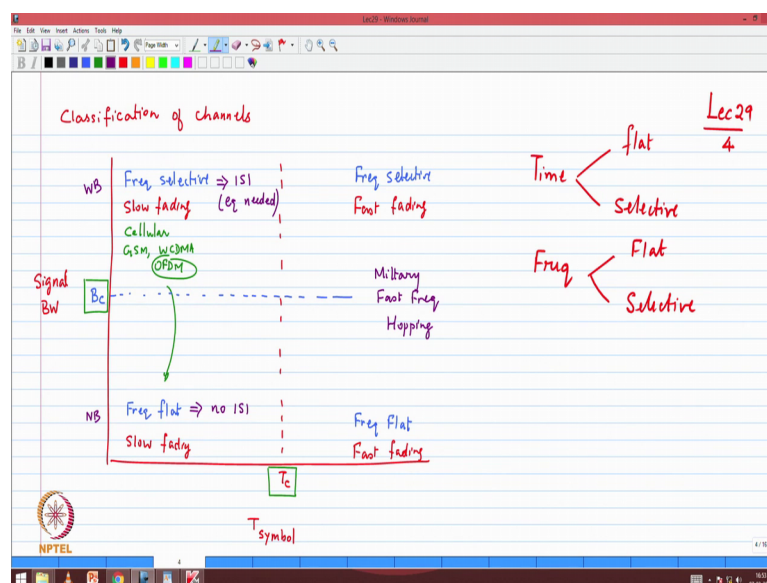
the channel or for the what duration of time you can assume that the channel is more or less constant and coherence bandwidth is the bandwidth over which the frequency response of the channel is correlated. And if you want to do frequency hopping you want to make sure that your frequency hops are greater than coherence bandwidth and your time channel estimates are done at a time period which is less than coherence time.

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So, basically you make sure that your estimating the channel as it changes the a pictorial representation helps us also greatly this is the time domain autocorrelation function we showed that it is a stationary function its Fourier transform is the Doppler spectrum bounded by plus minus maximum Doppler frequency the same autocorrelation function. If you said delta T to be 0 you find that we now have a power delay profile which can be characterize which can tell you the behavior of the time domain or the depressiveness; dispersive behavior of the channel which then leads us to a transform which tells us what the coherence bandwidth is.

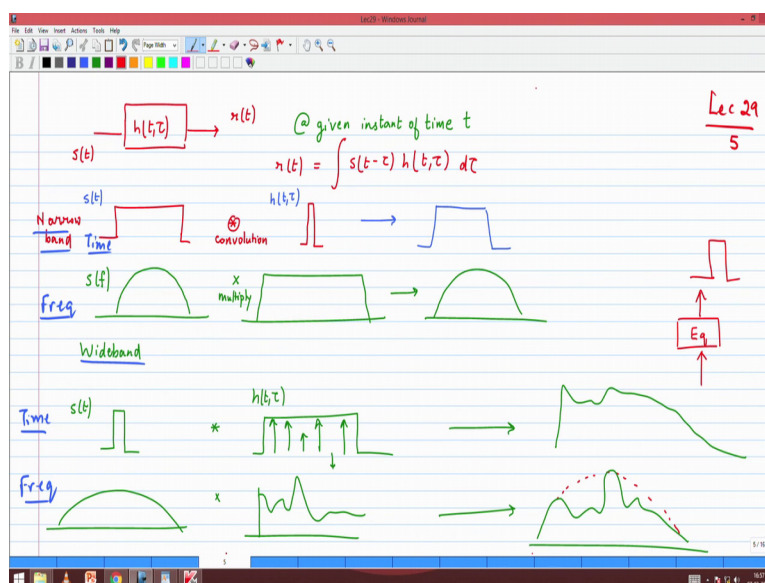
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Using these two parameters, the coherence time and coherence bandwidth, we did a four-quadrant classification of channels. We try to define when is a channel considered wide band, when is it considered narrow band. Again, a lot of it depends on the coherence bandwidth, and the coherence bandwidth in turn depends on the depressiveness of the channel.

So, again, the same signal, a GSM signal, can be treated as may seem to be a wide band signal for in under some channel conditions, may seem like a narrow band signal under some others. T_s , the symbol duration with respect to the coherence time, tells us whether your channel is slow fading or fast fading, and based on this characterization, we find that most of our systems of interest are on the west side of this partition, and the systems most likely to be encountered are in the north-west quadrant, which says that we are looking at frequency selective, but slow fading channels. So, equalizers are needed, but it is such that these are channels which we can do channel tracking and then build an efficient coherent receiver.

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So, the multiplicative fading you have to do convolution in frequency whereas, the depressiveness of the channel results in a convolution operation in the time domain which means that it is a multiplication and the frequency domain. So, if it is a narrow band signal the depressiveness of the channel is much less than the duration of a symbol slight distortion of the of the transmitted symbol and therefore, the spectrum more or less is transmitted without much change.

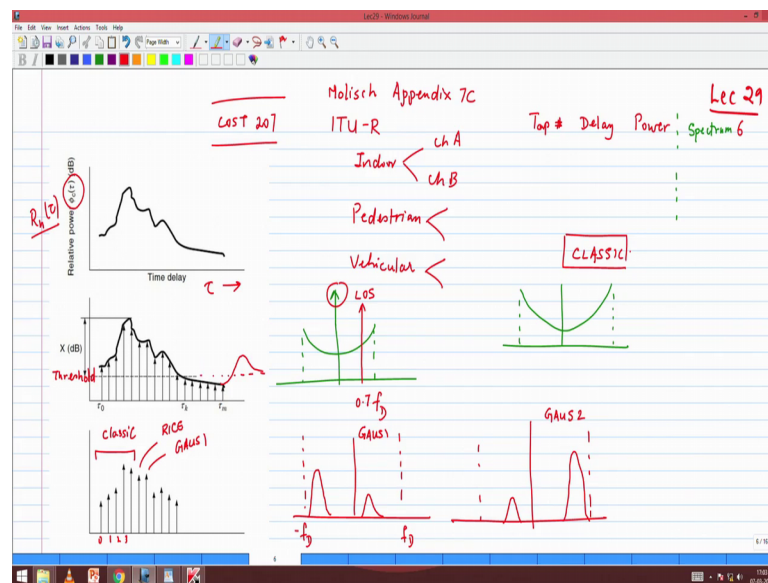
So, these are the flat fading type of scenarios where it is easy for us to build the receivers on the other hand wide band signals the channel depressiveness is much longer than the duration of a symbol which means that the symbol gets extended and distorted and successive symbols will overlap causing you inter symbol interference. This can be represented in the frequency domain as a spectrum of the input signal plus a frequency selective behavior of the channel which distorts the frequency spectrum. So, quick question; what is the job of an equalizer.

Student: (Refer Time: 05:05).

What is the job of an equalizer? So, equalizer has to basically restore this. So, half let me use a different color. So, if I pass this signal this convolved signal through an equalizer. So, through an equalizer if I pass it through an equalizer I want to get back I want to get back something that looks like this frequency domain I must do the inverse. So, that I get back my received spectrum.

So, the job of an equalizer is to restore the or undo the distortion introduced by the by the channel either in the time domain or in the frequency domain this is how we study in digital communications the only difference in a wireless channel is that this distortion keeps changing as the function of time. So, your equalizer also has to keep changing because it has adapted to the distortion and undo the distortion at every stage. So, that is a very good summary of the different channels that that we have encountered so far.

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So, from a practical stand point I am sure that a when you get a chance you will read Molisch chapters appendix 7 C Molisch appendix 7 C and it has got 2 sets of channel models it has got the ITU-R which gives us several channels indoor there is a channel a and then there is a channel b and then you have pedestrian; pedestrian you have channel A B and you have vehicular channel A B and actually there are several variance of this. But the key things that you are looking for is the tap number that is column number one tap number second is the delay of the tap number third is the power that is present power delay profile will that will tell you how much of that and the fourth one which I want you to think about is the spectrum Doppler spectrum.

So, for the only Doppler spectrum that we have studied is the Doppler spectrum corresponding to a Rayleigh. So, basically this is the Doppler spectrum that we have studied. So, today we will we will see how this is going to play a part in it, but the different channel models that are present sometimes specify a spectrum that you should

use for that particular tap. So, for example, tap 0 may be specified as a Ricean; Ricean distribution Ricean distribution is this correct Ricean distribution is that correct this is this is the line of sight component this is the line of sight component it is like a constant that is always present and this is the Doppler spectrum for the Rayleigh component.

So, basically we need to we need to keep in mind that that we are able to interpret the spectra that is given to us Ricean component it can be at 0 it can be at DC it can also be at some specified Doppler frequency Doppler frequency as well. So, you can see sometimes they will specify that the Ricean is at 0.7 of f_D max or f_D . So, some of those you can see are given to you

Now, there is a spectrum called the Gauss spectrum this is by the way this is called the classic spectrum that is the also known as the Jake's spectrum there is a Gauss 1 which has the following behavior this is minus f_D max or minus f_D this is f_D it has this type of a response can you interpret this for me what is this power spectrum telling you there is a strong component near minus f_D and A_{naught} . So, medium strength component, but it is not all frequencies are not present there are some dominant paths that are present. So, these are slightly different environments from that. So, the this is called Gauss 1 Gauss 1 there is a Gauss 2 which is the complement of that its got a small component in the negative side strong component on the positive side again it is the complement of that.

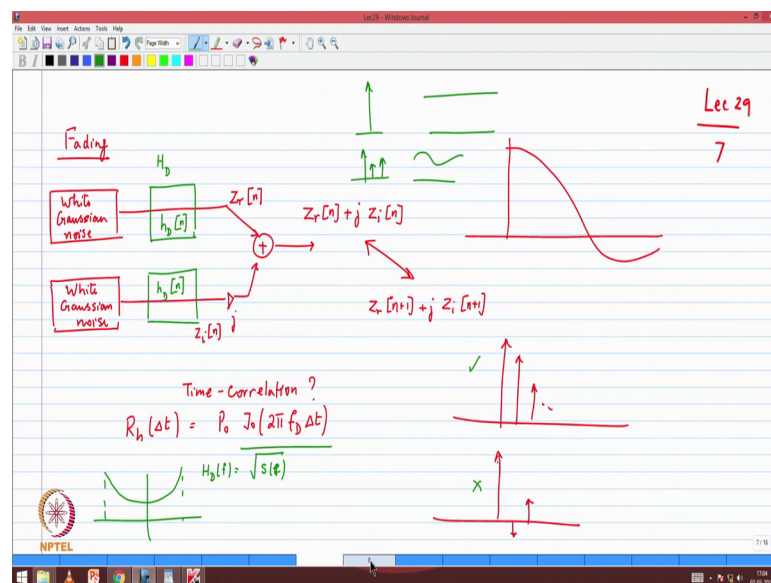
So, some of the taps may have classic some of them may have line of sight some of them may have Gauss 1 some of them may have Gauss 2 you have to implement them appropriately, but how do how does how does the underlying process work we basically this is the power spectrum R_h of τ ignore this notation we have been using R_h of τ versus τ you have a power delay profile that is present you set a threshold this is the threshold anything below this we are not interested.

So, if the channel goes below the threshold and remains below the threshold then that is the point at which we declare that the channel response has ended, but as long as there is another portion of the of the power delay profile supposing this where to come up again sometimes it can happen, which means that you you have to take into account these this portion as well. So, anything that is above the threshold you have to take into

account you sample it based on your time resolution and then you can then call this as tap number 0 1 2 3 and. So, on that is how the channels are presented.

So, sometimes they will say that the first four taps are classic; that means, it corresponds to the Jake's spectrum then they can call this as a Ricean then this as Gauss 1. So, so that is how ITU models are specified the cost 207. So, please do read and make sure that you are comfortable with the explanations given and what it means. So, basically they have taken a power delay profile you have sampled it you have gotten the distribution of the power if you did not specify anything then your interpretation is that it has to be the classic spectrum default is classic; that means, no line of sight there is Rayleigh statistics on each of those taps and the fading is independent is usually the assumption that is made, but if they specify one of the other modes or the spectra then we have to appropriately account for that in our simulation any questions.

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So, that pretty much summarizes the complete landscape of wide sense stationary uncorrelated scattering models how to interpret them how to characterize them and how to implement them, but now we will get into the computers implementation of that, but this frame work is very important for us to do the implantation any questions . So, then we go back to asking the question that where we needed in the last part of the last lecture. So, if I were to want to generate a fading channel Z_r of n plus j times Z_i of n , I need to generate a sequence of independent complex Gaussian random variables. Now

that is what this process will generate for me, but I know that I must have the in the time domain there is a autocorrelation R_h of δT , I must get a some p not times j not $2\pi f$ $D \delta T$, I cannot get independent samples from instant to instant.

So, this is this is not allowed this is independent from time instant time instant what I need is this type of, but how do I force my Gaussian noise to have that time correlation. So, this is where signal processing helps us correlation in time corresponds to coloration in frequency because if I have an autocorrelation which is an impulse response; that means, the there is no correlation in the frequency domain it is flat. So, in if I have a correlation that has got some time domain behavior then I would see some coloration in my in the spectral domain.

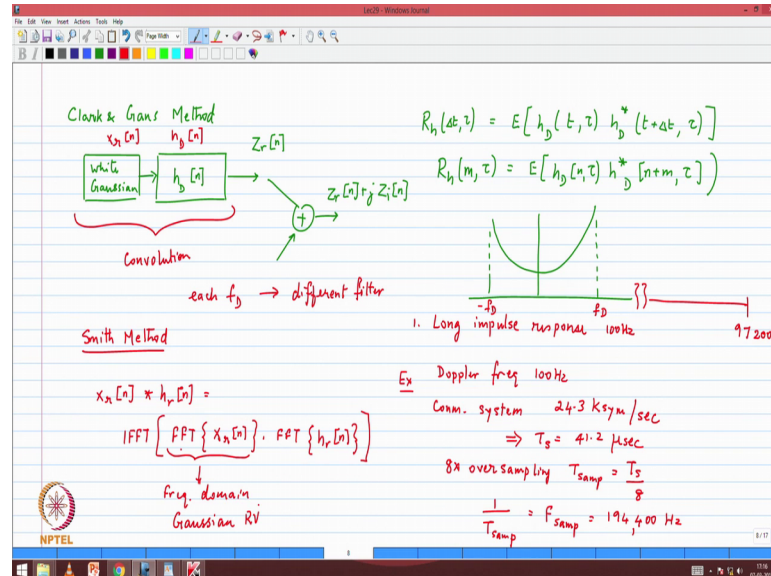
So, that is what we and this correlation in the time domain has to be a Bessel function. So, which says that either I can somehow try to include a Bessel correlation into my white noise generator an easier thing to be is to generate the white noise pass it through a filter which is got a spectrum that is that has this shape and your guaranteed that your output signal has the appropriate correlation because you have taken care of it in the frequency domain.

By the way what you need to take into account that the frequency response of this filter h Doppler of f should be square root of s of s of f right that is that the notation that we have used no s of ρ yeah. So, whatever is the; so basically the square root of the power spectrum is what you should get because when you take the output power spectrum it will be input power spectrum multiply by the power spectrum of the filter that you have used which will give you the Doppler spectrum s square of the power spectrum of the filter.

So, this is this is the way we would we would generate so; obviously, we need to introduce a filter here a filter here and these filters are the Doppler filters h , I will just write it as $h D$. So, we can write it as $h D$ of n if you think of it as a sampled sequence impulse response and the same filter has to be used here also because both the real and imaginary parts have to have the corresponding spectrum and then you combine them as real and imaginary which are independent of each other and together we will get the desired result desired behavior in our result. So, the process of generating the desired

response is as follows my method is to generate white noise pass it through a filter and then obtain the output.

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So, the first method that we will be we will be studying is called the Clark and Gans method Clark and Gans method again referred to Rapaport for a good discussion, but we will cover the key elements in our discussion today. So, basically what we are going to do is take white Gaussian noise white Gaussian noise white Gaussian random variable and white basically means its uncorrelated from time instant to time instant pass it through a appropriate filter h Doppler of n and this would be the real part and correspondingly obtain the imaginary part and we get Z r of n plus j times j, i of n that is that is the Clark and Gan's model.

Now, I want you to think of the following aspects of the of the filter by the way this will guarantee for us that the following property will hold R h of delta T comma tau is such that the whatever is generated should satisfy expected value of h D of delta T comma tau h D star this is T comma tau this is T plus delta T comma tau this has to has to be the Bessel function that we are interested in.

Now, the continuous time is the basis, but actually what we implement will be a discretized version of it. So, do not get confused why I am writing n here this is the discrete implementation the underlying process is the is the continuous time continues time process you can also write it in the following form just for clarity if this is m

comma tau this can be written as $h D \text{ of } n \text{ tau } h \text{ star } D n \text{ plus } n \text{ comma tau}$. So, basically there is a discrete equivalent of it, but you generate think of it as a continuous time process which is filtered and then we sample it then the all the properties that we are interested in are available for us in a clear fashion at first observation that I want you to make is can you tell me something about this filter what can you tell me about the impulse response of this filter.

Student: Real.

Real that is correct very good anything else it is a very Sharpe filter look at look at how steep it falls basically. So, when you have very Sharpe filters what is your impulse response behavior.

Student: (Refer Time: 19:27).

It will be very long. So, which means that in order to generate a reasonable sampling or representation of the signal you are going to have to filter a lot of data before you can get a usable segment of information. So, the first observation is that the long impulse response. So, please write it down with the correct interpretation and explanations I am going to just write down non impulsive response which means that you are going to have the filtering process is going to be cumbersome and therefore, long impulsive response is a problem for us and the reason it is a problem for us is because this will involve this process is a convolution in time and convolution will require a lot of data and because the impulse response is very long you would appreciate the fact that; that means, there are there is a transient that is present with a output process. So, you have to wait for the usable portion of the signal. So, that is again a challenge another observation that is very important for us this is minus $f D$ to $f D$.

Now, I want you to get a feel for what type of a filter this is basically it looks like a low pass filter little bit of a non uniform filter, but it is a passes the low frequencies, but what is very important is that what is the bandwidth of the filter compared to your sampling frequency. So, let us take an example because that is usually how we can appreciate the order of magnitude let us say that the Doppler frequency is 100 hertz Doppler frequency is 100 hertz. Now you will take system communication system which has the following symbol rate 24.3 kilo symbols per second its happens to be 1 of the 2 g systems 24.3 kilo symbols per second. So, which leads to a symbol duration T_{symbol} of 41.2

microseconds and as you are familiar with from the computer implementation most of the time our receivers will do 8 X over sampling. So, my sampling period T_{sampling} is T_{symbol} divided by 8 because that is my I am (Refer Time: 22:02) 8 samples per symbol and $1/T_{\text{sampling}}$ frequency is f_{sampling} that is my sampling frequency that corresponds to 194400 hertz; so, 194 kilohertz.

So; that means, if I were to show it on a digital scale π would be one-half of the sampling frequency is quite far away this is 100 hertz the one-half of the sampling frequency is 97200, did I get that right yeah. So, basically this is the very very narrow filter very narrow filter and it has got its own set of challenges as we will see in the moment, but the important point to know is this is how Clark and Gans proposed and this was how it was implemented and used for quite a good period of time.

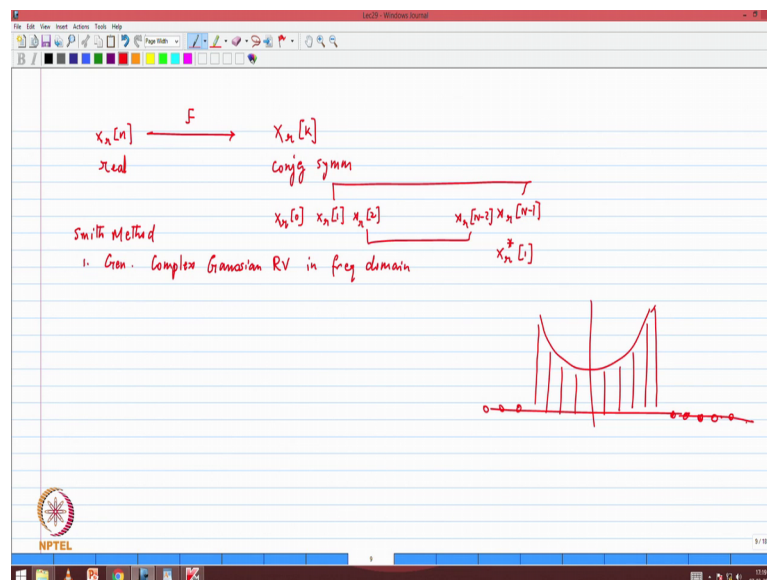
It was used before another method was proposed that was called the smith method the smith method was a way of saying I think this is very complicated because first of all you have to generate a long impulse response and you have to filter a lot of data and also keep in mind that for each f_D each f_D it corresponds to a different filter write it is you have to basically generate depending on what Doppler you want to different filter what I mean by different filter is the different impulse response. So, it is not a fixed or standard you cannot precompute all of the Doppler frequencies that you want because your Doppler frequency changes based on your environment. So, there are some limitations of this filter.

So, smith method basically said this is the best way to do it we do not know of any other way of doing it can we do it more efficiently. So, basically the smith method came along and said here is what we are trying to do. So, if this is we call this as X_r of n this as h_D of n what we are trying to do is X_r of n convolved with h_R of n that is what we are interested in. So, this can be written as the inverse FFT of FFT of X_r of n multiplied by FFT of h_R of n I think this is what we already discussed before the start of the lecture convolution in time can be implemented using the following frequency domain operation.

Now, this is where the other question leads to now what are X_r of n there are a sequence of uncorrelated Gaussian random variables. So, if I take a linear transformation the FFT of that then what comes out is another set of Gaussian random variables. So, the

important point to note is that I do not need to do this I can generate this directly in the frequency domain frequency domain Gaussian random variables can be generated; however, we need to keep in track one important element Gaussian random variables can be done and that important observation is as follows.

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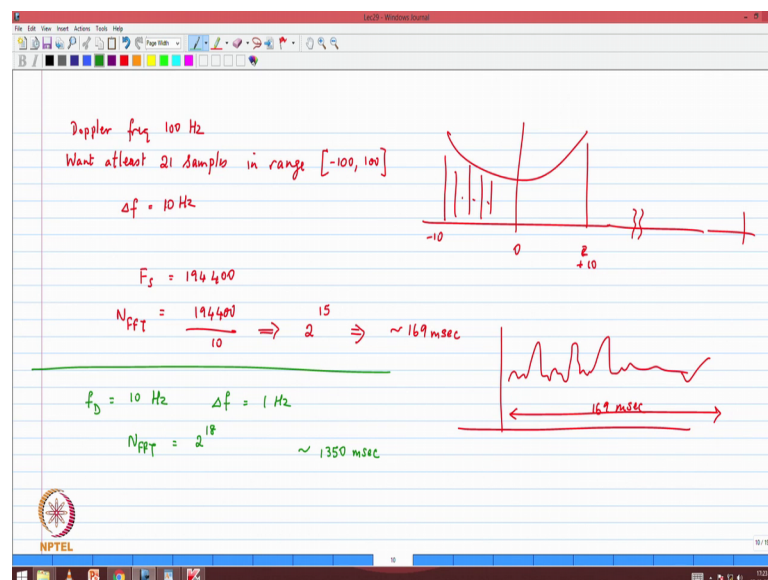
So, because x_n of n is real the Fourier transform is going to be a conjugate symmetric X_n of k basically will have conjugate symmetry conjugate symmetry. So, now, instead of doing x_n of n and then doing taking the Fourier transform we are going to generate Gaussian random variables already in the frequency domain and the way we would do that is as follows. So, you would generate x_n of 0 then you would generate x_n of 1 and x_n of n minus 1 they are not independent of each other these 2 are complex conjugates this is nothing, but x_n conjugate of one x_n or x_n of 0 is a real valued random variable x_n of 1 is a complex number its conjugate must be at the last.

Similarly, x_n of 2 and x_n of n minus 2 are also related you 2. So, basically you must ensure conjugate symmetries presence. So, that when you do the inverse Fourier transform you will get a real sequence and, but apart from this is this is very simple you just generate directly the Gaussian complex Gaussian. So, the smith method smith method says generate complex Gaussian random variables complex Gaussian random variables in the frequency domain in frequency domain what should you do for the

filtering process very straight forward sample it you get this as your sequence multiply the FFT of 1.

And there are lots of 0s remember because this is a there are lots of 0s present in the system in your Fourier transform of your filter. So, lots of 0s they will multiply they will remove out and then take the inverse Fourier transform and then get the X r the output we are interested in. So, I hope you are familiar enough with the how to do linear convolution using FFTs and the smith method basically said we will do the convolution in the frequency domain with the added observation that the complex Gaussian random variables can be generated directly in the frequency domain and you do not have to do the transformations.

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Now, the important aspect that we observe what is the limitations here come here is where the problem will come and I hope you will be able to appreciate this aspect of it. So, go back to our example Doppler frequency 100 hertz Doppler frequency 100 hertz, now I let us say that we want to have a certain resolution of our; of the FFT process. So, let us say we assume that we want this condition we want at least 21 samples 21 samples in the range minus100 to100 in the range minus 100 hertz to 100 hertz; that means, your number of samples number of samples. So, this is 0 up to 10 on either side minus 10 to plus 10. So, 21 samples are present at least why because only then I will get some

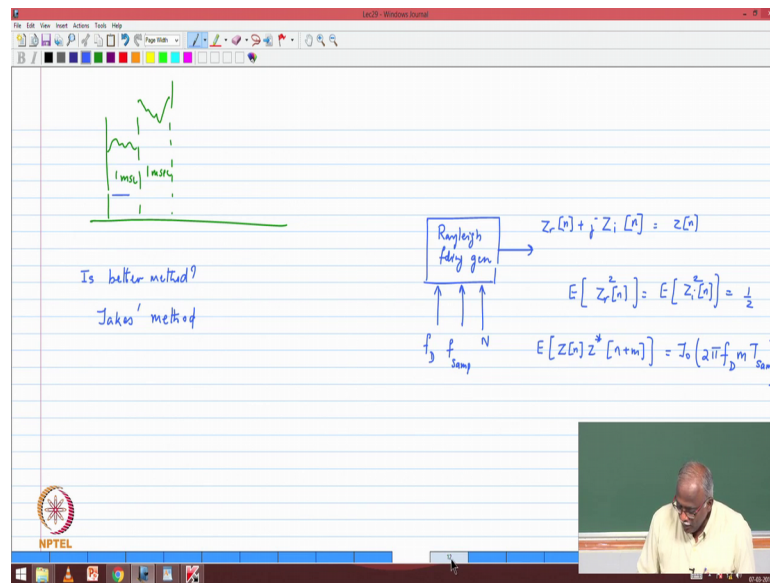
resolution of my because all the shaping is being done by these samples and keep in mind that you know there is a lot of 0s that are present in your filter Fourier transform.

So, I want to do this. So, that I can get good capture the Doppler's spectrum in an accurate manner. So, this tells me that my Δf ; that means, the spacing between my FFT samples FFT in the FFT coefficients has to be 10 hertz correct basically and only if I have a spacing of 10 hertz I will get this type of resolution now what is the size of the FFT that is very important. Now the size of the FFT depends on your sampling frequency and we already said that the sampling frequency is 194; 194 kilohertz 194400. So, the size of the FFT size of the FFT is 194400 divided by 10 which implies you want to take a power of 2 it would have to be 2 to the power of 15 would be the basically that would be the size of the FFT no problem this corresponding to at this sampling rate it corresponds to approximately 169 milliseconds.

So, one set of data that you will generate corresponds to the fading pattern at 100 kilohertz of 1s of 169 milliseconds no problem this is this is the smith method and we said we are done it very efficiently and therefore, we are in good shape. Now supposing I change the problem on you and I say that the maximum Doppler is 10 hertz what is your Δf Δf becomes 1 hertz. So, what is your FFT size, it is now 10 times basically it is 10 times larger. So, the size of the FFT now will become 2 power 18 that is one that will satisfy the number and this will generate for you 1350 milliseconds of data.

Now, you may still not see that there is a problem or you know may still wonder you know why are you why is it a problem I in one case I generated 169 milliseconds another case same set of data I am still interested, but I just change the Doppler frequency you told me size of the FFT has to change. Now at the sequence is changed now the problem comes for from the following reason because if I tell you that my environment is such that I am hopping every one millisecond.

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So, what am I expecting from you a fading pattern that last 1 millisecond and then the next millisecond because of hoping it is a different channel altogether the next 1 millisecond I am telling you I want to simulate a frequency hop system what will you tell me sorry you know I will give you 1350 milliseconds of this thing you throw away 1349 milliseconds and keep 1 millisecond.

So, you see that your basically there is a lot of wastage in our competition that we are doing. So, the question that we ask is yes many times we want to have a fading that is of this type right 1 millisecond duration correlated fading then I do a hop therefore, another 1 millisecond of uncorrelated correlated fading. So, like that you want to generate the question that we ask is there a better method if I want 1 millisecond I generate 1 millisecond not 1000 milliseconds and throw away 999 and 99 milliseconds. So, is there better method and that method was actually discovered and proposed by William Jake's and therefore, has been known as the Jake's method which is the heart of what we will be doing in today's lecture.

So, ah, but basically this is what we are we are trying to do we are trying to come up with a box which we will call as a Rayleigh fading generator Rayleigh fading generator what all do I need to give inputs to this one I need to give the Doppler frequency I need to give the sampling frequency. So, what is the spacing between my samples of the fading that I am asking for and the third one is what is the length how many samples that

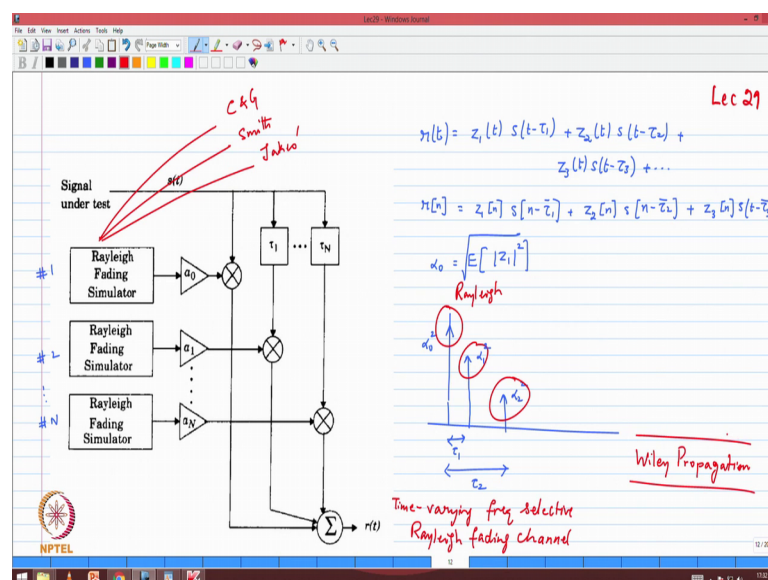
do I want 100; I want 1000 or 10000 I need to specify and the output should be Z_r of n plus j times Z_i of n which has the correct autocorrelation function that that we are interested in.

So, the properties that this one should satisfy expected value of Z_r of n whole square is equal to expected value of Z_i of n whole square should be equal to one-half write. So, that then they together they will give you unit variance as far as the channel response is concerned the Doppler; the Doppler's properties expected we call this as Z of n expected value of Z of n Z^* of n plus m Z^* of n plus m this should satisfy the following property it should behave like the Bessel function $2\pi f D$ and what else what else Δt , but now it is.

Student: m.

M that is all times T , Sam do not forget that basically m is just an index, but in order to show the extent of the correlation it has to be a time variable and therefore, the sampling m samples is nothing, but m times T samples in seconds now these property should be satisfied and we should get a box and basically Rayleigh Jake's method is one that tells us how to get such a box and how to how to generate that.

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Now, assuming that you have been able to generate this what is the next step the step is now basically we are trying to simulate a channel we say that I want to simulate a

channel where the following is the response $Z_1(t)$ of T s of $T - \tau_1$ $Z_2(t)$ of T s of $T - \tau_2$ plus $Z_3(t)$ of T s of $T - \tau_3$ dot dot dot this is the continuous time version you can write down the discrete time version as $Z_1(n)$ s of $n - \tau_1$ we have we have to look at the actual description of the delays plus $Z_2(n)$ s of $n - \tau_2$ let me just put it as bar basically you have to quantize it to the appropriately $Z_3(n)$ s of $T - \tau_3$ bar and goes on.

But how do we actually implement this in the. So, you what you would do is call multiple times the Rayleigh fading simulator one two; however, many you have this is the n th multipath component each of them is generating an independent Rayleigh fading according to the statistics that are have been specified now each of them has got a different power level. So, this multiplicative factor is alpha not is expected value of mod Z_1 square. So, basically no it is square root. So, it is an amplitude scaling. So, when I take the power level, it will scale to get the corresponding. So, this is a unit variance random process generator multiplied by a scaling to get the appropriate power levels and so on and so forth you get the corresponding statistics.

So, and you take the input signal multiply it with the with the corresponding gain for the first term second coefficient has a delay τ_1 you multiply it and then you add all of them to get the received signal. So, this particular model has is implementing this the following process it is got a tap at 0 delay which has got a power level of alpha not square this is in the power delay profile and then there is a second tap which is at a delay of τ_1 which is got alpha 1 square as the average power then there is a third multipath component which is at a delay of τ_2 which has got alpha 2 square as the. So, this is this is what this figure is implementing each of these are Rayleigh this is Rayleigh has got Rayleigh statistics this is got Rayleigh statistics this is got Rayleigh statistics and. So, you have. So, all of your channel impulse response is each of those coefficients has got a of Rayleigh statistics.

Now, the question that. So, once you have a Rayleigh fading generator you can call multiple instances of them to generate independent or in different Rayleigh random variables and then you multiplied with the appropriate delays scale it to get the appropriate power levels that you are interested in. So, basically this is the power levels that you are interested in these are the delays then you combine all of them. So, this is

what is at the end of the day what you have obtained is a time varying frequency selective Rayleigh fading channel Rayleigh fading channel.

So, this is our this is what we need in our simulation and that is what we have been able to obtain now the key question is Rayleigh fading simulator I know we can do Clark and Gans method 1, then there is the smith method then third one is the Jake's method and the Jake's method is that we are going to be looking at in the next lecture again it is a non intuitive model. So, the way we are going to do it is rather than try to motivate how Jake's came up with it we will present you the Jake's model and then show that it satisfies the time domain correlation it. So, it satisfies all of the properties needed for Rayleigh fading, but in order to know what those properties are definitely you would recommend that you would take a look at the propagation handout that has been given to you propagation Wiley propagation handout and that will tell you the model and also tells you what are the basic properties of the Jake's model so that you can then appreciate it when we discuss it in the class.

So, there is a handout which is been uploaded in model you can take a look at it and that talks about the Jake's model it is a non intuitive model, but it is a beautiful model because it satisfies all of the requirements of the Rayleigh fading statistics basically all of the properties that we are interested in and satisfies them in a very very nice manner very compact manner and today almost all researches in wireless who work with fading channels a particularly Rayleigh fading channels would be working with a Jake's model.

So, one of the things that we will do in the next computer assignment is actually have you programmed the Jake's model and makes sure that you are able to verify the statistics that are that are obtained through the Jake's model we will pick it up there in tomorrow's class.

Thank you very much.